MIDDLE SCHOOL STUDENTS' INFORMAL INFERENCES ABOUT FALLING RAINDROPS

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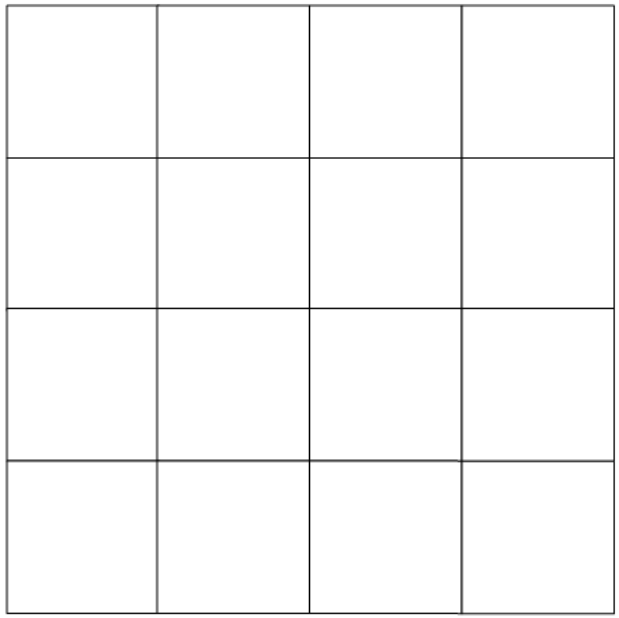
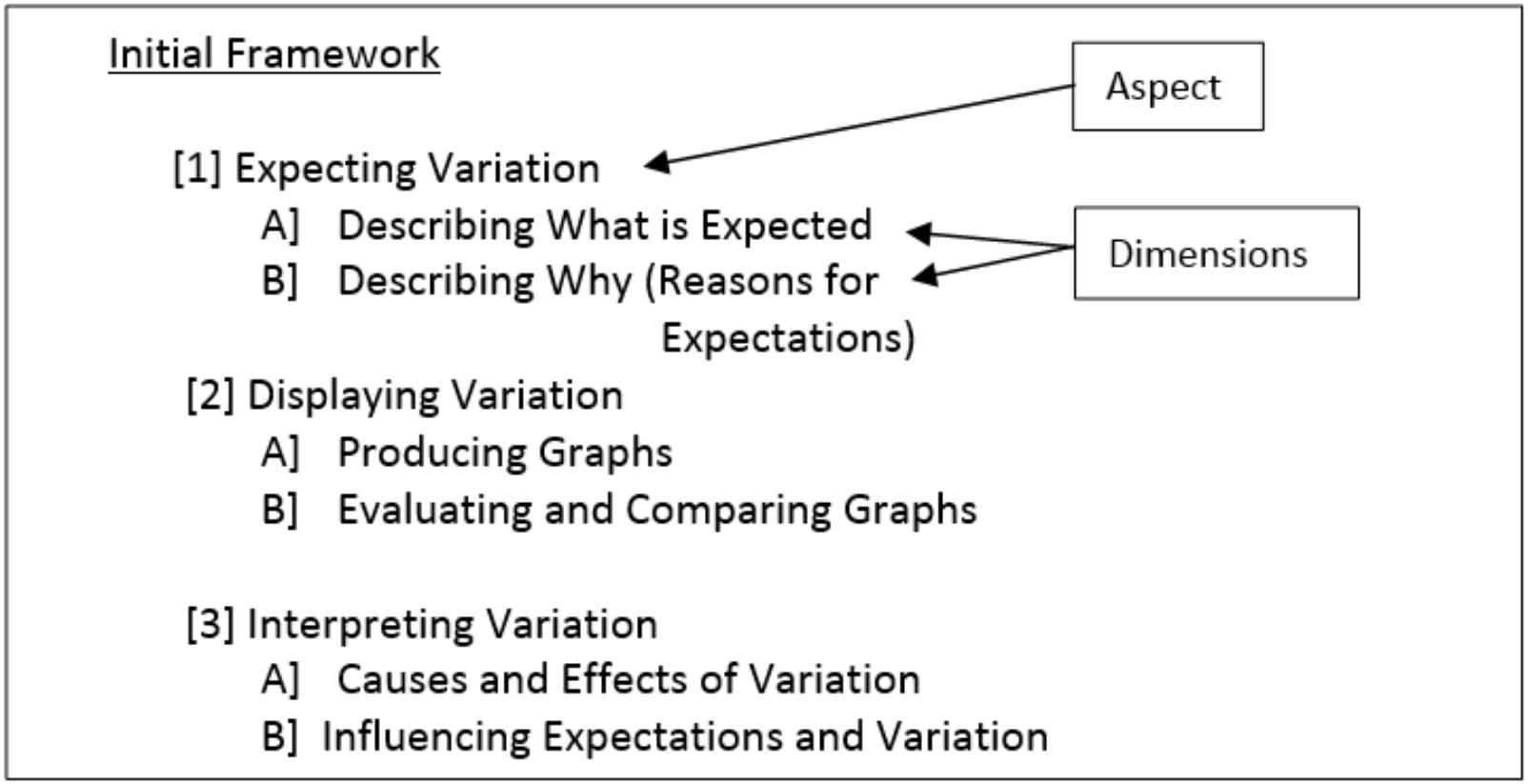
INITIAL TASK

There are sixteen equal-sized square tiles on the rooftop garden patio.

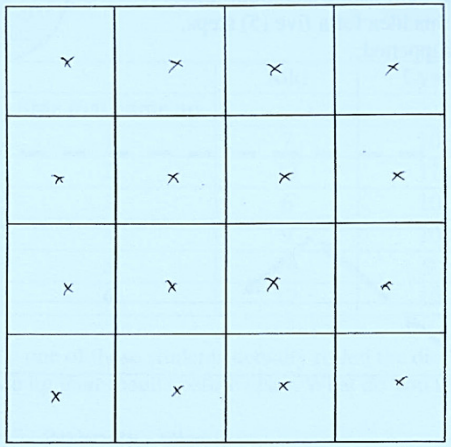
It starts to snow, and after while sixteen snowflakes have landed on the patio. Place an “x” to mark where you think the sixteen snowflakes might have landed.



Explain why you made the choices you did.


Initial Framework
Aspect
[1] Expecting Variation
A] Describing What is Expected
B] Describing Why (Reasons for Expectations)
Dimensions
[2] Displaying Variation
A] Producing Graphs
B] Evaluating and Comparing Graphs
[3] Interpreting Variation
A] Causes and Effects of Variation
B] Influencing Expectations and Variation

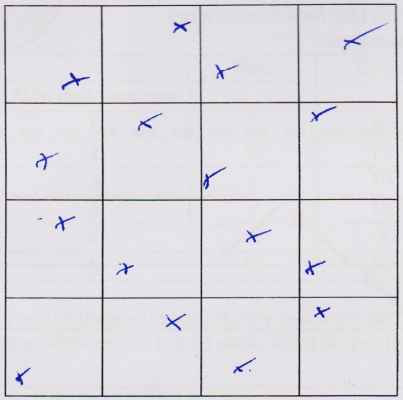
[D] There are sixteen equal-sized square tiles on the rooftop garden patio. It starts to rain, and after while sixteen raindrops have landed on the patio. Place an "x" to mark where you think the sixteen raindrops might have landed.



Explain why you made the choices you did.

Because they would be all spread out. Usually when it rains, raindrops don't all fall on the exact same ~~step~~ spot.

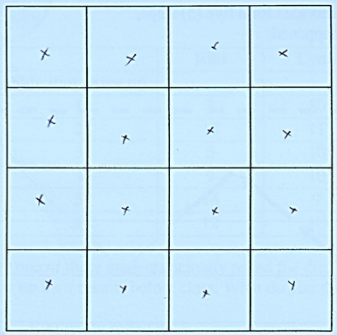
[D] There are sixteen equal-sized square tiles on the rooftop garden patio. It starts to rain, and after while sixteen raindrops have landed on the patio. Place an "x" to mark where you think the sixteen raindrops might have landed.



Explain why you made the choices you did.

the choices I made were random choices. I did put an x in each box, as it is most likely that one will fall in each box, however where in the box is random. I tried to not put it in any systematic order.

[D] There are sixteen equal-sized square tiles on the rooftop garden patio. It starts to rain, and after while sixteen raindrops have landed on the patio. Place an "x" to mark where you think the sixteen raindrops might have landed.

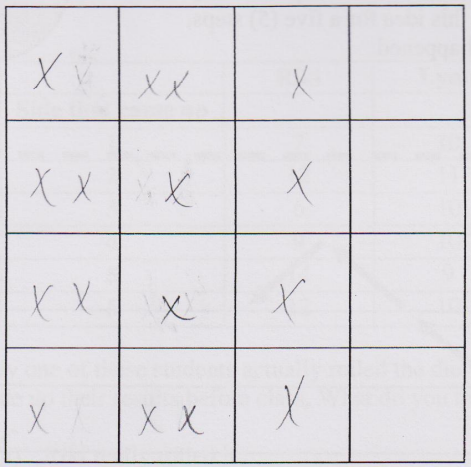


Explain why you made the choices you did.

If the rain was falling straight down (and the rain drops were evenly spaced) there is an equal chance that the rain drops are going to fall on each square.

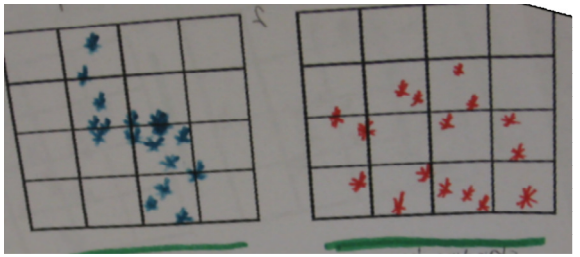
This wouldn't apply if the rain was falling at an angle or the drops weren't evenly spaced.

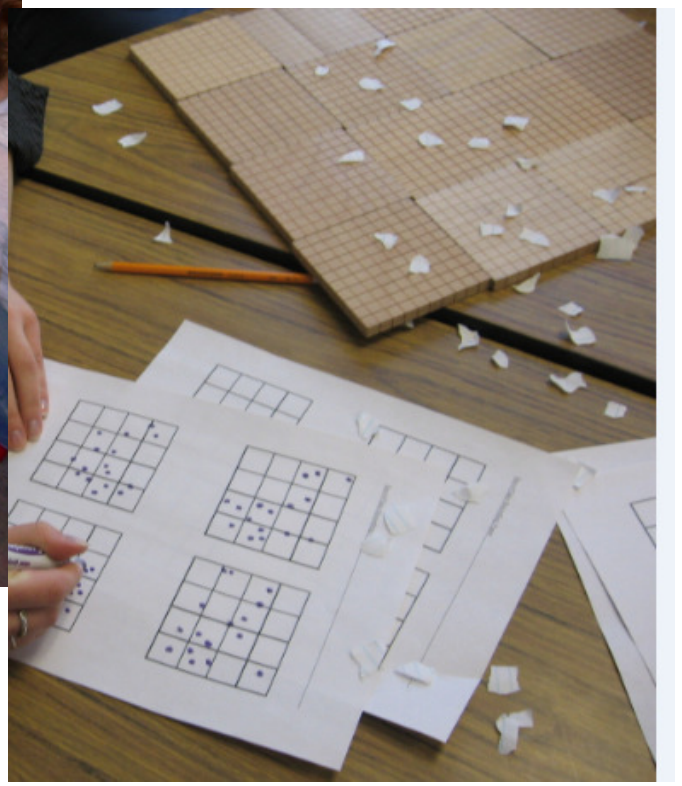
[D] There are sixteen equal-sized square tiles on the rooftop garden patio. It starts to rain, and after while sixteen raindrops have landed on the patio. Place an "x" to mark where you think the sixteen raindrops might have landed.

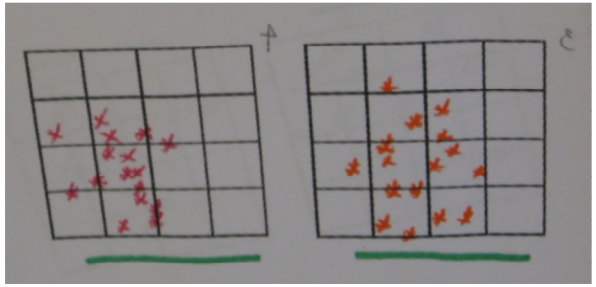


Explain why you made the choices you did.

I did this with my mind on where the rain is coming from (and what angle) and this is one of the factors, whereas theoretically there would be one rain drop on each square

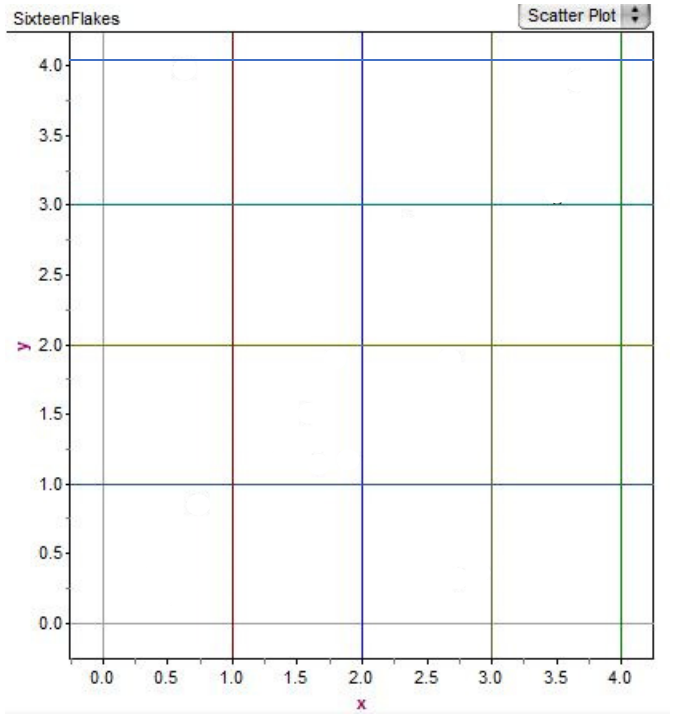


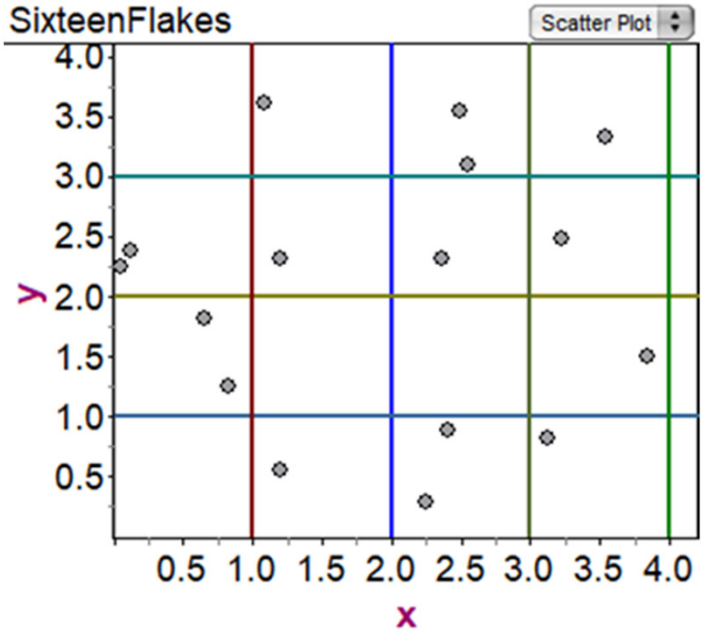




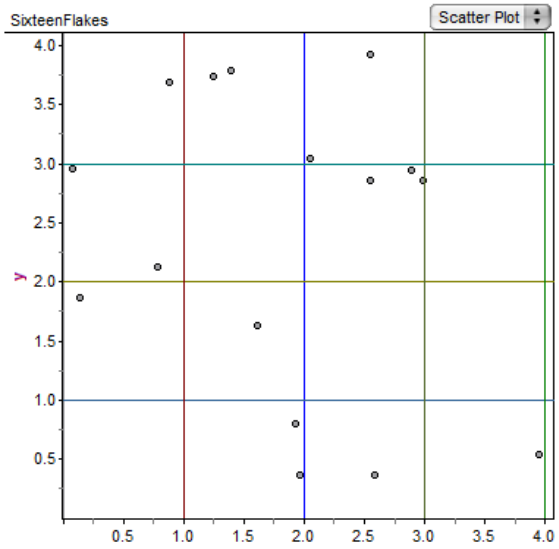
How do the actual results compare with our predictions?

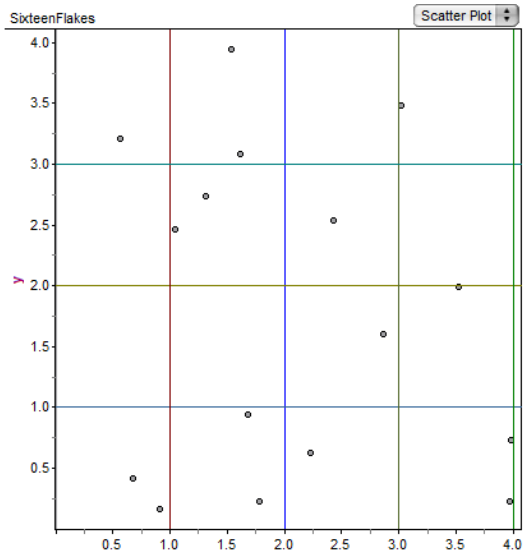
USING FATHOM TO GENERATE A TRIAL

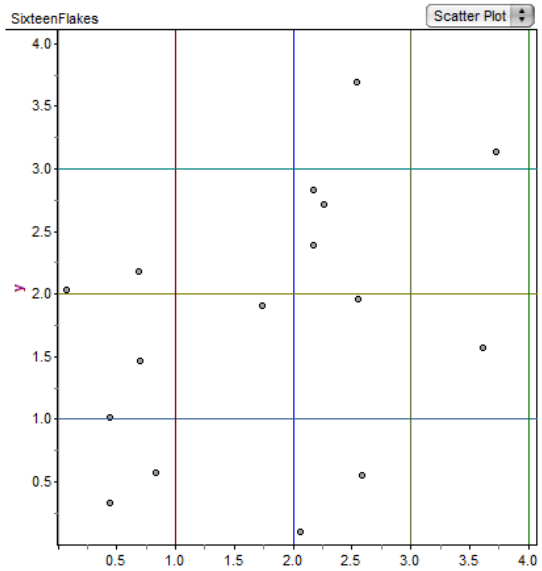


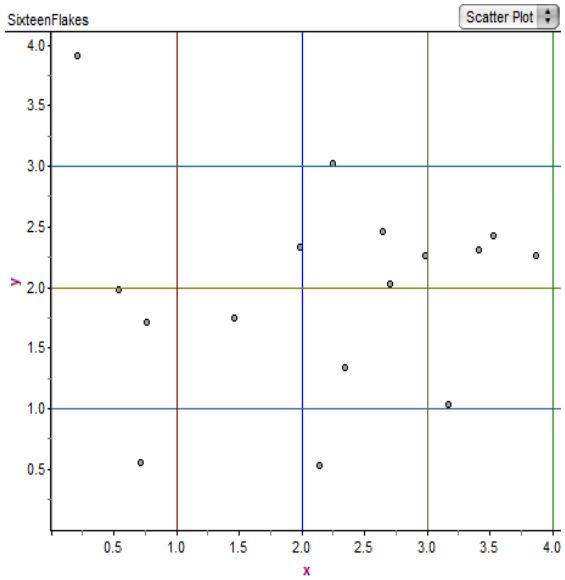


MANY TRIALS


SixteenFlakes
Scatter Plot
y
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0


SixteenFlakes
Scatter Plot
y
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0


SixteenFlakes
Scatter Plot
y
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

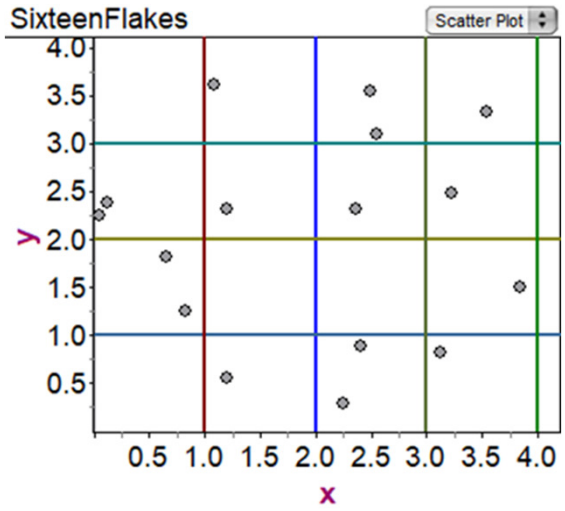

SixteenFlakes
Scatter Plot
y
x
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

SOME NEW QUESTIONS

1. How many empty squares do we expect to see?

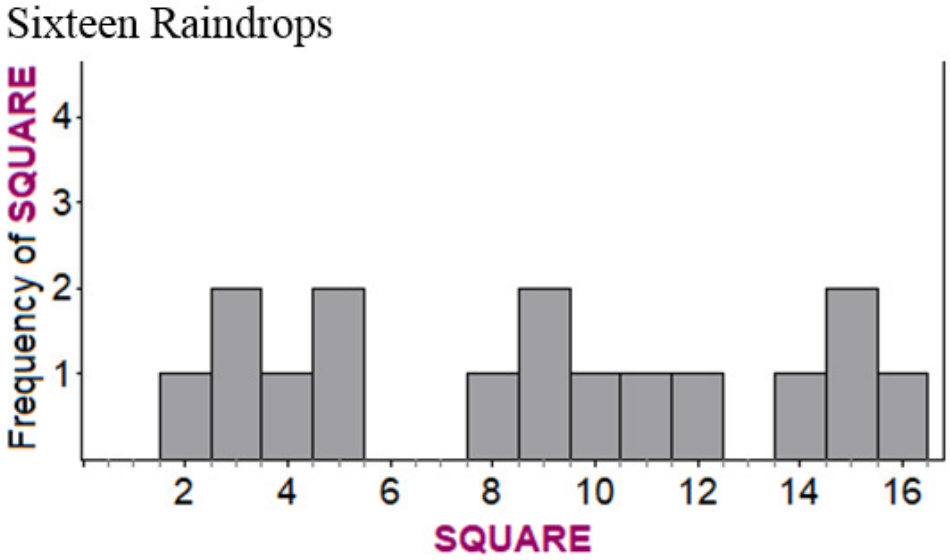
2. What's the most # of raindrops we expect to see on a tile?

3. Is there likely to be an entire empty row (or column) on the patio ?



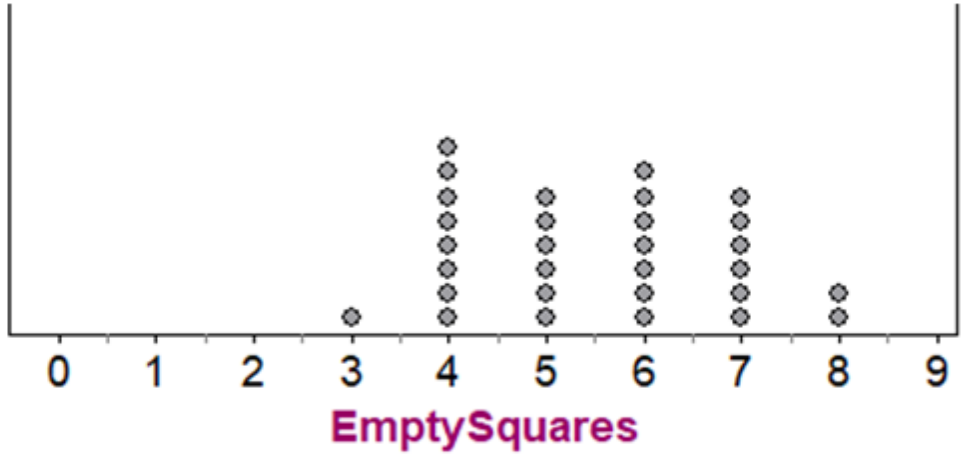
The labels for the 16 squares in the 4 x 4 Grid are as follows:

13	14	15	16
09	10	11	12
05	06	07	08
01	02	03	04

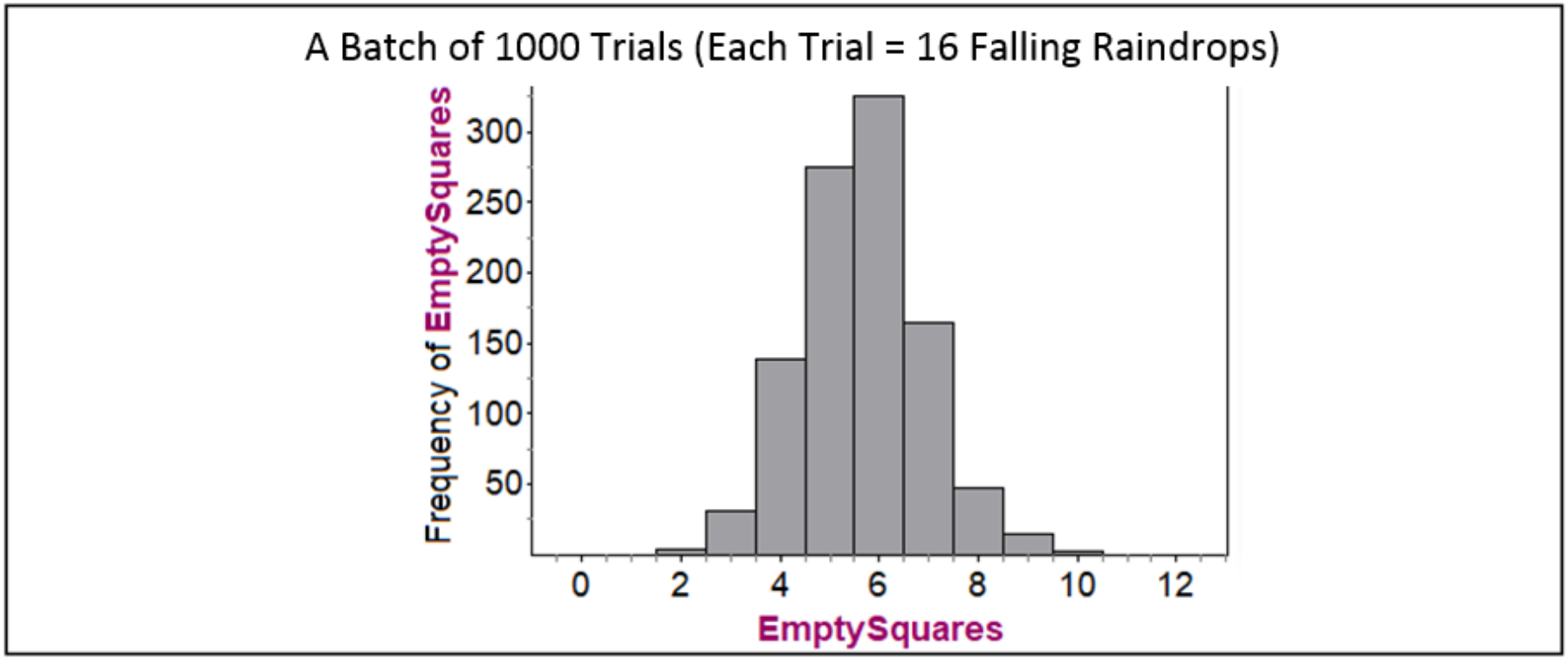


A Batch of 30 Trials (Each Trial = 16 Falling Raindrops)

	EmptySquares
26	5
27	4
28	6
29	4
30	4



Counting the empty squares in each of thirty trials



Counting the empty squares in each of 1000 trials

Note: Engel & Sedlmeier (2005) ascribed the “Novice” label to students whose predictions included between one and three empty squares, and the label of “Expert” to those between four and eight empty squares.

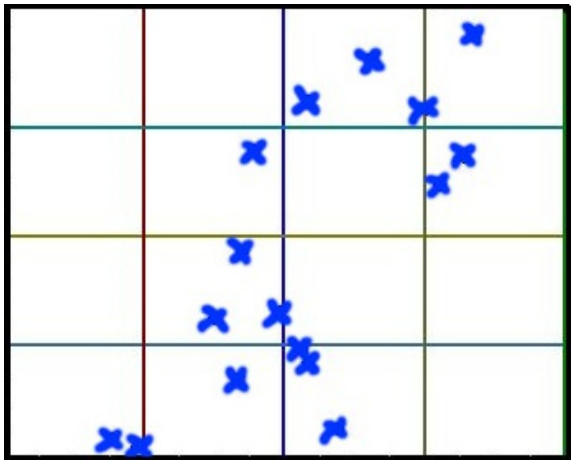
REAL OR FAKE?

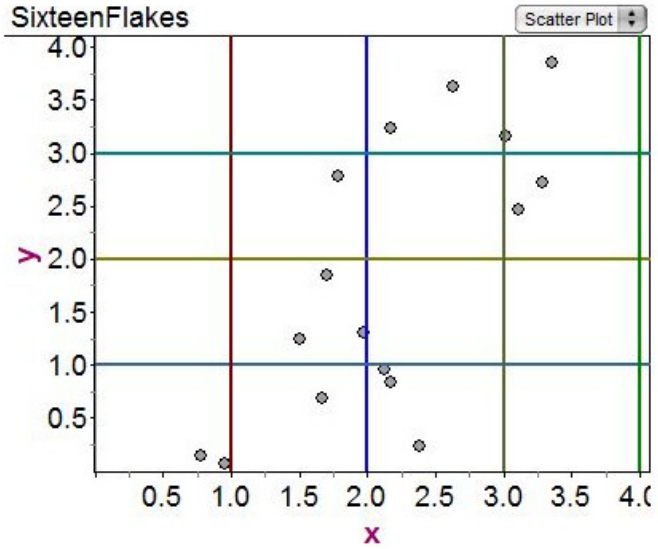
To simulate 16 snowflakes falling on a (4 x 4) patio, students were asked to go home and do this experiment:

> ***Toss 16 pennies randomly onto a (4 x 4) grid, and mark down where they landed. Bring in your marked grid tomorrow.***

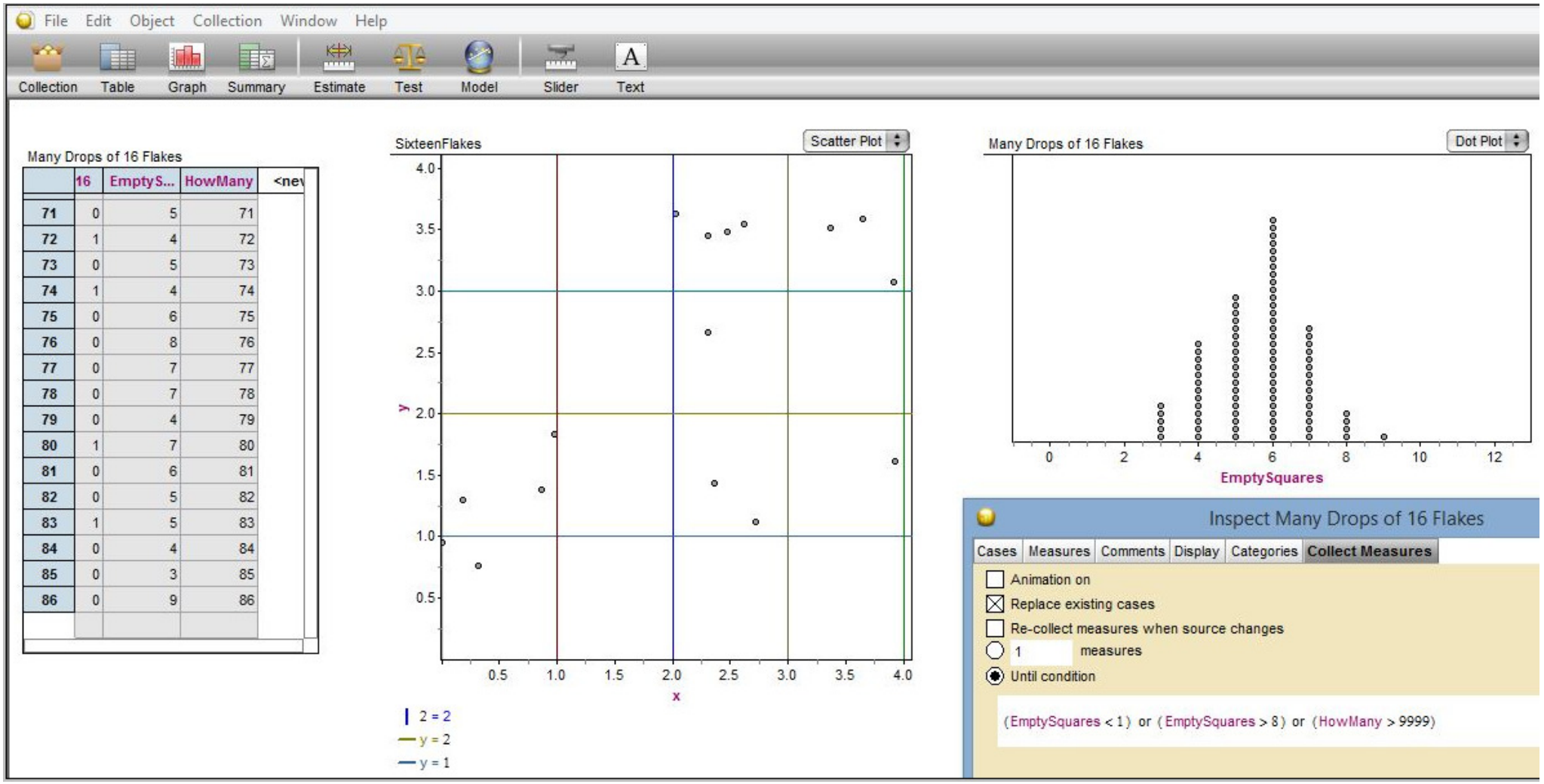
When the teacher got back the "results", she suspected some students did NOT actually do the experiment, but just made sixteen marks in what they hoped was a random way.

What do you think about the supposed "result"' shown above: Is it from a REAL experiment (randomly tossing 16 coins onto a grid), or did the student just FAKE these "results" ? Why do you think this?





HOW UNLIKELY?

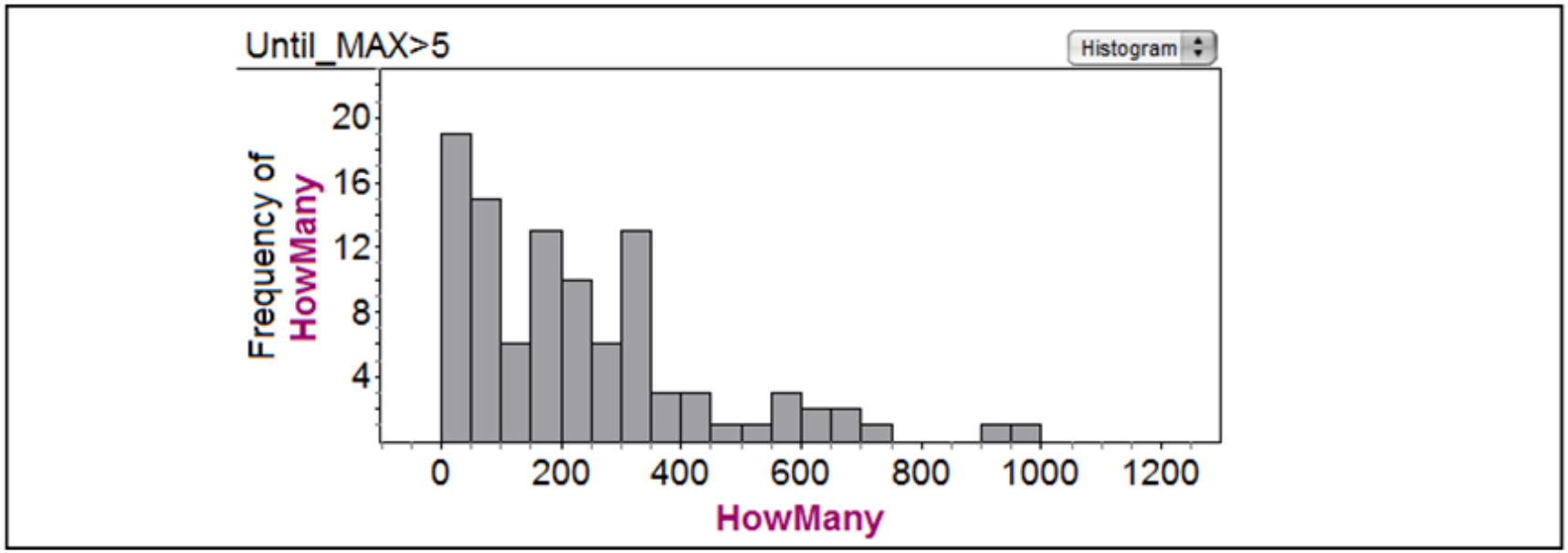


SOME (MORE) NEW QUESTIONS

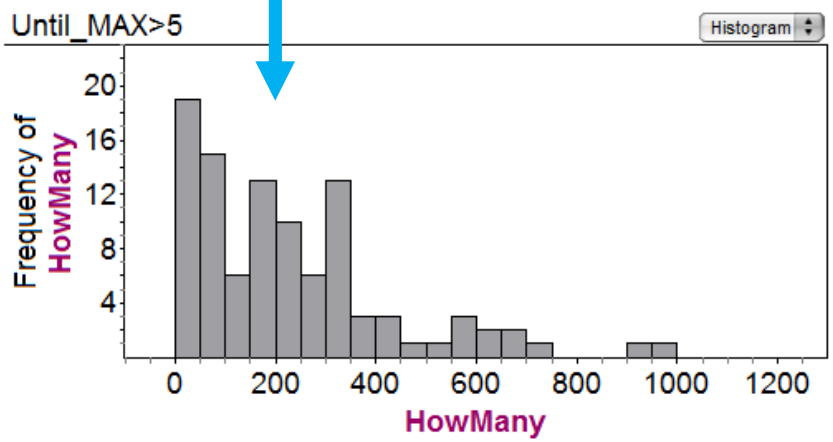
4. How likely is it that any given trial has exactly 1 raindrop on each square?

5. How many squares in any given trial are likely to contain exactly 2 raindrops?

6. What's the likelihood any given trial will have 6 raindrops or more on a square?



100 Experiments of "How many trials to get a square with 6 or more drops?"



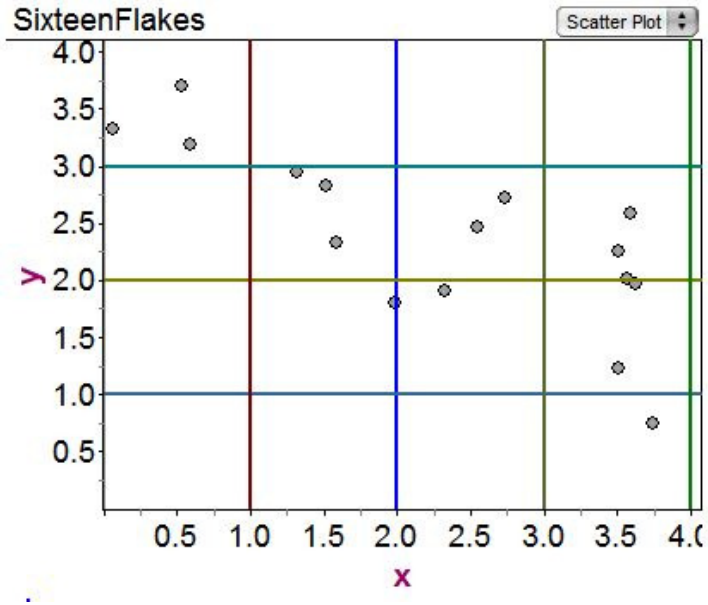
Mean is about 230 Trials

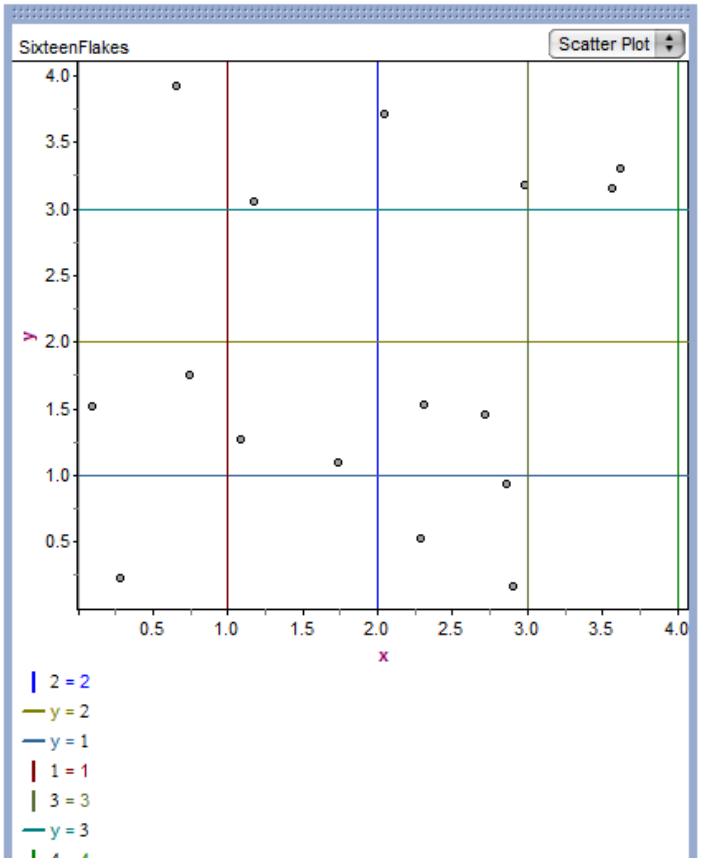
Reciprocal of 230 is 1/230 ≈ 0.0043

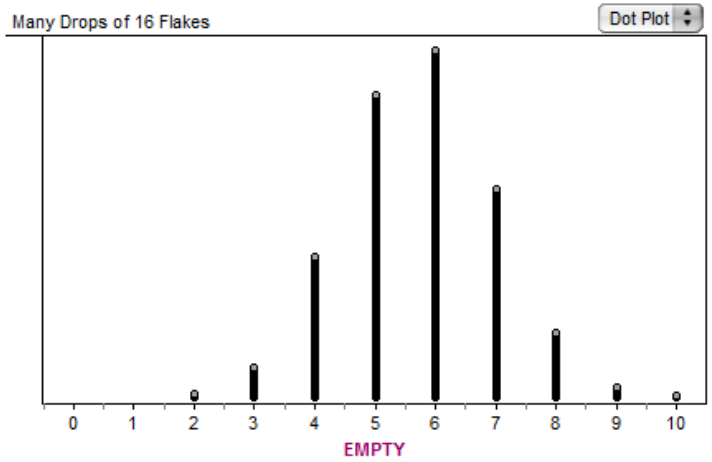
Conjecture for probability that a
Given trial has 6 or more raindrops
In a square?

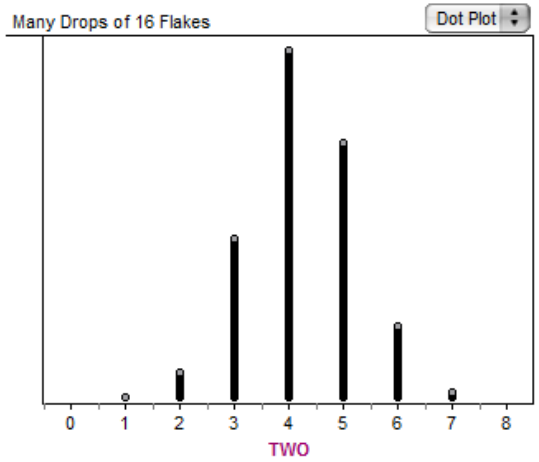
Last Temptation:

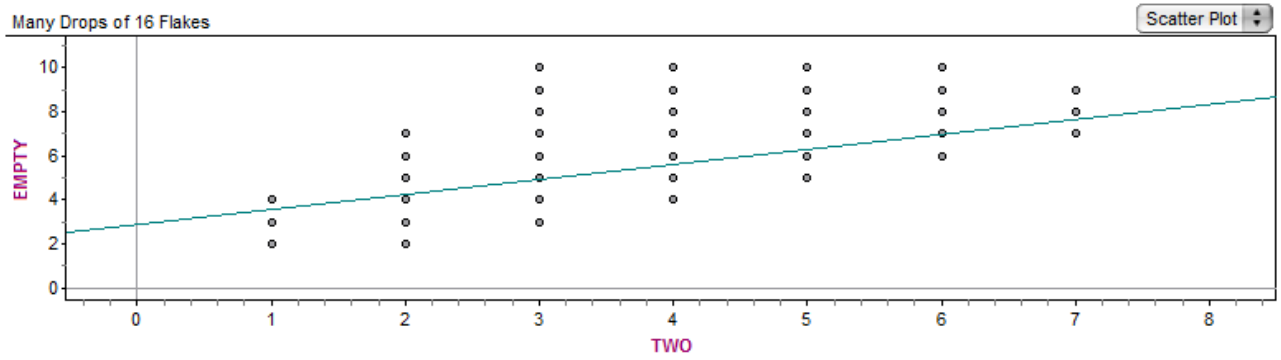
A correlation between “Number of Empty Squares”
and “Maximum Number of Raindrops on a Square?”




SixteenFlakes
Scatter Plot
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
y
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
x
2 = 2
y = 2
y = 1
1 = 1
3 = 3
y = 3


Many Drops of 16 Flakes
Dot Plot
0
1
2
3
4
5
6
7
8
9
10
EMPTY


Many Drops of 16 Flakes
Dot Plot
0
1
2
3
4
5
6
7
8
TWO


Many Drops of 16 Flakes
Scatter Plot
10
8
6
4
2
0
EMPTY
0
1
2
3
4
5
6
7
8
TWO

SOME THOUGHTS IN CONCLUSION

First, students markedly changed their predictions of where sixteen raindrops might land, as they were exposed to ever-increasing amounts of experimental data.

Second, students seemed better able to integrate a reasoning about variability in making informal inferences about hypothetical results.

Third, and perhaps most intriguing, students generated further questions that were based on what they noticed, and what they wondered about, in the face of large amounts of simulated data.

Thank You!

(Terima Kasih)