UNIVERSITÄT PADERBORN
Die Universität der Informationsgesellschaft

DESIGN OF A TEACHING UNIT TO DEVELOP PRIMARY SCHOOL STUDENTS´ REASONING ABOUT UNCERTAINTY IN MULTI-STEP CHANCE EXPERIMENTS

Daniel Frischemeier & Rolf Biehler
University of Paderborn

Presentation at IASE Satelllite Conference 2019,
Kuala Lumpur

Overview

1. Introduction

2. **Design and realization of a teaching unit to enhance statistical reasoning about uncertainty in primary school**

3. Explorative study: „Primary school students‘ statistical reasoning when conducting chance experiments with TinkerPlots“

4. Summary & Outlook

Introduction

Starting point

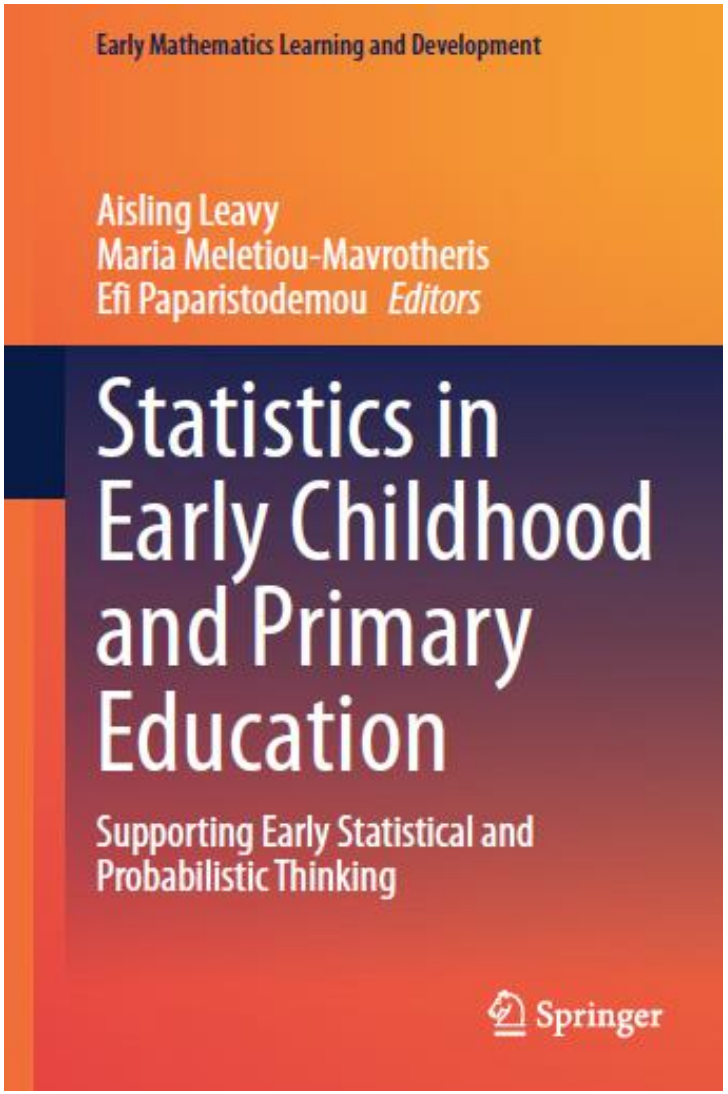
- Citizen need to be statistical literate to be able to participate in public decision processes
- But: Statistical literacy is a long-term process (Gal, 2004)
  → therefore: first steps can / should be done in primary school

- From a German perspective statistics has received increased attention in primary school curriculum (Hasemann & Mirwald, 2012)

  → Recommended activities:
  - posing statistical questions, collecting data, getting to know how to represent data and interpreting representations of data
  - interpretation and comparison of chances of certain events in one-step chance experiments

The need for Early Statistical Thinking

„Today‘s students need to learn to work and think with data and chance from an early age, so they begin to prepare for the data-driven society in which they live.“ (Ben-Zvi, 2018, vii)



Leavy, A., Meletiou-Mavrotheris, M., & Paparistodemou, E. (2018). *Statistics in Early Childhood and Primary Education: Supporting Early Statistical and Probabilistic Thinking*. Singapore: Springer.

A conceptualization of randomness

Batanero (2015) distinguishes between three conceptualizations of randomness:

(i) Randomness as equiprobability,

(ii) randomness as stability of frequencies,

(iii) subjective view of randomness.

Conceptualizations of randomness

- In “randomness as equiprobability” probability is defined in the sense of Laplace as “the number of favorable cases to a particular event divided by the number of all cases possible in that experiment, provided all the possible cases are equiprobable” (Batanero, 2015, p. 36).

- “Randomness as stability of frequencies” is related to the empirical law of large numbers, probability is defined according to Batanero (2015, p. 37) as “the hypothetical number towards which the relative frequency tends”.

- The subjective view “considered probability as a personal degree of belief that depends on a person´s knowledge or experience” (Batanero, 2015, p. 37).

Conceptualizations of randomness

- In “randomness as equiprobability” probability is defined in the sense of Laplace as “the number of favorable cases to a particular event divided by the number of all cases possible in that experiment, provided all the possible cases are equiprobable” (Batanero, 2015, p. 36).

- “Randomness as stability of frequencies” is related to the empirical law of large numbers, probability is defined according to Batanero (2015, p. 37) as “the hypothetical number towards which the relative frequency tends”.

- The subjective view “considered probability as a personal degree of belief that depends on a person´s knowledge or experience” (Batanero, 2015, p. 37).

Conceptualizations of randomness

- In “randomness as equiprobability” probability is defined in the sense of Laplace as “the number of favorable cases to a particular event divided by the number of all cases possible in that experiment, provided all the possible cases are equiprobable” (Batanero, 2015, p. 36).

- “Randomness as stability of frequencies” is related to the empirical law of large numbers, probability is defined according to Batanero (2015, p. 37) as “the hypothetical number towards which the relative frequency tends”.

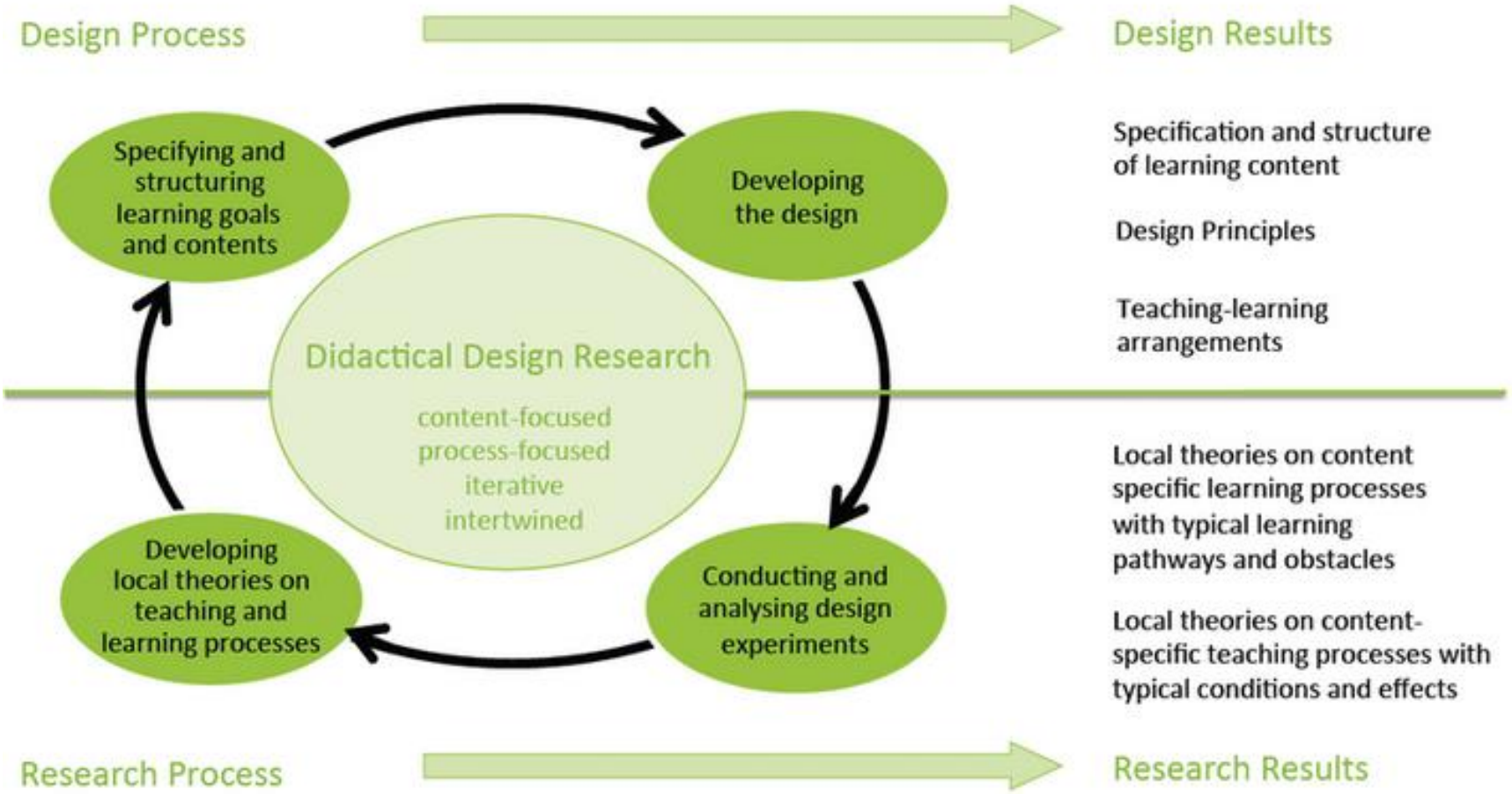
- The subjective view “considered probability as a personal degree of belief that depends on a person´s knowledge or experience” (Batanero, 2015, p. 37).

Goals of this project

Design of a teaching unit which covers each of the three conceptualizations of randomness to develop primary school students´ reasoning about uncertainty

Research Methodology

Didactical Design Research (Prediger, & Zwetzschler, 2013).

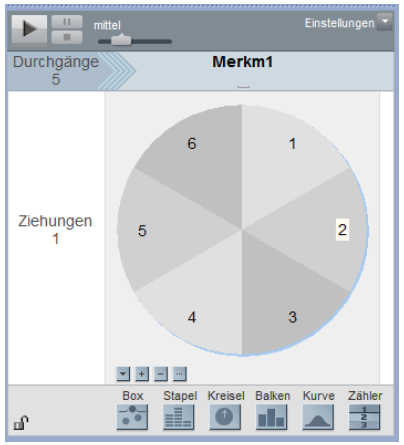


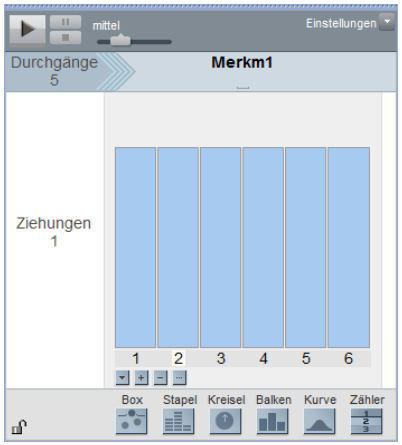
Research study

- First cycle of Design research
- Primary school in rural area in Germany
- 7 lessons (45 minutes each)
  - Data: field notes of the teacher, documents of students, videos
- Participants: 20 students, grade 4 (10-11 years old)
  - no specific pre-knowledge in statistics apart from collecting data in tallies and creating and reading bar charts and reading pie charts.

Design ideas for the teaching learning environment

- **Taking into account all three conceptualizations of randomness** of Batanero (2015) to develop the reasoning about uncertainty among grade 4 students.
- **Elements of the „Statistical Reasoning Learning Environment“** (Garfield & Ben-Zvi , 2008), e.g.
  - **Peer learning settings, promoting classroom discourse**
  - **Using educational software TinkerPlots** (Konold 2007, Garfield & Ben-Zvi 2008, Ben-Zvi & Pfannkuch 2011) → TP Sampler





Seven lessons of the teaching learning environment

No	Content of lesson
1	Pupils learn about how to read and interpret statistical bar graphs and pie charts
2	Pupils estimate and compare probabilities of events by classifying them in “certain”, “possible”, “unlikely” and “impossible”.
3	Pupils conduct hands-on experiment “throw of a dice” and collect data to get the insight that the probability for each side of the dice is equal.
4	Pupils are introduced in the sampler of TinkerPlots and get first experiences of the empirical law of large numbers by simulating the throw of a coin with the TinkerPlots sampler
5	Pupils conduct hands-on experiment “throw of two dice”, collect the sum of the two dice for each throw and collect data to use the frequencies of occurrence of the different sums to make statements with regard to the probability of the events of different sums
6-7	Pupils conduct TinkerPlots simulation “throw of two dice” and collect data to compare the probability of events. Pupils try to find explanations why some sums appear more frequent than other sums when throwing two dice.

Seven lessons of the teaching learning environment

No	Content of lesson
1	Pupils learn about how to read and interpret statistical bar graphs and pie charts
2	Pupils estimate and compare probabilities of events by classifying them in “certain”, “possible”, “unlikely” and “impossible”.
3	Pupils conduct hands-on experiment “throw of a dice” and collect data to get the insight that the probability for each side of the dice is equal.
4	Pupils are introduced in the sampler of TinkerPlots and get first experiences of the empirical law of large numbers by simulating the throw of a coin with the TinkerPlots sampler
5	Pupils conduct hands-on experiment “throw of two dice”, collect the sum of the two dice for each throw and collect data to use the frequencies of occurrence of the different sums to make statements with regard to the probability of the events of different sums
6-7	Pupils conduct TinkerPlots simulation “throw of two dice” and collect data to compare the probability of events. Pupils try to find explanations why some sums appear more frequent than other sums when throwing two dice.

Seven lessons of the teaching learning environment

No	Content of lesson
1	Pupils learn about how to read and interpret statistical bar graphs and pie charts
2	Pupils estimate and compare probabilities of events by classifying them in "certain", "possible", "unlikely" and "impossible".
3	Pupils conduct hands-on experiment "throw of a dice" and collect data to get the insight that the probability for each side of the dice is equal.
4	Pupils are introduced in the sampler of TinkerPlots and get first experiences of the empirical law of large numbers by simulating the throw of a coin with the TinkerPlots sampler
5	Pupils conduct hands-on experiment "throw of two dice", collect the sum of the two dice for each throw and collect data to use the frequencies of occurrence of the different sums to make statements with regard to the probability of the events of different sums
6-7	Pupils conduct TinkerPlots simulation "throw of two dice" and collect data to compare the probability of events. Pupils try to find explanations why some sums appear more frequent than other sums when throwing two dice.

Contents of the second lesson

- Students evaluate events of several chance experiments using the expressions “certain”, “probable”, “unlikely” and “impossible”.
- Teacher offers a strip with a scale from “impossible” to “certain” which the students use to show and to visualize their expectation with regard to specific events.
- Then: Teacher expands the scale and adds frequency descriptions like “never”, “seldom”, “often” and “always” for each terminology

Seven lessons of the teaching learning environment

No	Content of lesson
1	Pupils learn about how to read and interpret statistical bar graphs and pie charts
2	Pupils estimate and compare probabilities of events by classifying them in “certain”, “possible”, “unlikely” and “impossible”.
3	Pupils conduct hands-on experiment “throw of a dice” and collect data to get the insight that the probability for each side of the dice is equal.
4	Pupils are introduced in the sampler of TinkerPlots and get first experiences of the empirical law of large numbers by simulating the throw of a coin with the TinkerPlots sampler
5	Pupils conduct hands-on experiment “throw of two dice”, collect the sum of the two dice for each throw and collect data to use the frequencies of occurrence of the different sums to make statements with regard to the probability of the events of different sums
6-7	Pupils conduct TinkerPlots simulation “throw of two dice” and collect data to compare the probability of events. Pupils try to find explanations why some sums appear more frequent than other sums when throwing two dice.

Contents of the third lesson

Pupils conduct hands-on experiment “throw of a dice” and collect data to get the insight that the probability for each side of the dice is equal

Students work in pairs (10 pairs) to throw a dice fifty times and document the frequencies of the occurrences of each side (hands-on activities)

→ Teacher uses notations like “In 79 of 500 cases the side 1 has occurred” or “in 78 of 500 cases the side 2 has occurred” to make the frequencies comparable

→ Insight for the students: the probability for each side of the dice is equal.

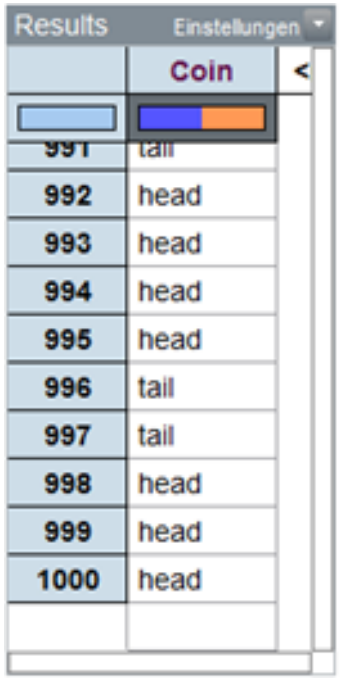
Seven lessons of the teaching learning environment

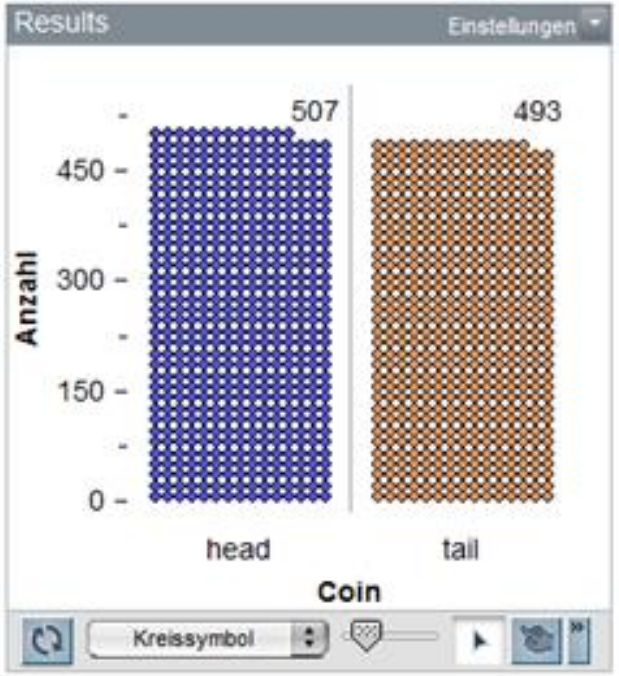
No	Content of lesson
1	Pupils learn about how to read and interpret statistical bar graphs and pie charts
2	Pupils estimate and compare probabilities of events by classifying them in “certain”, “possible”, “unlikely” and “impossible”.
3	Pupils conduct hands-on experiment “throw of a dice” and collect data to get the insight that the probability for each side of the dice is equal.
4	Pupils are introduced in the sampler of TinkerPlots and get first experiences of the empirical law of large numbers by simulating the throw of a coin with the TinkerPlots sampler
5	Pupils conduct hands-on experiment “throw of two dice”, collect the sum of the two dice for each throw and collect data to use the frequencies of occurrence of the different sums to make statements with regard to the probability of the events of different sums
6-7	Pupils conduct TinkerPlots simulation “throw of two dice” and collect data to compare the probability of events. Pupils try to find explanations why some sums appear more frequent than other sums when throwing two dice.

Contents of the fourth lesson

Students are introduced in the simulation of chance experiments with the TinkerPlots sampler.

→ Teacher demonstrates how to realize the “toss of a coin” in the TinkerPlots sampler





Seven lessons of the teaching learning environment

No	Content of lesson
1	Pupils learn about how to read and interpret statistical bar graphs and pie charts
2	Pupils estimate and compare probabilities of events by classifying them in “certain”, “possible”, “unlikely” and “impossible”.
3	Pupils conduct hands-on experiment “throw of a dice” and collect data to get the insight that the probability for each side of the dice is equal.
4	Pupils are introduced in the sampler of TinkerPlots and get first experiences of the empirical law of large numbers by simulating the throw of a coin with the TinkerPlots sampler
5	Pupils conduct hands-on experiment “throw of two dice”, collect the sum of the two dice for each throw and collect data to use the frequencies of occurrence of the different sums to make statements with regard to the probability of the events of different sums
6-7	Pupils conduct TinkerPlots simulation “throw of two dice” and collect data to compare the probability of events. Pupils try to find explanations why some sums appear more frequent than other sums when throwing two dice.

Contents of the fifth lesson

- Teacher chooses a more complex chance experiment (“throw of two dice”) → multi-step chance experiments.
- Task for the students: “Find out which sum is more likely to appear when throwing two dice.”
- Students are paired in groups and each group was given two dice and the task to throw the two dice fifty times and to document their outcomes → teacher collects the results of all ten groups

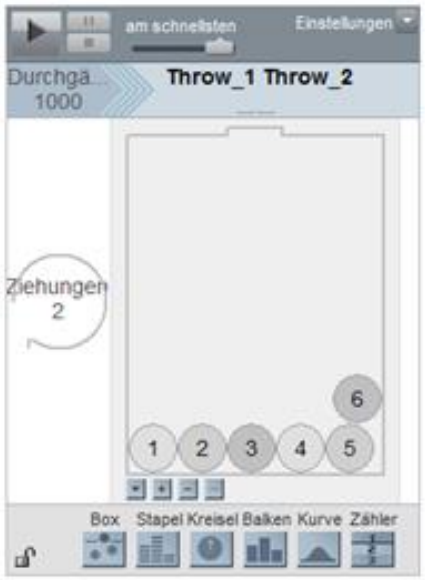
The first insight for the students in lesson 5 is that the different sums of the throw of two dice are not equiprobable

Seven lessons of the teaching learning environment

No	Content of lesson
1	Pupils learn about how to read and interpret statistical bar graphs and pie charts
2	Pupils estimate and compare probabilities of events by classifying them in “certain”, “possible”, “unlikely” and “impossible”.
3	Pupils conduct hands-on experiment “throw of a dice” and collect data to get the insight that the probability for each side of the dice is equal.
4	Pupils are introduced in the sampler of TinkerPlots and get first experiences of the empirical law of large numbers by simulating the throw of a coin with the TinkerPlots sampler
5	Pupils conduct hands-on experiment “throw of two dice”, collect the sum of the two dice for each throw and collect data to use the frequencies of occurrence of the different sums to make statements with regard to the probability of the events of different sums
6-7	Pupils conduct TinkerPlots simulation “throw of two dice” and collect data to compare the probability of events. Pupils try to find explanations why some sums appear more frequent than other sums when throwing two dice.

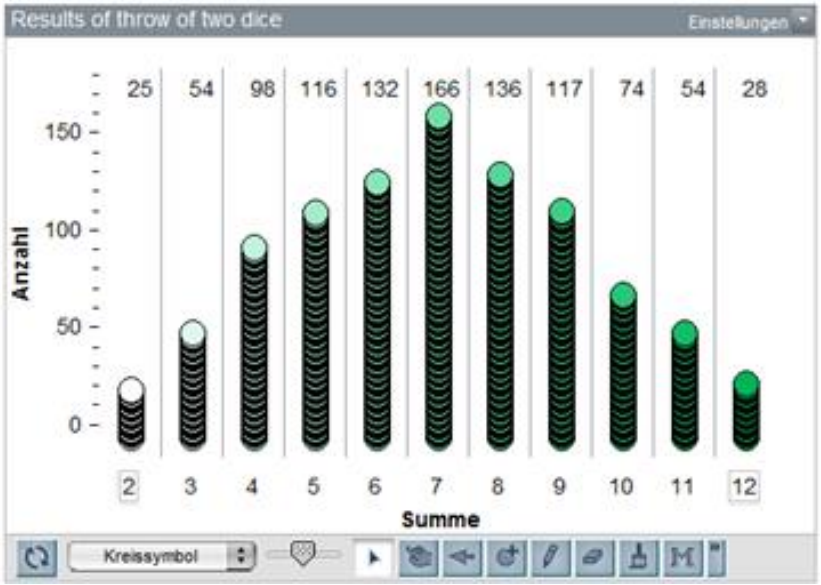
Contents of the sixth and seventh lesson

- The teacher reflects the results from the hands-on activity in the fifth lesson on the chance experiment “sum of two dice” and demonstrates how to set up this chance experiment in the TinkerPlots sampler



Results of throw of two dice — Einstellungen

	Gesamt	Throw_1	Throw_2	Summe
655	4;2	4	2	6
656	4;2	4	2	6
657	4;4	4	4	8
658	4;5	4	5	9
659	5;6	5	6	11
660	2;2	2	2	4
661	2;3	2	3	5
662	1;3	1	3	4
663	5;2	5	2	7
664	1;5	1	5	6
665	2;6	2	6	8
666	5;6	5	6	11



Contents of the sixth and seventh lesson

- The teacher repeats the simulation in TinkerPlots several times and the students observe that the mode of the distribution is the sum “7”.
- The students have the idea to consider the possible outcomes of the throw of two dice for each of the sums (2-12) → identify the outcomes for each sum and collect their results
- The students explore that the sum “7” has the most outcomes (1+6, 6+1, 2+5, 5+2, 3+4, 4+3) and therefore occurs more often in the experiment than for example the sum “3” (with the two outcomes 1+2 and 2+1) or the sum “2” (with the outcome 1+1).
- The students also find out the outcomes for each sum and that there are 36 outcomes in total (each of them are equiprobable).

Contents of the sixth and seventh lesson

- Comparisons of probabilities of certain events like “the sum 6 is more likely to show up than the sum 3”
  → because there are five favorable out of 36 outcomes for sum “6” but only two favorable out of 36 outcomes for sum “3”.

First observations from the teaching-learning environment

- Students were very engaged - especially with the simulation of chance experiments (hands-on, TinkerPlots)
- Elements of the three conceptualizations of randomness (see Batanero, 2015) can be already implemented in primary school classroom.
- TinkerPlots sampler seems to be a powerful educational tool to facilitate the modelling and the data production process when simulating chance experiments in primary school
  → Some students needed support (especially when setting up the model)
  → Challenge: interpretation of the produced data

Explorative Study: „Primary school students' statistical reasoning when conducting chance experiments with TinkerPlots“

Research question

In which way does the teaching unit develop the competence of pupils with regard to compare probabilities of events of multi-step chance experiments?

Data, participants and data analysis

We collected data on different levels:

(a) written pre/post-tests,

(b) working notes on tasks and activities after each lesson and

(c) interviews after the teaching unit.

Data, participants and data analysis

We collected data on different levels:

(a) written pre/post-tests,

(b) working notes on tasks and activities after each lesson and

(c) interviews after the teaching unit.

Written pre/post tests were handed out at the beginning and at the end of the teaching unit

→ Six tasks.

→ Posttest is identical to the pretest.

→ 19 students have participated in the pre- and posttest.

→ We will only concentrate on the tasks which focus on the comparison of probabilities of events of multi-step chance experiments (tasks 5 and 6).

Tasks

Task (5) You and your friend Tim play a game where you toss a coin twice. You win if you get tail both times. Tim wins if he gets tail and head when tossing the coin twice. Do you both have the same chances of winning? Explain!

Task (6) Anna and Tom are playing a game. They throw a dice twice. If the sum of the two dice is "7", Anna wins. If the sum is 3 or 12, Tom wins. Do Anna and Tom have the same chance of winning? Explain!

Data analysis procedure & Results

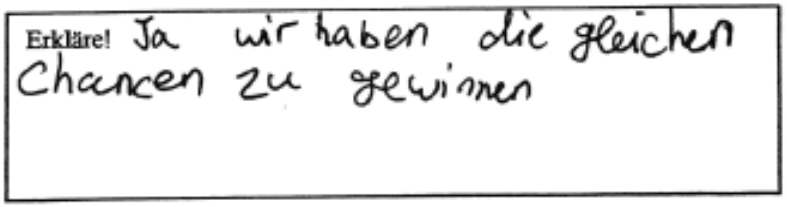
1. Coding the correctness of the statements of each participant in pre-test and post-test
2. Frequency analysis and calculation of the percentages of correct statements of each task in comparison to pre- and posttest.

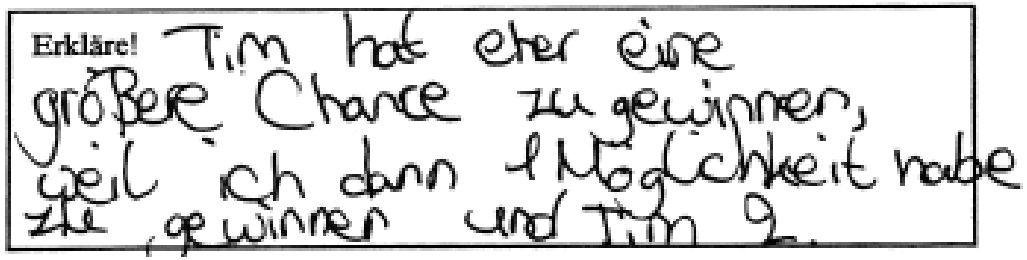
Item	% pretest *n*	% posttest *n*	Difference *n*
5	0.0% *0*	47.4% *9*	+47.4 + *9*
6	9.5% *2*	52.6% *10*	+43.1 + *8*

Exemplary statements of students (task 5)

Task (5): You and your friend Tim play a game where you toss a coin twice. You win if you get tail both times. Tim wins if he gets tail and head when tossing the coin twice. Do you both have the same chances of winning? Explain!

"Yes, we have an equal chance of winning." (statement in pre-test)



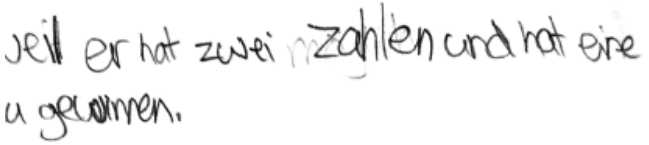


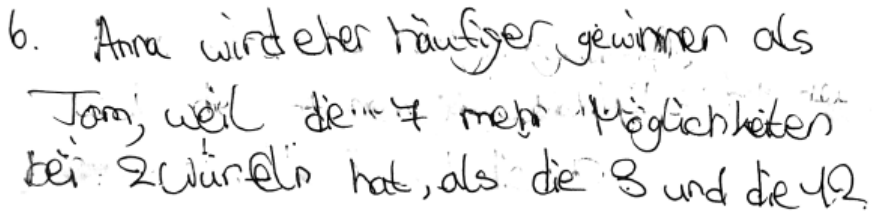
"Tim has a better chance to win because I have one chance to win and Tim has two." (statement in post-test)

Exemplary statements of students (task 6)

Task (6): Anna and Tom are playing a game. They throw a dice twice. If the sum of the two dice is "7", Anna wins. If the sum is 3 or 12, Tom wins. Do Anna and Tom have the same chance of winning? Explain!

"Tom will win because he has two numbers and has a better chance to win." (statement in pre-test)





"Anna will win more often than Tom, because the 7 has more possibilities with two dice than the 3 and 12." (statement in post-test)

Summary & Outlook

Summary

- Elements of the three conceptualizations of randomness (see Batanero, 2015) can be already implemented in primary school classroom.
- The performance of the students from pre-test to post-test has increased considerably
- The selected statements in the post-test show that the students´ reasoning about the interpretation of events in multi-step chance experiments has developed in a positive way
  - Some students are now able to use, e.g. elements of the approach of Laplace to explain their statements when comparing the probability of events.

First „Implications“ (although not in a sense of experimental evidence)

- Statistical reasoning should be initiated as early as possible → in primary school
- The TinkerPlots sampler seems to be a powerful educational tool to facilitate the modelling and the data production process when simulating chance experiments in primary school → challenge: interpretation of the produced data (decision making based on data)
- The students should conduct the chance experiments as hand-on activities first and then use software like TinkerPlots for the modelling and data production process.

Outlook

- The teaching can also be adapted for implementation in secondary school classroom.
- The other data (working notes and interview data) collected in the frame of this research project are still under analysis.
  - Interview: insight into the cognitive processes of primary school students when modelling one/multi-step chance experiment with TinkerPlots

Literature

- Batanero, C. (2015). Understanding Randomness: Challenges for Research and Teaching. In K. Krainer & N. Vondrova (Eds.), Proceedings of the 9th Congress of the European Society for Research in Mathematics Education (pp. 34–49). Charles University in Prague: Faculty of Education and ERME.
- Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design Experiments in Educational Research. Educational Researcher, 32(1), 9-13.
- Garfield, J., & Ben-Zvi, D. (2008). Developing Students' Statistical Reasoning. Connecting Research and Teaching Practice. The Netherlands: Springer.
- Hasemann, K., & Mirwald, E. (2012). Daten, Häufigkeit Und Wahrscheinlichkeit. In G. Walther, M. van den Heuvel-Panhuizen, D. Granzer, & O. Köller (Eds.), Bildungsstandards Für Die Grundschule: Mathematik Konkret (pp. 141-161). Berlin: Cornelsen Scriptor.
- Jackes-Schulte, A. (2016). Design, Durchführung Und Evaluation Einer Unterrichtsreihe Zur Entwicklung Der Kompetenz "Gewinnchancen Beim Doppelten Würfelwurf Einzuschätzen" in Jahrgangsstufe 4. (Bachelor of Education), University of Paderborn.
- Konold, C., & Miller, C. (2011). Tinkerplots 2.0. Emeryville, CA: Key Curriculum Press.
- Leavy, A., Meletiou-Mavrotheris, M., & Paparistodemou, E. (2018). Statistics in Early Childhood and Primary Education: Supporting Early Statistical and Probabilistic Thinking. Singapore: Springer.
- Plückebaum, K. (2018). Simulieren Von Zufallsexperimenten in Der Primarstufe Mit Der Software Tinkerplots. (Master of Education), Universität Paderborn, Paderborn.
- Weustenfeld, W. (2007). Die Augensumme Zweier Würfel Voraussagen: Alles Nur Eine Frage Von Glück Oder Pech. Stochastik in der Schule, 27(3), 2-15.

Thank you very much for your attention!