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Decision Making Based on Data

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Obstacles in the evolution of secondary school students' mental models of reasoning on decision making

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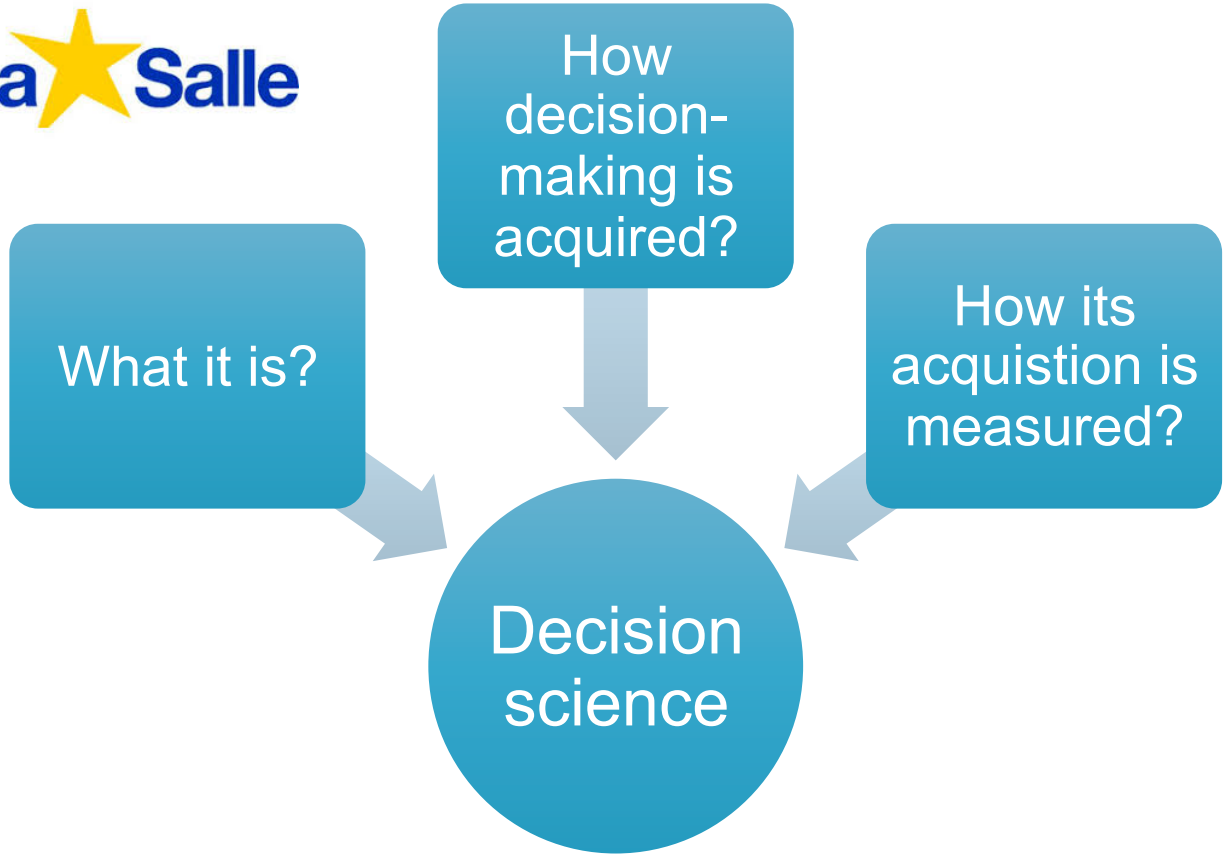
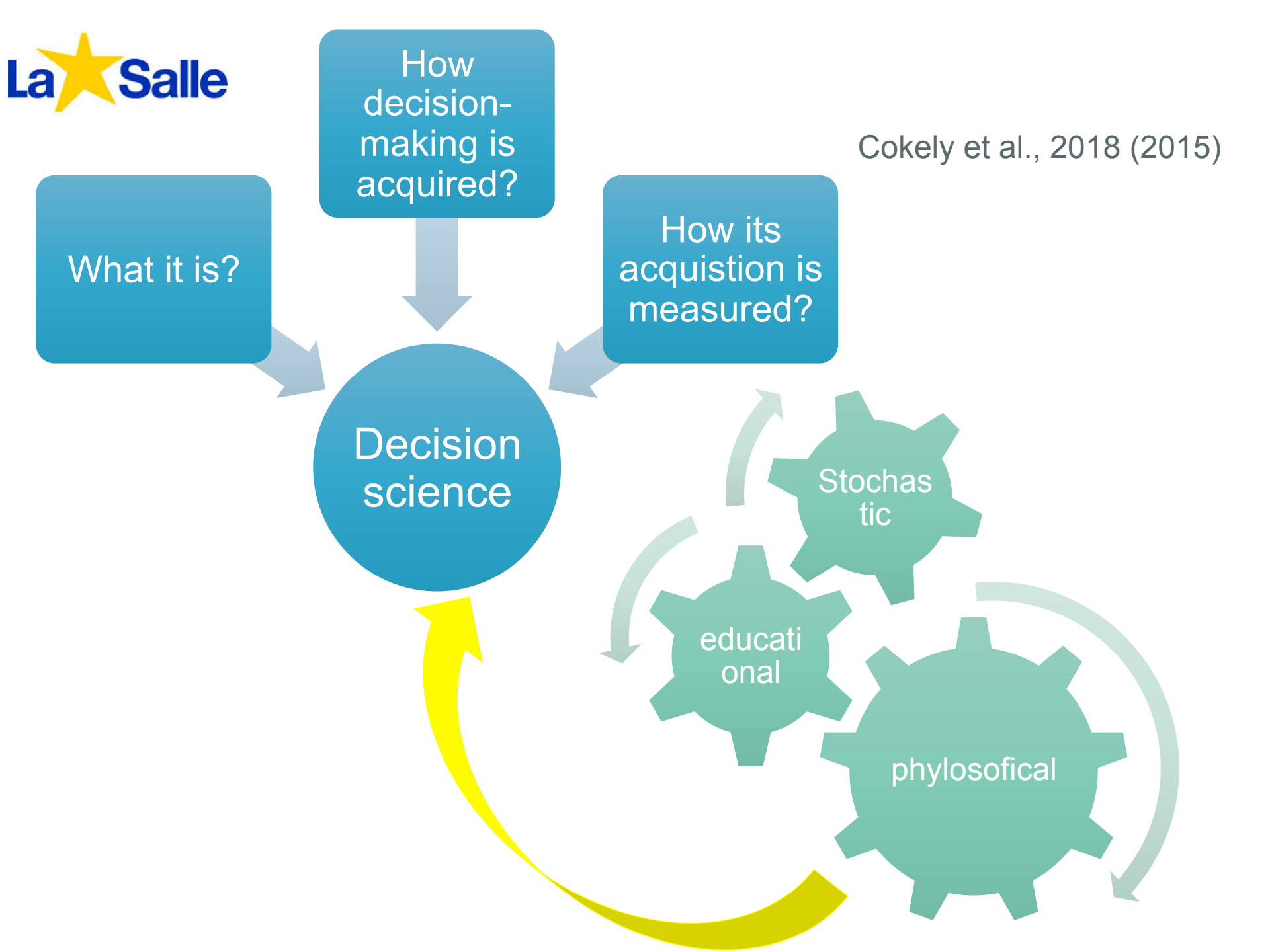


Cokely et al., 2018 (2015)



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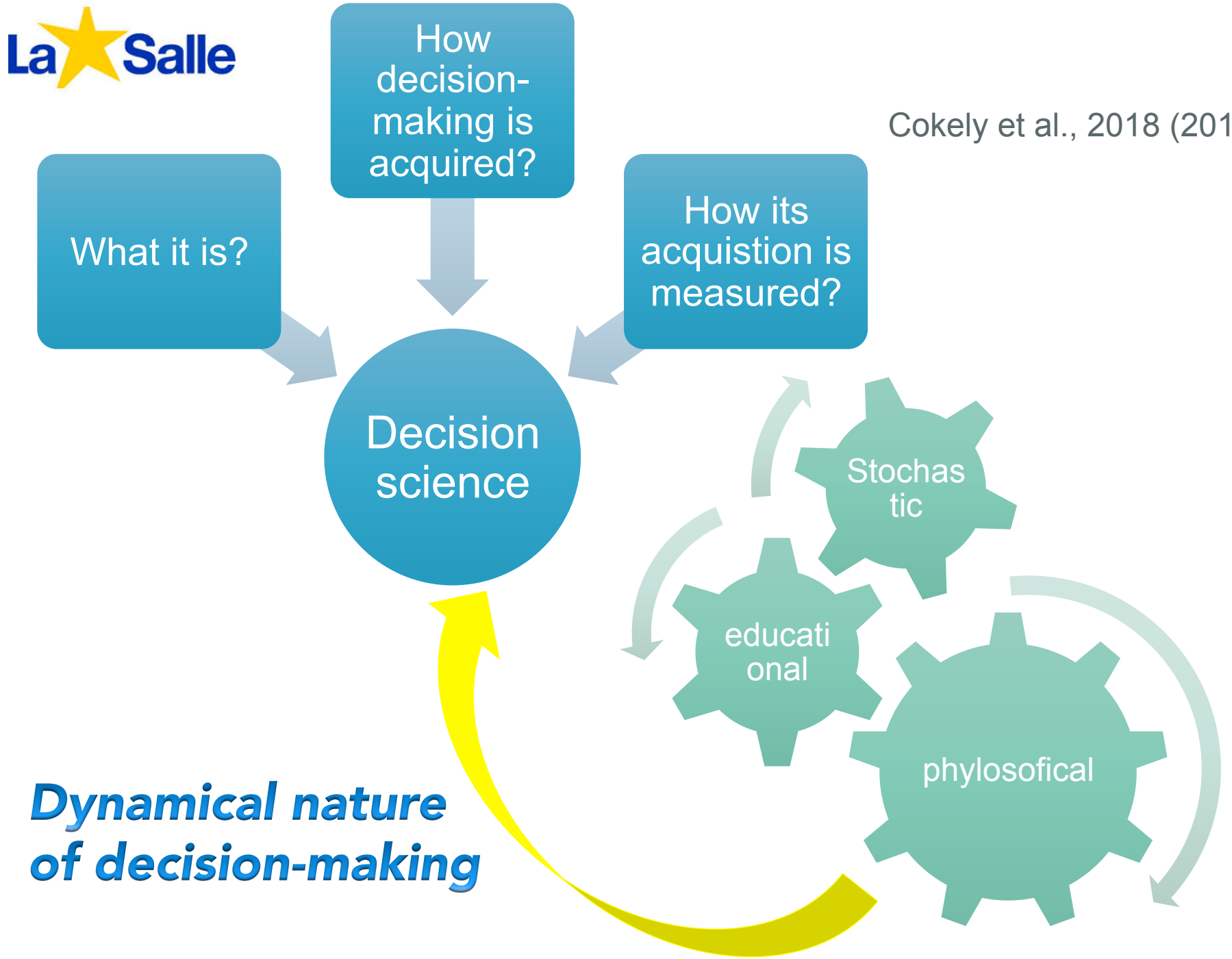

Cokely et al., 2018 (2015)

Dynamical nature of decision-making

Busemeyer (2015)

Dynamical nature of decision making

The decision-maker sequentially samples evaluations based on partial cognitive models of decision-making for preferences that estimate the utility of an action until the preference for one action exceeds a threshold

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Evolution in the cognitive models that the person develops for decision making

Busemeyer (2015)

Dynamical nature of decision making

The decision-maker sequentially samples evaluations based on partial cognitive models of decision-making for preferences that estimate the utility of an action until the preference for one action exceeds a threshold

Evolution in the cognitive models that the person develops for decision making

The dynamical nature of the decision-making process to minimize the risk to lose when Secondary school students play a game of chance

Integers Addition Bingo
Game of chance task

Partida Tiro Tiradas = 25

bola1 = −4 bola2 = 4

Suma los valores de la bolas

- [x] Frecuencia absoluta
- [x] Frecuencia relativa
- [x] Probabilidad teórica

Integers Addition Bingo
Game of chance task

Partida Tiro Tiradas = 25

INTEGERS ADDITION BINGO				
-7	-5	-2	+0	+1
+1	+3	+5	+9	+10

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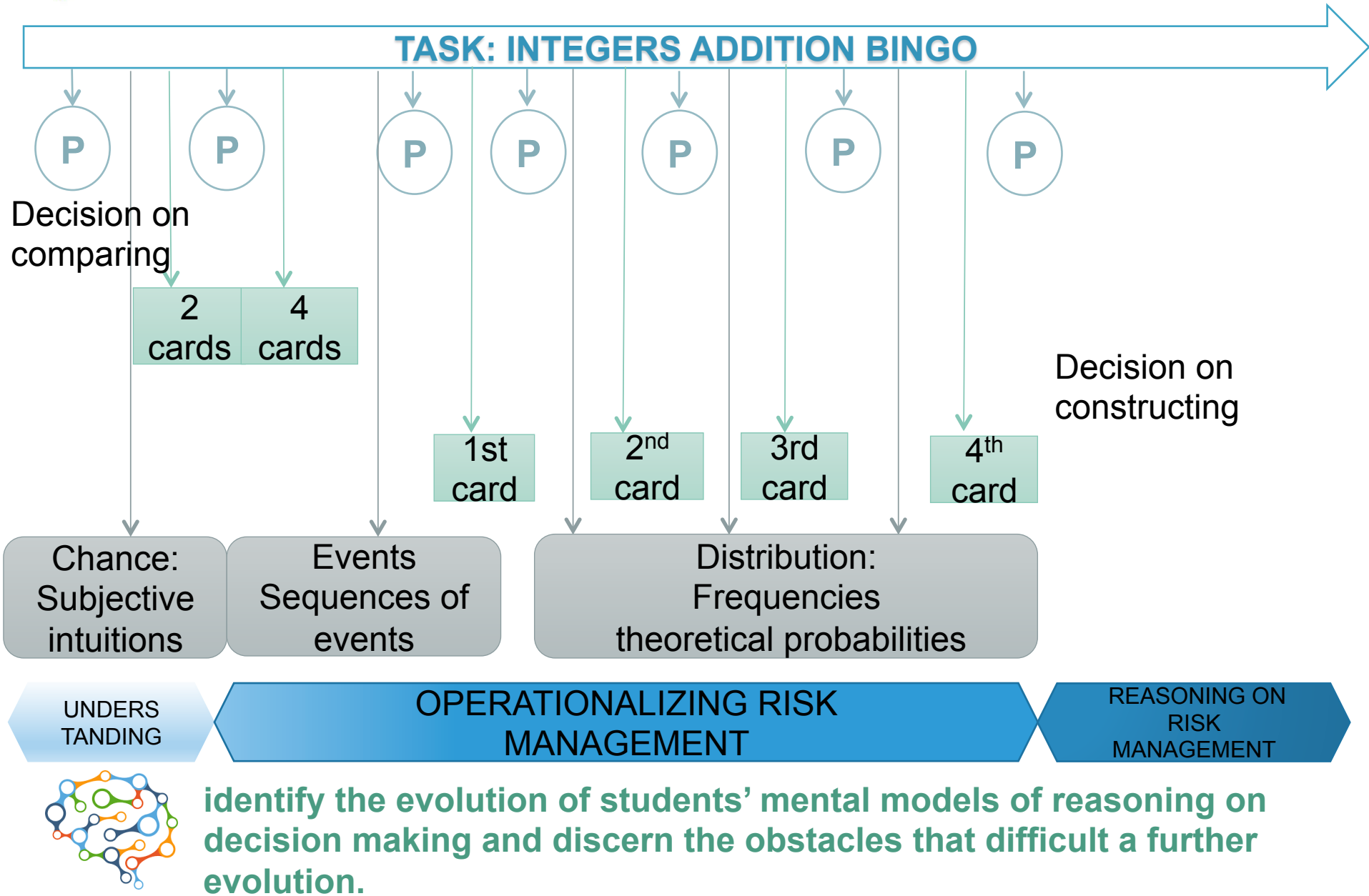
- If the number result of the addition is on their card, they mark the result on it.
- The first student who have marked all the numbers of his/her card wins the game.

Integers Addition Bingo
Game of chance task



identify the evolution of students' mental models of reasoning on decision making and discern the obstacles that difficult a further evolution.

Obstacles in the evolution of decision-making

Cokely et al., (2018)

Obstacles in the evolution of decision-making

Probabilistic science

Cokely et al., (2018)

Blaise Pascal
Pierre Fermat

1654

Decision science

Gambling problem

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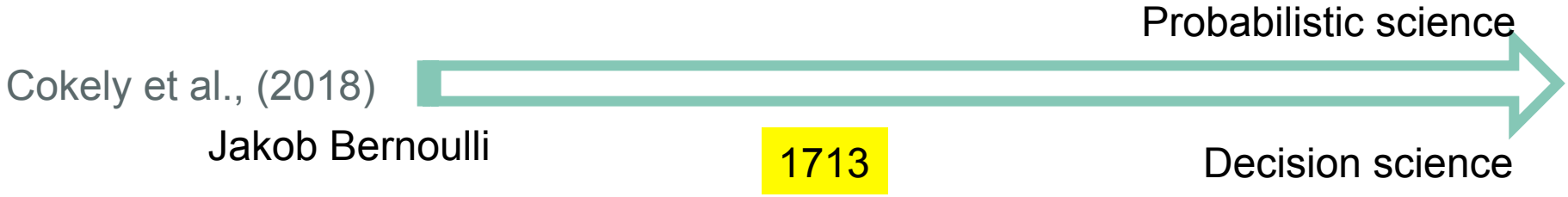
Decisions based on personal preferences

Deterministic thinking

Decisions rationally bounded on classical probabilistic postulates

Obstacles in the evolution of decision-making



Law of large numbers

Obstacles in the evolution of decision-making

Probabilistic science

Cokely et al., (2018)

Jakob Bernoulli

1713

Decision science

Law of large numbers

- Insufficiency of data to conclude on the experimental probability of a situation
- Insufficiency of the condition of independence of events to conclude about the experimental probability Borovcnick and Kapadia (2014)

Obstacles in the evolution of decision-making

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Decision science

Law of large numbers

- Insufficiency of data to conclude on the experimental probability of a situation
- Insufficiency of the condition of independence of events to conclude about the experimental probability Borovcnick and Kapadia (2014)

Distinguish:
Events and outcomes
(Savard, 2014)

Understand:
Proportional relationship
(Saldanha and Liu, 2014)

Understand:
The relationship between a classical a priori and a frequentist model of probability in estimating the probability of random outcomes

Obstacles in the evolution of decision-making

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1713

Decision science

Law of large numbers

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-7	-5	-2	+0	+1
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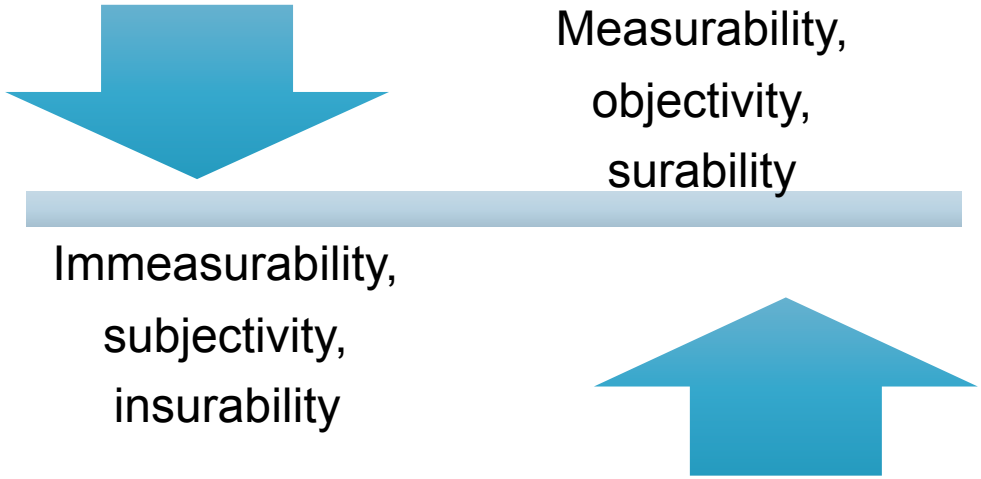
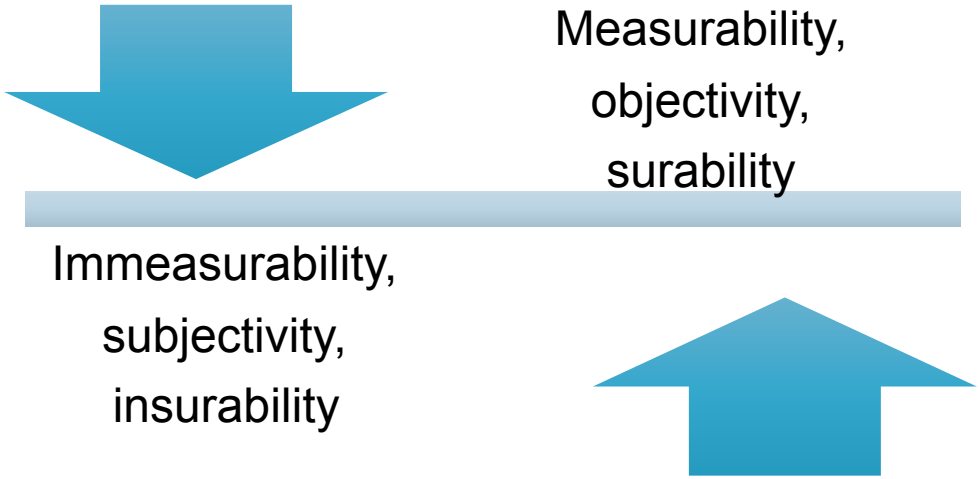
Decisions rationally bounded on classical probabilistic postulates

Insufficiency of data
Condition of independence

Decisions rationally bounded on the probabilities known


Measurability,
objectivity,
surability
Immeasurability,
subjectivity,
insurability

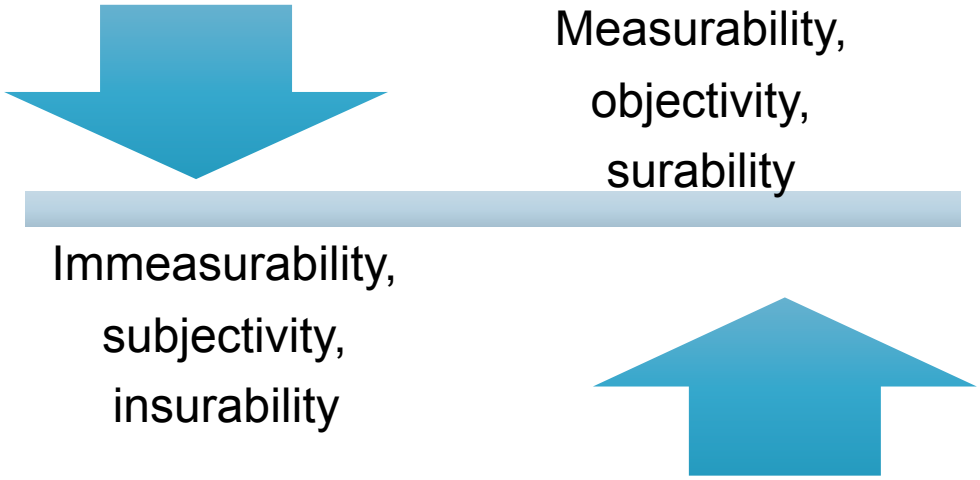


ignorance

complete and perfect information
on the uncertainty of the situation

Stochastically
Thinking ⇔ Deciding

Knight (1921)



ignorance ⟷ complete and perfect information on the uncertainty of the situation

Stochastically Thinking ⇔ Deciding

Knight (1921)

When decisions are done with a partial knowledge of the situation

Decisions under uncertainty

When all the alternative possibilities are known and the probability of the occurrence can be accurately ascertained

Decisions of risk

UNCERTAINTY

ignorance

complete and perfect information on the uncertainty

Recognising postulates

the probability of the occurrence can be accurately ascertained

PROBABILITY

UNCERTAINTY

ignorance

complete and perfect information on the uncertainty

Based on personal preference

Recognising postulates

the probability of the occurrence can be accurately ascertained

PROBABILITY

UNCERTAINTY

ignorance

complete and perfect information on the uncertainty

Based on personal preference

Rationally bounded

Under uncertainty

Of risk

Recognising postulates

the probability of the occurrence can be accurately ascertained

PROBABILITY

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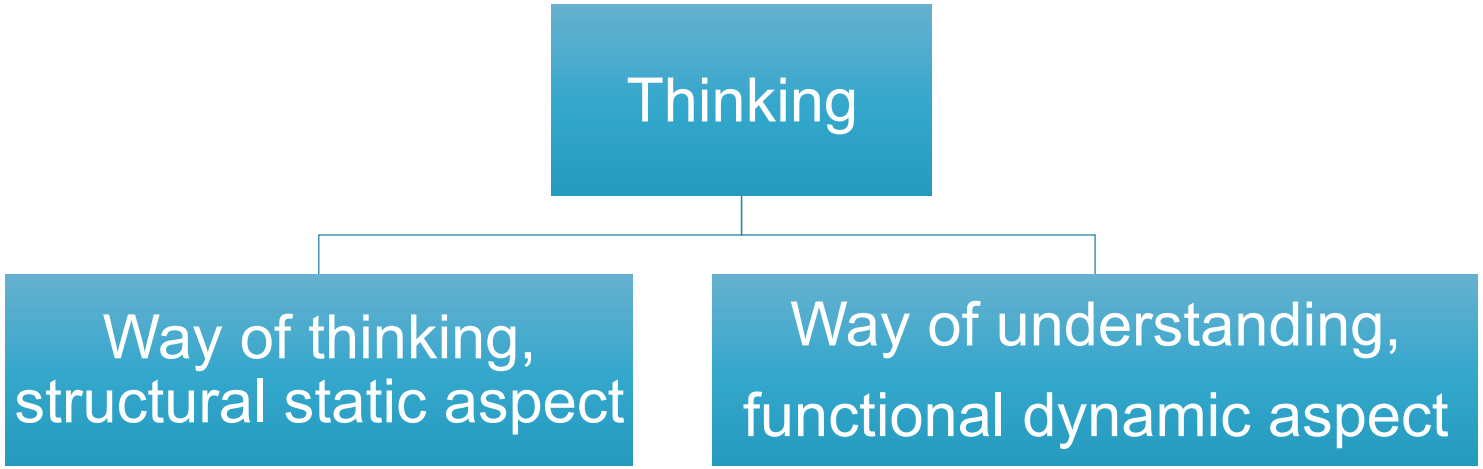
Recognising postulates

the probability of the occurrence can be accurately ascertained

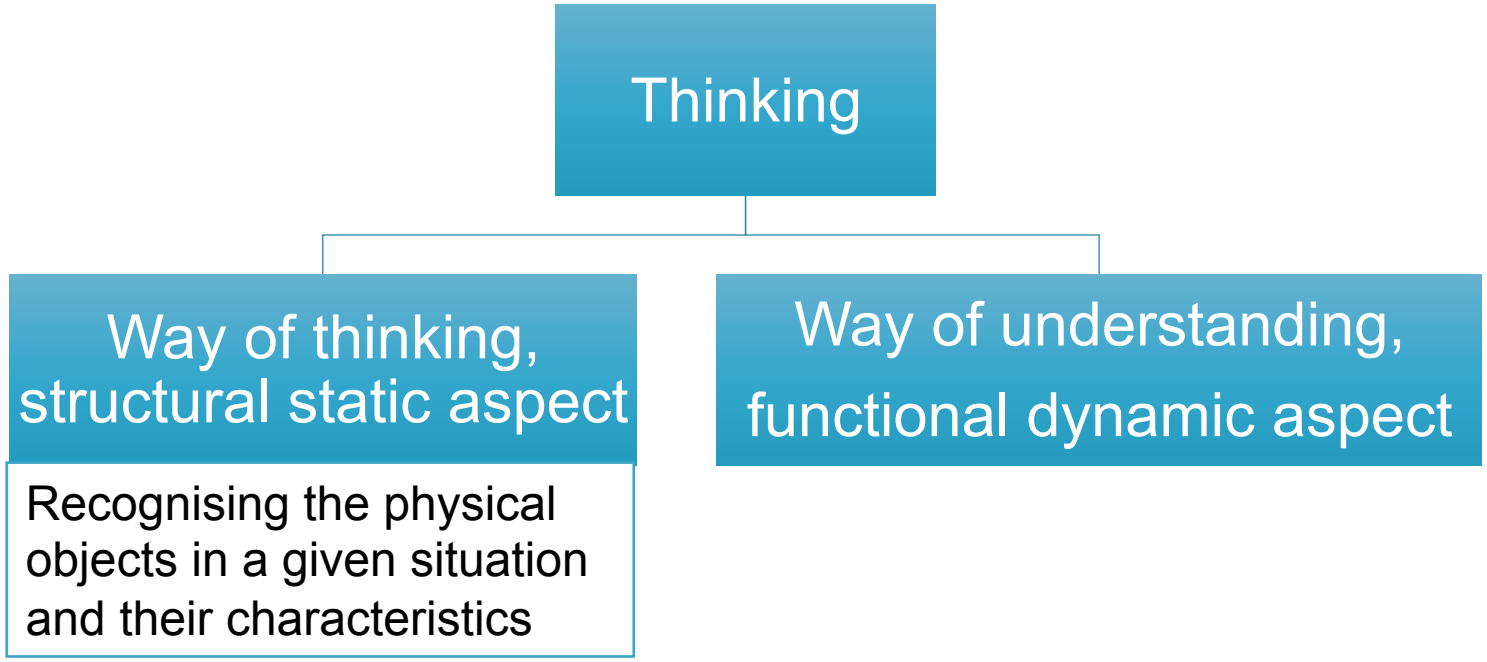
PROBABILITY

- No ascription to general mental models of reasoning in decision-making
- Situationally contextualized to a task

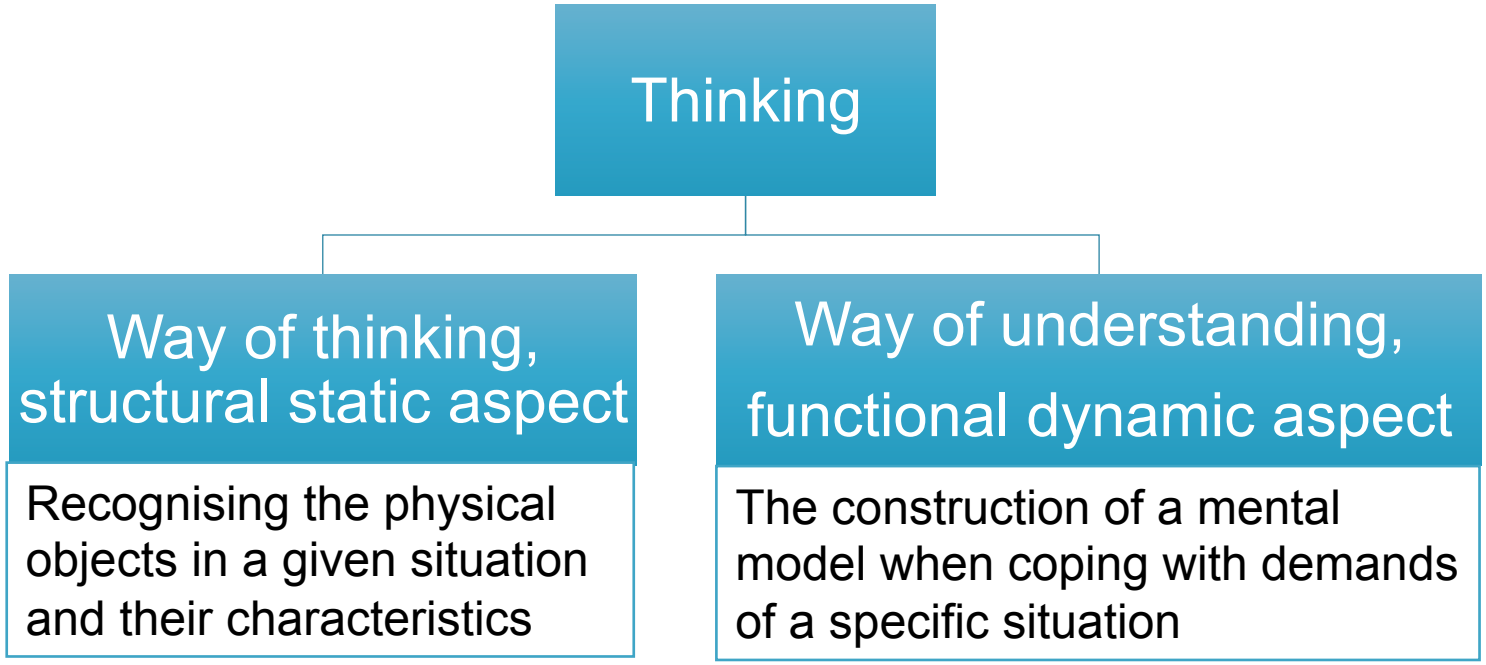
Theory of mental models



Theory of mental models



Theory of mental models



Theory of mental models

Thinking

Way of thinking, structural static aspect	Way of understanding, functional dynamic aspect
Recognising the physical objects in a given situation and their characteristics	The construction of a mental model when coping with demands of a specific situation

SOLO

Structure of Observed Learning Outcomes

Biggs and Collins (1982)

Theory of mental models

Thinking

Way of thinking, structural static aspect	Way of understanding, functional dynamic aspect
Recognising the physical objects in a given situation and their characteristics	The construction of a mental model when coping with demands of a specific situation

SOLO

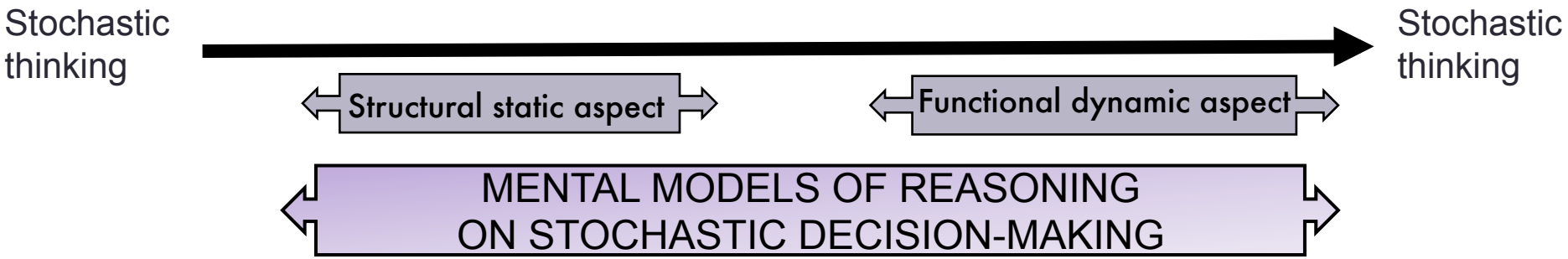
Structure of Observed Learning Outcomes

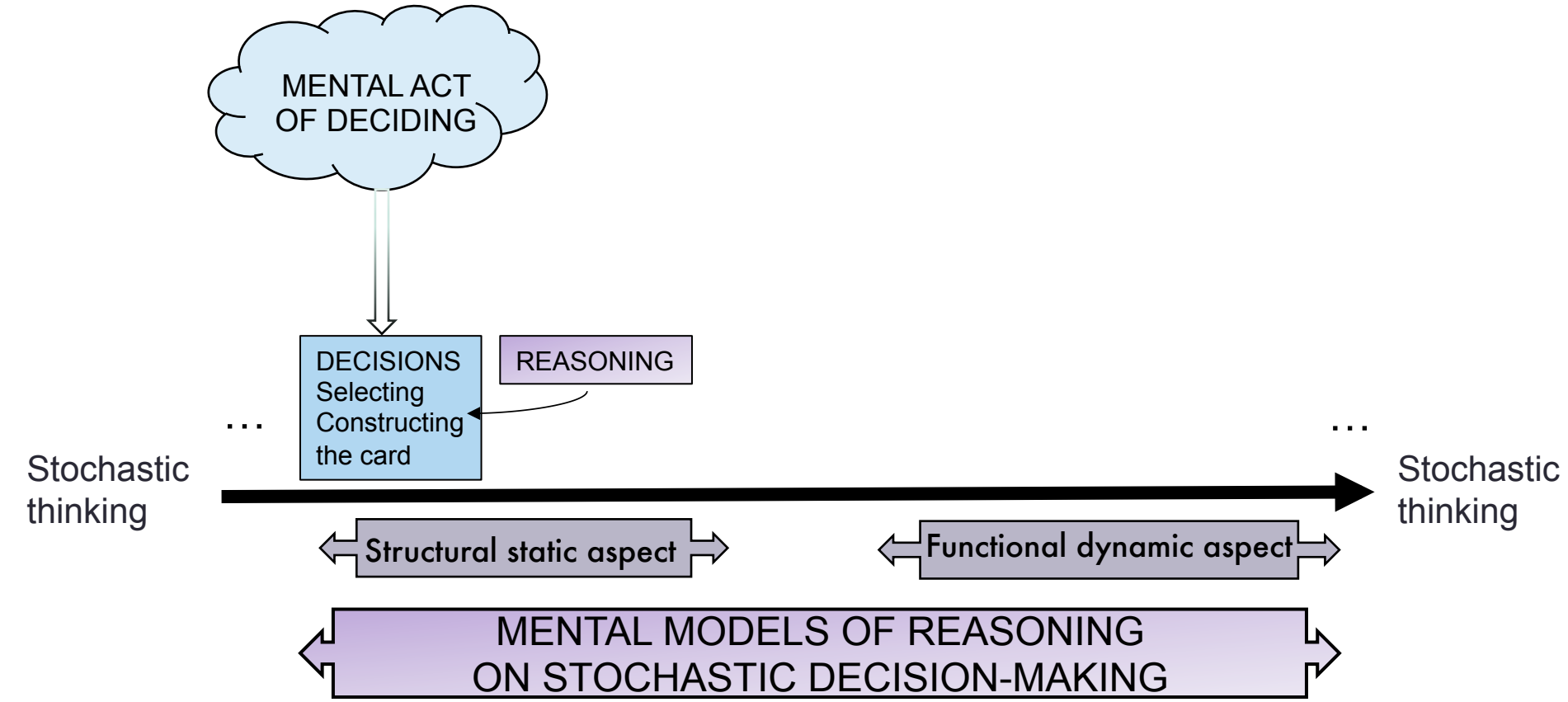
Biggs and Collins (1982)

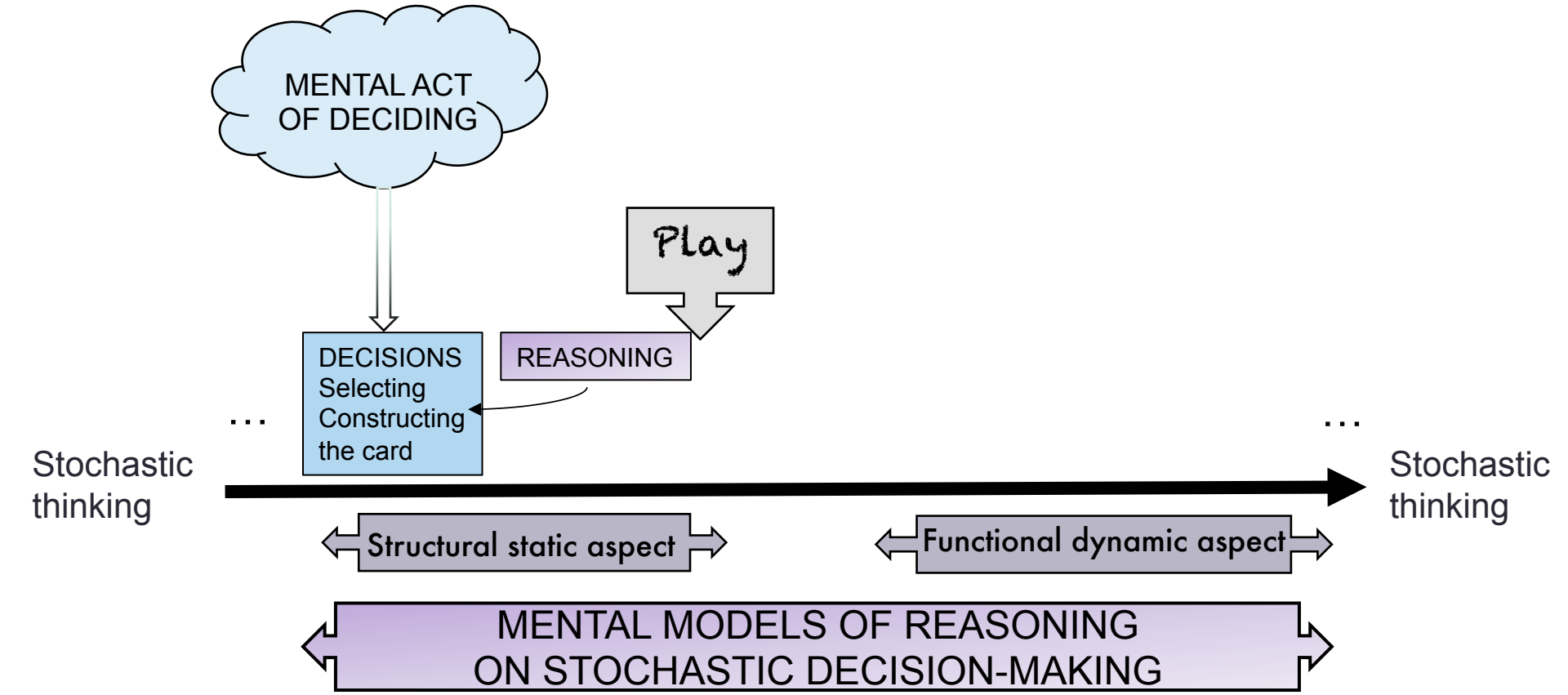
Mooney, Langrall and Hertel, (2014)

Students' probabilistic thinking moves from being idiosyncratic to proportional in nature.


Stochastic thinking
…
…
Stochastic thinking


Stochastic thinking
Stochastic thinking
Structural static aspect
Functional dynamic aspect
MENTAL MODELS OF REASONING
ON STOCHASTIC DECISION-MAKING


MENTAL ACT
OF DECIDING
DECISIONS
Selecting
Constructing
the card
REASONING
...
...
Stochastic
thinking
Stochastic
thinking
Structural static aspect
Functional dynamic aspect
MENTAL MODELS OF REASONING
ON STOCHASTIC DECISION-MAKING


MENTAL ACT
OF DECIDING
Play
DECISIONS
Selecting
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the card
REASONING
...
...
Stochastic
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Stochastic
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Structural static aspect
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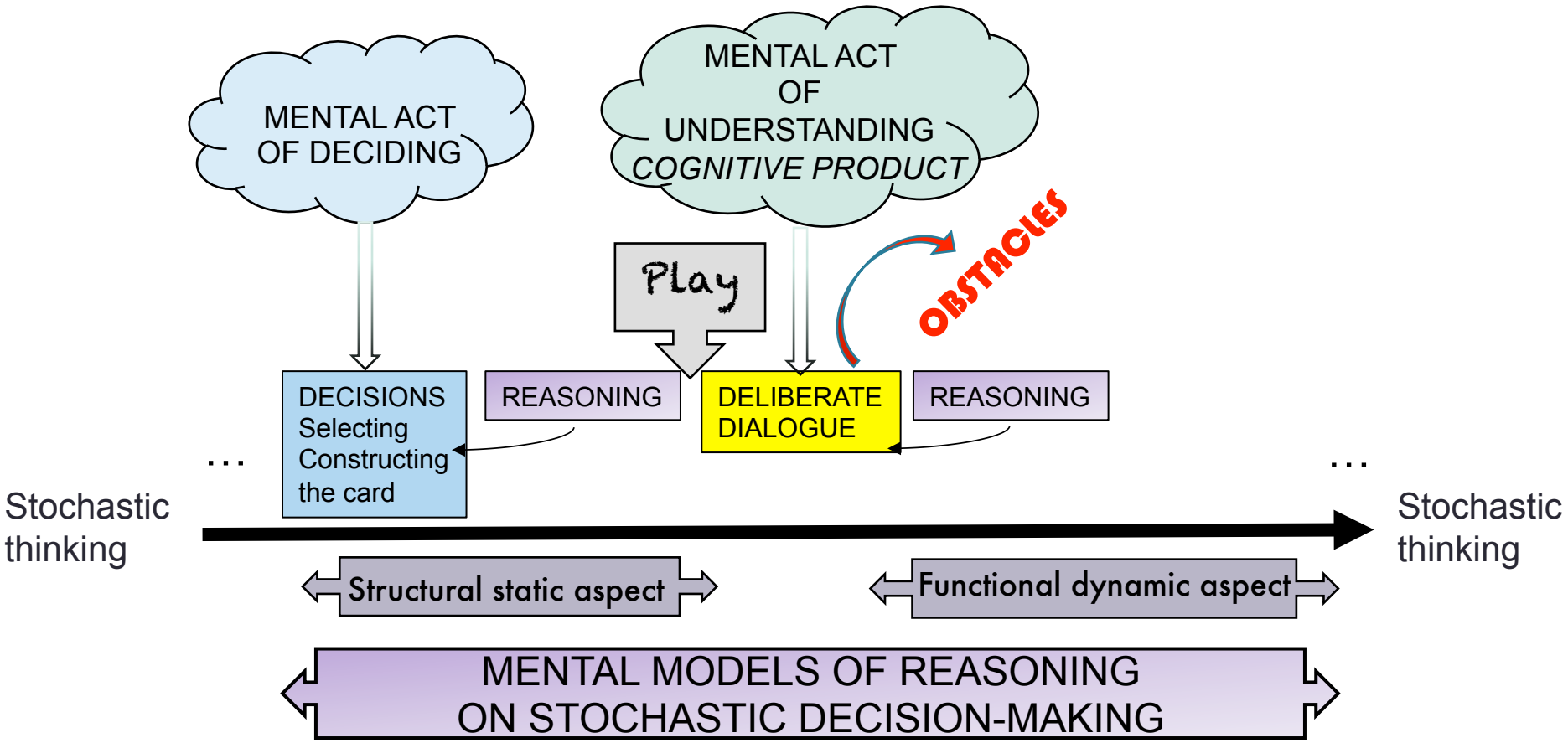
MENTAL MODELS OF REASONING
ON STOCHASTIC DECISION-MAKING


MENTAL ACT
OF DECIDING
MENTAL ACT
OF
UNDERSTANDING
COGNITIVE PRODUCT
Play
OBSTACLES
DECISIONS
Selecting
Constructing
the card
REASONING
DELIBERATE
DIALOGUE
REASONING
...
...
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MENTAL MODELS OF REASONING
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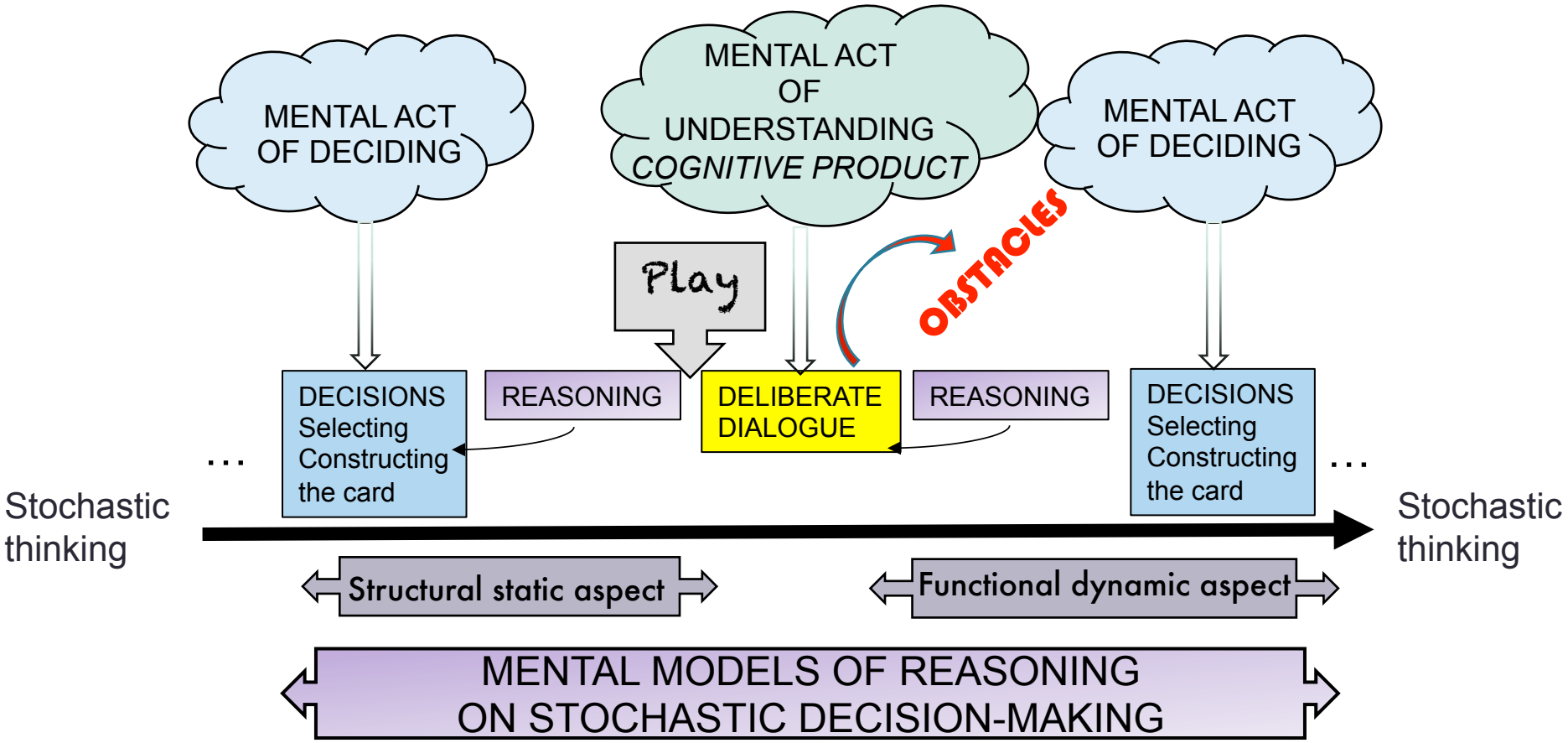
MENTAL ACT OF DECIDING
MENTAL ACT OF UNDERSTANDING COGNITIVE PRODUCT
MENTAL ACT OF DECIDING
Play
OBSTACLES
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REASONING
DELIBERATE DIALOGUE
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Stochastic thinking
Stochastic thinking
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Functional dynamic aspect
MENTAL MODELS OF REASONING ON STOCHASTIC DECISION-MAKING

Serradó (2018)

Mental levels of reasoning on decision-making situationally-provoked as a result of students engagement in IAB task

Serradó (2018)

Mental levels of reasoning on decision-making situationally-provoked as a result of students engagement in IAB task

- Based on personal preferences
- No recognition uncertainty

Pre-structural

Serradó (2018)

Mental levels of reasoning on decision-making situationally-provoked as a result of students engagement in IAB task

- Based on personal preferences
- No recognition uncertainty

Pre-structural

- Under uncertainty
- Relative frequency distribution

Uni-structural

Serradó (2018)

Mental levels of reasoning on decision-making situationally-provoked as a result of students engagement in IAB task

Pre-structural

- Based on personal preferences
- No recognition uncertainty

Uni-structural

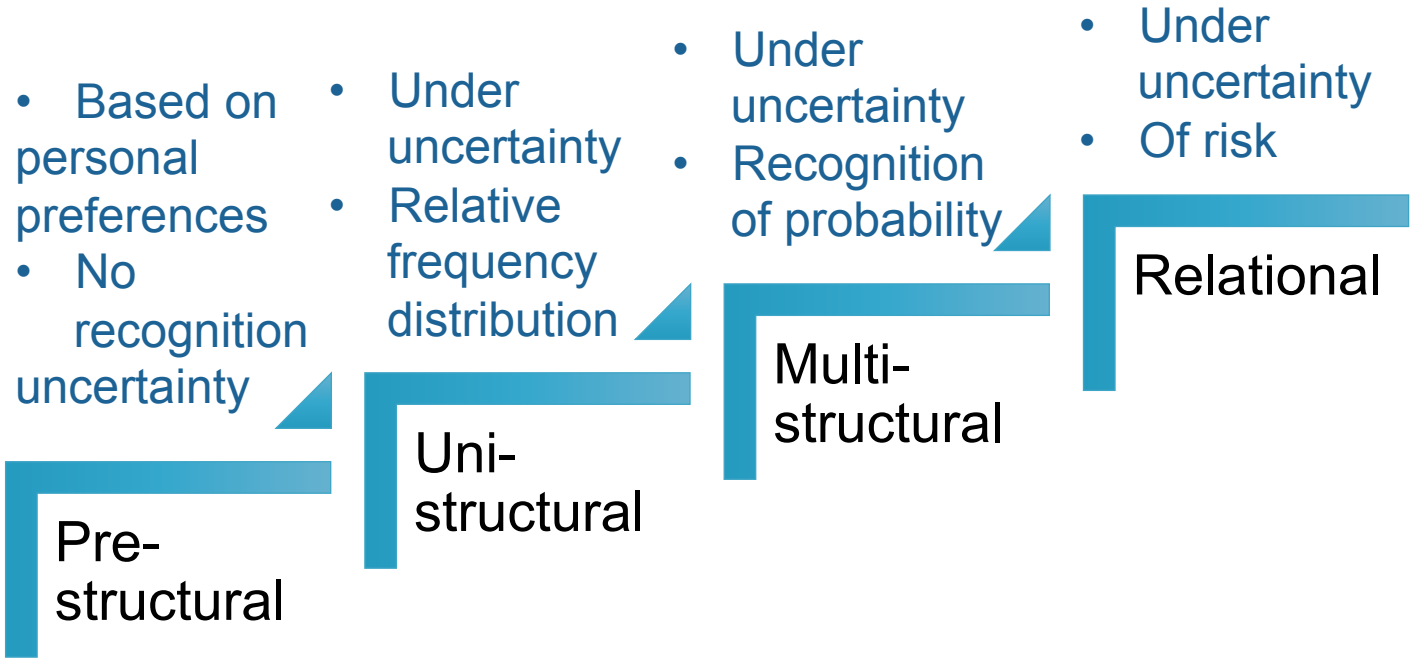
- Under uncertainty
- Relative frequency distribution

Multi-structural

- Under uncertainty
- Recognition of probability

Serradó (2018)

Mental levels of reasoning on decision-making situationally-provoked as a result of students engagement in IAB task



Serradó (2018)

Mental levels of reasoning on decision-making situationally-provoked as a result of students engagement in IAB task

- Based on personal preferences
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Pre-structural

- Under uncertainty
- Relative frequency distribution

Uni-structural

- Under uncertainty
- Recognition of probability

Multi-structural

- Under uncertainty
- Of risk

Relational

Extended abstract

- Deterministic way of thinking
- Difficulties in discerning between the randomness of the generator, the events and sequences of events
- Lack of previous knowledge about measures of centre for frequency distributions

Design based research

Design based research

2015

Task design

Implementation

48 Grade 7
(ages 12-14)

Design based research

2015

Task design

Implementation

48 Grade 7
(ages 12-14)

Retrospective analysis

- Deterministic way of thinking
- Difficulties between the randomness of the generator, the events and sequences of events
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Design based research

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Retrospective analysis

- Deterministic way of thinking
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2016

Task redesign

Questions:

- Differences: randomly generated and the random events obtained by the addition of the random-generated numbers
- Independence of events

Deliberate dialogue uncertainty of pseudo-generated numbers

Design based research

2015

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Deliberate dialogue uncertainty of pseudo-generated numbers

2018

Implementation

28 Grade 10
(ages 16-17)

Students evolution and obstacles that emerged

2015 → 2018

S	2015 evolution level	2018 evolution level	Obstacle
Ta	Uni-structural	Multi-structural	Deterministic way of thinking
A	Initiating a relational	Relational	Lack of proportional thinking
NC	Multi-structural	Initiating an extended abstract	Ignorance of the relationship between the theoretical and the frequentist model of probability

Evolution from uni-structural to multi-structural

2015

Ta

Students' evolution

2018

"The values of the relative frequency. The total of the relative frequency that is the probability, and the equal probable outcomes"

Misunderstanding of the relationship between a frequentist model and a theoretical classical model of probability

(Borovnick and Kapadia, 2014)
Insufficiency of data to measure the values of the relative frequency

Evolution from uni-structural to multi-structural

Ta Students' evolution

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Misunderstanding of the relationship between a frequentist model and a theoretical classical model of probability

(Borovnick and Kapadia, 2014)
Insufficiency of data to measure the values of the relative frequency

"Thinking on the numbers with higher probability of appearance and the ones that have repeated more before"

Analysis of sequences of 65, 55 and 41 events
Stochastic way of thinking based on short-term perception

2018

(Borovnick and Kapadia, 2014)
To understand the law of large numbers
Serradó (2019)
Experiment that is random has a unique formulation

Evolution from uni-structural to multi-structural

Ta Students' evolution

2018

130	F	I think that the game is random. Because the addition, you always know that two plus two is four. The probability is of the addition.
131	T	[Teacher interrupts the students] I have asked about the randomness and not the probability.
132	F	Because every [ball] has the same… the same… Ummm!
133	ML.	I think the addition of the [value] of the two balls.
134	T	You think that the addition is random, why?
135	ML.	Because it is what it is going to appear.
136	Ta	*Random is the card that you select, because the numbers that you write on the card are the numbers that you want to appear. And, the ones that…*

Misunderstanding of the random nature of the IAB pseudo-generator

Evolution from uni-structural to multi-structural

Ta Students' evolution

2018

"Those that I think can come out with more certainty, because they are closer to the theoretical probability. […] And, you do not have the certainty that the same number always comes out"

Certainty in short runs

Saldanha and Liu (2014):

- probability is considered a model that is chosen for a certain situation with the purpose of approximating phenomena and gaining information.
- the model is expected to be more powerful predictor of outcomes over the long run than a deterministic analysis

Deterministic way of thinking

Evolution from uni-structural to multi-structural

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Deterministic way of thinking

Deterministic nature given to the IAB pseudo-generator a possible obstacle for internalizing the relationship between events and results.

SOLO (Structure of Observed Learning Outcomes, Biggs and Collins, 1982)
Students' probabilistic thinking (Mooney et. al, 2014)

Ta evolution and obstacles that emerged

2015 2018

Pre-structural → Uni-structural → Multi-structural

SOLO (Structure of Observed Learning Outcomes, Biggs and Collins, 1982)
Students' probabilistic thinking (Mooney et. al, 2014)

Ta evolution and obstacles that emerged

2015

Pre-structural → Uni-structural

2018

Uni-structural → Multi-structural

- Recognises:
  - Frequential model
  - Classical model
- Make predictions
- Execute decisions

SOLO (Structure of Observed Learning Outcomes, Biggs and Collins, 1982)
Students' probabilistic thinking (Mooney et. al, 2014)

Ta evolution and obstacles that emerged

2015 2018

Pre-structural → Uni-structural → Multi-structural

- Decisions under uncertainty
  - Frequential model
  - Classical model
- Make predictions
- Execute decisions

- Non idiosyncratic way of thinking:
- Deterministic nature given to the IAB pseudo-random generator
- Misunderstanding randomness of long run events

Evolution from initiating a relational to relational

2015

A Students' evolution

Saldanha and Liu (2014):
Prediction of outcomes

Predictive potential of probability.
Reasoned using:

- Modal clumps
- Symmetry of the distribution of probabilities
- Density of the frequencies
- Initial understanding of the difference between probability and frequency

Evolution from initiating a relational to relational

2015

A Students' evolution

Saldanha and Liu (2014):
Prediction of outcomes

Predictive potential of probability.
Reasoned using:

- Modal clumps
- Symmetry of the distribution of probabilities
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- Initial understanding of the difference between probability and frequency

"One is what could happen and the other is what happened"

Evolution from initiating a relational to relational

A Students' evolution

2015

Saldanha and Liu (2014):
Prediction of outcomes

Predictive potential of probability.
Reasoned using:

- Modal clumps
- Symmetry of the distribution of probabilities
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- Initial understanding of the difference between probability and frequency

2018

"If I had more runs, the relative frequency would have increased and it would be the [distribution more symmetric to the theoretical probability"

Increment of the number of runs

SOLO (Structure of Observed Learning Outcomes, Biggs and Collins, 1982)
Students' probabilistic thinking (Mooney et. al, 2014)

A evolution and obstacles that emerged

2015 2018

Pre-structural → Initiated relational → Relational

SOLO (Structure of Observed Learning Outcomes, Biggs and Collins, 1982)
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A evolution and obstacles that emerged

2015 → 2018

Pre-structural → Initiated relational → Relational

Decisions of risk based on

- Predictive nature of IAB random generator
- Need of more runs to conclude about the predictive nature of the experimental frequentist model of probability

SOLO (Structure of Observed Learning Outcomes, Biggs and Collins, 1982)
Students' probabilistic thinking (Mooney et. al, 2014)

A evolution and obstacles that emerged

2015 → 2018

Pre-structural → Initiated relational → Relational

Integrate stochastic thinking

- Predictive nature of IAB random generator
- Need of more runs to conclude about the predictive nature of the experimental frequentist model of probability

Proportional thinking

Evolution from multi-structural to initiating extended abstract

2018

Proportional thinking when increasing the number of runs

271	T	Let's our mind fly! Think about what could happen if instead of 66 throws, we would have 350. I know that Fran is the winner, but that the game would have needed 350. What do you think it could have happened?
272	JR.	Would the zero have appeared more times?
273	T	That the zero could have appeared more times.
274	C	*That the relative frequency would have been smaller, because if you make a quotient with more numbers. This must be smaller.*
275	T	But, he says that it would appear more times.
276	N	*Then it would be bigger.*
277	T	Would it be bigger or not?

Students' evolution

Evolution from multi-structural to initiating extended abstract

2018

Reasoning on the law of large numbers with a frequentist approach

N: "How many more times it repeats, it will tend to stabilize more"

Card construction to minimize the risk to lose

N: "The symmetry of the theoretical probability and the modal clumps of the stabilized relative frequency"

N C Students' evolution

Evolution from multi-structural to initiating extended abstract

2018

Reasoning on the law of large numbers with a frequentist approach

N: "How many more times it repeats, it will tend to stabilize more"

Card construction to minimize the risk to lose

N: "The symmetry of the theoretical probability and the modal clumps of the stabilized relative frequency"

Approximation to the theoretical classical probability

C: "When it starts to be very stabilized, the difference will be less appreciated, but it sill continue to stabilize. The relative frequency will never be equal to the theoretical probability, but it stabilizes the values"

Card construction to minimize the risk to lose

C: "I will use numbers close to the median [of the stabilized relative frequency distribution], because I do not know exactly the values that are going to appear"

N C Students' evolution

SOLO (Structure of Observed Learning Outcomes, Biggs and Collins, 1982)
Students' probabilistic thinking (Mooney et. al, 2014)

A evolution and obstacles that emerged

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Pre-structural → Multi-structural → Initiation extended abstract

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A evolution and obstacles that emerged

2015

Pre-structural → Multi-structural

2018

→ Initiation extended abstract

Decisions based on the evaluation of the probabilities on hand.
First insights about the relationship between the stabilized relative frequencies distribution and the theoretical classical distribution.

SOLO (Structure of Observed Learning Outcomes, Biggs and Collins, 1982)
Students' probabilistic thinking (Mooney et. al, 2014)

A evolution and obstacles that emerged

2015

Pre-structural → Multi-structural

2018

Initiation extended abstract

Decisions based on the evaluation of the probabilities on hand.
First insights about the relationship between the stabilized relative frequencies distribution and the theoretical classical distribution.

How simulation of sequences of events using the theoretical model lead to evaluate more cases and made, consequently, decisions of risk

Conclusions

- From a theoretical point of view:
  - Insights about the parallel evolution of theory of decision making and probability
  - Theoretical framework:
    - Decisions based on personal preferences
    - Decisions rationally bounded:
      - In the uncertainty of the situation
      - Of risk

Conclusions. Evolution of dedisions

Pre-structural
Personal preference

→ Uni-structural
Rationally bound
uncertainty
EFM distribution

→ Multi-structural
Rationally bound
Uncertainty
EFM distribution or
CTM distribution

Deterministic thinking

Pre-structural
Personal preference

→ Initiating a relational level
Decisions based on predictive
nature of probability

→ Relational
Decisions of risk
No connection
EFM and CTM

Lack of Proportional thinking

Pre-structural
Personal preference

→ Multi-structural
Rationally bound
Uncertainty
EFM distribution
or CTM distribution

→ Initiation extended abstract
Decisions of risk
Insights of law of large
numbers

EFM⇔CTM

Conclusions

Limitations:

- reasoning on decision making is situationally provoked
  - IAB of chance selected
  - The task implemented
- The design based research methodology used

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Decision Making Based on Data

13 – 16 August 2019, Kuala Lumpur, Malaysia

Thank you!

Obstacles in the evolution of secondary school students' mental models of reasoning on decision making

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