Teaching Statistics in Schools throughout the World

PART 1
Europe

CHAPTER 1

Statistical Education in Schools in England and Wales

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This chapter describes the present situation, as well as the background development and current trends, in statistical education at school level in England and Wales. The situations in Scotland, and in Northern Ireland, are somewhat different as a result of separate and distinct styles of structure and administration. A brief review of the Scottish scene is given in the Supplement below.

Before discussing specifically the national position regarding the teaching of statistics it is necessary to spell out a number of general matters about the pattern of school-level education in England and Wales, its administration, the process of curriculum revision and the style and implications of public examination structure. These aspects are dealt with in section 1.1 prior to the main discussion of *statistical* education, *per se*.

1.1 INTRODUCTION: SCHOOL-LEVEL EDUCATION IN ENGLAND AND WALES

School education is compulsory in England and Wales for children of ages 5 to 16, inclusive. In practice, the majority of children enter full-time schooling (not nursery or kindergarten) *before* their 5th birthday, and about 35% of the age groups are still continuing school education at ages 17 and 18 on a voluntary basis studying for higher level examinations or following technical/commercial courses. A further 10% of each age group are continuing in the same vein at Colleges of Further Education. In the age range 18–20, about 16% are in full-time education: 2% in schools, 8% in Further Education Colleges (including Polytechnics) and 6% in Universities. The latter two groups are involved in Higher Education working (predominantly) for a degree, or for professional qualifications.

1.1.1 *Different categories of school*

The division of the schools into different categories over the age range 5–16 years is predominantly as shown in Table 1.1

Although the overall responsibility for school provision resides with the Department of Education and Science (DES) of the UK Government, the DES plays only a limited role at the local level on such matters as administration, finance, organisation, curriculum, etc. The provision in each locality is

Ages	Type of school	
5–8	Infants	Primary
8–11	Junior	
11–16		Secondary

Table 1.1 Categories of school.

under the control of a Local Education Authority (LEA) which finances the schools from central government funds augmented by local income levied as rates on domestic and industrial property. The LEA is responsible for the state schools in its area, and local variations occur in many respects. Some LEA's adopt (or allow) a different categorisation to that shown in Table 1.1 either for all, or for some, of the schools under their jurisdiction. Two noteworthy distinctions are

(i) transference to *secondary* schools at age 12 rather than 11.
(ii) *First* school : 5–9 years
Middle school : 9–12 years
Upper school : 12–16 years

But a large majority of state schools fall in the divisions shown in Table 1.1.

Over the last 15 years there has been a dramatic change of emphasis at the *secondary* level. There *had for some time been* a policy of directing 11 year olds either to Secondary Modern Schools or to Grammar Schools on the basis of their performance on the '11+ examination': the Grammar Schools were predominantly 'streamed' and directed their teaching to the attainment of good examination performance and progression (for many in the upper streams at least) after two years in the *sixth form* (16–18 year olds) to university, teacher training colleges, etc. The Secondary Modern Schools provided a less academic environment and pupils left at age 15 or 16 years.

Dissatisfaction with the early categorisation implied by the 11+ examination, and a desire for rationalisation in use of resources, resulted in discontinuation of the 11+ examination, and a single style of *'comprehensive'* secondary school which would cater for all abilities and interests with ease of transference within the school to suit the individual's needs. Notwithstanding differing political and educational attitudes, most secondary school pupils now attend non-fee-paying state comprehensive schools. Some of the grammar schools not totally funded by LEA's left the state system and joined the independent schools who were not prepared to offer comprehensive facilities and have become independent, fee-paying schools. About 8% of children attend such schools. Some LEA's were not prepared to reorganise on comprehensive lines, and in these areas the selective system remains as the local pattern for *state* secondary schools. Roughly 74% of pupils are in state comprehensive schools, whilst the remaining 18% are in state selective schools.

Thus, in the main, secondary school education is on a non-selective comprehensive basis, but the freedom of action of LEA's and of Head-Teachers in the organisation of their schools has lead to a variety of manifestations of

the comprehensive idiom. 'Streaming', 'banding', 'setting' of pupils takes place in different schools, whilst some maintain a 'mixed-ability' environment at least for the first three years. Resulting patterns undoubtedly reflect, to a degree, political and sociological principles as well as educational considerations.

There seems to be much less variation in the pattern and aim of teaching and school structure at the primary level.

1.1.2 *Syllabus and curriculum*

So far there has been no central (governmental) direction of the curriculum. In sharp contrast with many other countries, the choice of the material to be taught and its manner of presentation rests with the individual head-teacher and his/her staff. But the constraints of well-tried and accepted series of text-books, of public-examination syllabuses at the secondary level (see below, § 1.1.3) and the native good-will and good sense of most teachers militate against an arbitrary chaotic mix of material and method. The freedom does seem to be responsibly exercised and it does lay open the prospect of regular and thorough assessment and redirection. Curriculum reform within a school is *potentially* an easy prospect, whether it springs from individual initiatives within the school or from external agencies using the good offices of selected schools for development, assessment and validation. What is less easy, and we shall see this as particularly important for essentially radical innovation such as in Statistics, is the promotion of *nation-wide* curriculum change and the obtaining of adequate financial support for major programmes of curriculum reform.

The syllabus in the *primary schools* emphasises critical reading and creative writing (including spelling), early ideas in science and society and, of course, a sound basis in number work with encouragingly little inhibition about learning of tables. The arithmetic emphasises learning by play and discovery and often introduces some good practical problem environments for numerical calculation to appear relevant and meaningful. It is relatively free of the more formal excesses of 'modern mathematics'. Simple data collection, presentation and (rudimentary) interpretation is often found; the electronic calculator is usually kept safely at bay. Elementary geometry, through the study of shapes, is also introduced.

But the success of number-work tuition depends much on the ability of the individual teacher. The current concern about the poor level of numeracy of school leavers (taken up in more detail below) stretches back to the primary schools in one respect. It is only within the last year or so that primary school teachers have been required to possess even a rudimentary qualification (an O-level pass – see § 1.1.3) in mathematics.

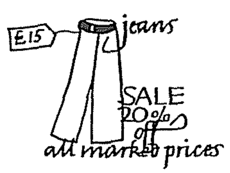
At the *secondary level* the syllabus is much broader. Mathematics and English language are compulsory throughout; in the first year or so the mix will also contain a foreign language (typically French), history, geography, physics, chemistry (perhaps biology), craft subjects, physical education and religious studies. With progression through the school some changes occur: another modern language, science, and a classical language may be introduced for the more able; additional craft subjects and social studies may replace some of the more formal subjects for the less able. In the final year of compulsory education (age 15 to 16) pupils will take up to about 10 public examinations at an appropriate level.

Those who continue their education beyond age 16 predominantly enter the school sixth form (or transfer to a sixth-form college, if this is the pattern in the local area) and do two-years study in a much more limited range of subjects at a higher level. They may alternatively attend a College of Further Education. Typically three subjects in cognate areas will be studied and appropriate higher level public examinations taken either as a terminal qualification or as preparation for higher education (teacher training, polytechnic or university). Subjects such as economics, computing, statistics may be studied in the sixth-form, without previous experience.

With recent changes in employment prospects and social attitudes a small, but growing, proportion of less able pupils are continuing beyond age 16 and studying technical or craft subjects, or perhaps trying to improve on their earlier examination performances.

There are two distinct styles of mathematics syllabus throughout the secondary school, either *traditional* or *modern* with roughly equal numbers of pupils being taught on each style. The modern syllabuses started to appear about 17 years ago with the aim of replacing what was seen as rather dull, rote, impractical material with more practically relevant and exciting topics. In the event much of the modern syllabus material has proved to be over-formal, highly concept-oriented and inapplicable (or at least unapplied). The traditional syllabuses seem to give greater emphasis on mathematical manipulation. There are moves in the examining boards (see § 1.1.3) and relevant national committees to harmonise the modern and traditional approaches.

Leaving aside philosophical attitudes to the different styles of syllabus, there can be little doubt about the justice of recently expressed concern over the poor level of numeracy of many 16-year-old school leavers. The Institute of Mathematics and its Applications conducted a survey where an elementary test of numeracy was administered (see Clarke, 1978). Pass levels of about 50% were found, in place of the 90% anticipated, on such questions as this.



A reduction of 20% is given on sale items. What is the sale price of a pair of jeans marked at £15?

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A government enquiry into the Teaching of Mathematics in Schools was recently carried out under the chairmanship of Dr. W.H. Cockcroft; its interesting conclusions (with comments on the teaching of statistics as well as mathematics) have recently been published (Cockcroft, 1982).

1.1.3 *Public examinations*

Reference has been made to different levels of public examination. At present there are three different levels; papers are set in each subject, at each level, on a range of syllabuses.

There is the General Certificate of Education at Ordinary level (GCE O-level) for (typically) the 16 year old, and at Advanced level (GCE A-level) for those who have undertaken two years further study. The GCE is administered by GCE Boards of which there are 8 in number distributed throughout England and Wales. For each Board and each subject there will be examinations set on a variety of well-discussed and publicised syllabuses. Great efforts are made to ensure comparability of standards. Any school thus has a wide choice of examination syllabuses to which it may work. The GCE has been in existence for about 25 years.

About 15 years ago the Certificate of Secondary Education (CSE) was introduced, administered by a separate network of regional examining boards. Designed for the less-able 16-year-old school leaver its top pass category (grade 1) is equivalent to the third level of pass (grade C) of the GCE O-level. The aim is to fix the grade 3 CSE pass at about the 50 percentile of the ability range of the school leaver. Whilst most candidates for the CSE enter under an externally set and marked examination paper system similar to that of the GCE, there is an alternative system (Mode 3) which is interesting, and appeals to some schools and subjects. Under Mode 3, the school determines the syllabus and assesses pupils on a continuous assessment basis, with external moderation by the CSE board. A small number of schools operate under Mode 2: externally set examination papers on internally constructed syllabuses.

Discussions, and trials, of alternative examination schemes are currently in progress. These include

(i) provision for studying 5 rather than 3 subjects in the first year of the sixth form, and
(ii) O-levels awarded on the basis of continuous assessment ('the 16+ examination').

But for the near future at least it appears that the CSE/GCE structure will remain.

Obviously the public examination syllabuses have a profound influence on the contents, timing and emphasis of school courses. In principle the schools, and others, can propose syllabuses for examination – especially for CSE and GCE O-level. But this is a time-consuming exercise and the common practice is to choose from the menu offered by the Boards. The GCE boards regularly reassess their syllabuses, with advice from schools, professional bodies etc., and frequently initiate deletion, revision or replacement as appropriate.

1.1.4 *Teacher training*

There are two routes to qualification as a teacher in England and Wales. Applicants with two A-level passes can obtain a place in a College of Education on a three-year course of training as a teacher. On successful completion of the course, certification as a qualified teacher is conferred by the DES. Those with appropriate ability and interest can obtain an ordinary degree (B.Ed.) or, after a fourth year's study an honours degree (B.Ed.) validated by a University. Alternatively, those with an honours degree in some specific subject may register for a years postgraduate course with the Education department of a University (or at some Colleges of Education) and obtain certification in this way. Whatever the method of training, the teacher is only formally recognised as a 'qualified teacher' by the DES after a successful probationary year of teaching.

It is only for the last 13 years or so that college-trained (as opposed to degree-based) teachers have had to spend three years in training. Previously the course of training lasted only two years.

Recently, falling school numbers and government restrictions on expenditure have substantially reduced the numbers of teachers in training. Many Colleges of Education have closed; except in one or two specific fields (e.g. mathematics, modern languages) newly qualified teachers find it difficult to obtain posts and large numbers have to seek employment outside the teaching profession. The average numbers of pupils per teacher are 24 in primary schools and 17 in secondary schools, although the widening social role of schools places additional burdens on teachers, and class sizes tend to be around 25–30.

1.2 STATISTICS IN THE PRESENT SCHOOL CURRICULUM

Some statistical and probabilistic material has been in the school curriculum for about the last 20 years. This ranges from the introduction of, and practice in, simple date collection exercises for 8–10 year-olds in the primary schools to rather formal probability theory and statistical inference and methods in the secondary school sixth-form. In the secondary schools, statistics is usually taught by the mathematics teacher as part of the mathematics programme. There are *some* GCE, O-level and A-level, syllabuses in Statistics as a full subject; on the other hand *almost all* O-level and A-level mathematics syllabuses contain some material described as 'statistics' at least as an option, but usually as an integral component. A recent survey (Rubra, Ed., 1976a) showed that over 80% of secondary schools claimed to be teaching some statistics, but only 3% of pupils are entered for CSE or O-level examinations specifically in statistics.

Over the last decade revision, or introduction, of science and social science syllabuses at O- and A-level has lead to the wide-spread incorporation of statistical ideas and methods. This is particularly true of biology, economics and geography. In the 1976 survey it appeared that about 25% of schools were teaching some statistics in geography courses, about 20% in biology courses, but these proportions are likely to be increasing.

In discussing the statistics in O- and A-level syllabuses (§ 1.2.2) it will be necessary to distinguish between Mathematics (and/or Statistics) syllabuses, and syllabuses in applied disciplines.

Important detailed information on the present statistics content of school courses is given in various reports of the Schools Council Project on Statistical Education, notably:

Holmes (1976a), *Probability and Statistics in the Primary School*
Holmes (1976d), *Statistics in GCE O-level Syllabuses*
Kapadia (1976c), *Statistics in CSE Syllabuses*

Other reports in the same series cover statistics in mathematics courses (Kapadia, 1976a), in humanities courses (Rubra and Kapadia, 1976), in science courses (Holmes, 1976b); references for teaching statistics (Holmes, 1976c); equipment, the media and other resources (Rubra, 1976b) and in-service and other courses for statistics teaching (Kapadia, 1976b).

Revised and edited versions of these various reports have been published jointly under the title *Statistics in Schools, 11–16: A Review* (Holmes *et al*, 1981.

Another detailed source of information and comment is the recent report of the Centre for Statistical Education (Barnett, *et al*, 1979a) entitled *Statistical Education and Training for 16–19 Year Olds.* Although partly outside the age range considered in this chapter paper, it contains much relevant material including chapters on statistics at O- and A-level, publications, teacher-training, employment, and curriculum development with discussion of the difficulties of teaching statistics, and recommendations and conclusions.

The Schools Council reports, and the report of the Centre for Statistical Education, are drawn on widely in the material of the following sections. Indeed, some of this material consists of direct quotation (with permission) from the report of the Centre for Statistical Education (Barnett, *et al*, 1979a).

1.2.1 *Statistics in primary schools*

The emphasis in the mathematics taught in primary schools is on the notions of measurement, counting and the representation of numerical conclusions in practical problems. This inevitably involves contact with simple statistical ideas including the collection of data, its summarisation and representation by graphical or pictorial method. Whilst there is undoubtedly a fair amount of basic statistical material (and some elementary probability ideas) taught in the primary schools the coverage is by no means uniform in respect of topics covered or time spent on the statistical material. Differences reflect the interests of individual teachers, the extent to which they have developed their own material and the text-books and other aids employed.

The major influences which produced the present concern for the *use* of number arose in the 1960's, notably in the work of Miss E.E. Biggs, one of Her Majesty's Inspectors, who was 'commissioned to mobilise forces to help spread and consolidate liberal ideas on the learning of mathematics by primary school children' (Holmes, 1976a), and of the Nuffield Mathematics Project which set up *teachers centres* and local discussion groups, and produced the Nuffield Guides (1967) for teachers.

The influences are highlighted in the survey for the Schools Council Primary Mathematics Project (see Ward, 1979) and the materials found to be currently in quite wide-spread use reflect the style and content of the prevailing statistical material.

Most statistical ideas involve the pupils in collecting their own data in situations relevant to their environment: for example, measurement and plotting of lengths of shadows introduces ideas of graphical representation and simple extrapolation. Sharing of sweets, or comparing total weights of groups of four children, involve notions of average and aggregation. Quadrat sampling of weeds in a lawn and rough prediction of total numbers of weeds, or sampling of different species in a pond, illustrate biological applications. Traffic censuses, and shopping surveys, are used as sources of data. Attempts are made at teaching proper interpretation of statements about data. Sometimes these may be over-ambitious; as for example in asking for comment on the statement:

'There have been 50 fewer car accidents in Burslem than in Newcastle during the last 5 years, so it is safer to drive in Burslem.'

Much of the statistical work involves constructing frequency distributions and representing them using concepts such as bar charts, pie charts etc. Informal inference from sample data is sometimes covered, but often the work stops at the representation stage. There is little attempt to explain sampling methods such as random sampling.

Far less work on probability is encountered. That which occurs is usually based on games using dice, spinners etc; often these are deliberately biased. Stabilisation of relative frequencies is sometimes illustrated. Some of the influences on the probability teaching came from BBC broadcast material (e.g. *BBC Mathematics Workshop*, with associated workcards) and from abroad (e.g. the games of Tamás Varga from Hungary and the booklet *Look on Luck* by IOWA, 1974, in Holland). Unfortunately, little effort is made to *link* statistical and probabilistic material.

1.2.2 *Statistics in secondary schools*

There is much more to report on the statistics content of secondary school courses. Some statistics is taught 'for interest' in courses (in mathematics, predominantly) which are not aimed at public examinations, but the vast majority is examination material at different levels, and under different subject designations (e.g. mathematics/statistics, geography, economics). To review the present situation, and its development, it is convenient to divide the coverage according to the double classification of level and subject. Thus we consider separately the statistics in mathematics/statistics courses and in courses in user subjects, at each of the levels CSE, O-level, A-level.

Statistical material has been present in the examination syllabuses from their outset; that is to say for about 25 years for O- and A-level and about 12 years for CSE. The first substantial component at A-level seems to have appeared in 1961 with the introduction by the Joint Matriculation Board (JMB) of the Pure Mathematics with Statistics syllabus in which each of the subjects occupied half of the study time. Dual O-levels (and other A-levels) followed, as did the gradual trend towards the incorporation of some statistics in almost all mathematics syllabuses. Statistics in user-subject syllabuses at

O- and A-level followed in the late 60's and has been proliferating in extent and coverage in recent years.

It is ironical, and by no means untypical of other syllabuses, that the pressures for the introduction of the JMB Pure Mathematics with Statistics arose from sources concerned that social-science, and science, oriented pupils not destined to study mathematics *per se* should have a grounding in useful statistical methods, *whilst in contrast with this stimulus the resulting syllabus had a very formal mathematical style and proved very difficult to candidates.*

CSE syllabuses. The present situation is reviewed by Kapadia (1976c).

Mode 1 examinations are set by the examining boards on their own syllabuses. There are 34 syllabuses in mathematics offered by the 14 boards all of which contain at least a small number of compulsory statistics topics (the 'modern' syllabuses also contain some probability). There are *two* syllabuses in *statistics* alone. Each of the following topics is included in the majority of the syllabuses:

- data collection and tabulation,
- pictograms, pie charts, bar charts,
- histograms, frequency polygons, cumulative frequency curves, quartiles,
- averages (mean, median, mode),
- probability.

The treatment of the material involves much data handling but is not really very statistical – the emphasis is on deterministic processing of the data with little concern for the notions of random variation or statistical reasoning. The coverage of probability tends to be cursory with little or no contact with the descriptive statistics.

The two mode 1 syllabuses in statistics *per se* include more topics but still stop short of any serious motivation or explanation of the basic nature of (informal) statistical inference. This is regrettable, but not inevitable! Such ideas can be successfully handled with properly structured material and sympathetic, empirically-oriented, teaching (see § 1.3.1).

The mode 3 syllabuses (constructed and administered by individual schools) offer scope for a wider, more *statistical*, coverage of statistics in that they allow the enthusiastic and knowledgeable teacher to choose his own material and method of presentation. Continuous assessment more easily accommodates project work, with the motivation and reinforcement this provides. There is not too much evidence that the mode 3 syllabuses at present in operation exploit these potential advantages although isolated exceptions are encountered. They tend too often to echo the mode 1 style, but we might hope that they will begin to be influenced by the material of relevant curriculum studies such as that of the Schools Council Project on Statistical Education (see § 1.3.1).

There is little statistical material at present in non-mathematical CSE syllabuses beyond general prescriptions (e.g. in geography or social/environmental studies) to include material which will 'encourage pupils to develop the skills of using simple statistics'.

O-level syllabuses. Detailed studies appear in Holmes (1976d) and Chapter 2 of Barnett *et al* (1979a).

Mathematics/Statistics syllabuses. Most O-level syllabuses in mathematics contain some statistical material. Those which have been in existence for a long time often only include some general item such as 'graphical representation of data', but about 20 syllabuses include much more substantial components. There are *three* syllabuses specifically in statistics offered by the Associated Examining Board, the Southern Universities Joint Board and the Welsh Joint Education Committee. Some syllabuses, such as the JMB Additional Mathematics (Pure Mathematics with Statistics), contain a 50% component on statistics. But this is at the so-called AO-level ('alternative ordinary' level), intended to provide additional syllabus material for pupils wishing to extend their O-level mathematics studies to further material *at the same level.* In practice, however, the level seems to be rather higher: intermediate between O- and A-level. There are three AO syllabuses in statistics alone, and two which contain 50% statistics.

It is difficult to review the statistics content of O-level syllabuses. The formal description of the topics included in the syllabus gives little indication of the required (or prevailing) emphasis in teaching those topics. Descriptions of aims, or notes for teachers, published along with syllabuses give some impression of what is required; examination papers give the clearest view of what the Board (or at least its examiner for the time being) is seeking by way of knowledge and understanding. So much will depend, however, on how the teacher interprets the syllabus. This reflects, in turn, his or her own understanding of the subject which is, sadly, often rather limited.

Holmes (1976d) indicates the topics included in the three statistics O-level syllabuses, the twenty or so O-level mathematics syllabuses which contain a substantial statistics component and the five AO-level syllabuses described above. There is a comprehensive syllabus review with about 100 syllabus items identified. The emphasis is seen to lie very much on descriptive statistics, involving more arithmetic manipulation than in the CSE syllabuses. There is relatively little attempt to cover general aspects of the origins and uses of statistical data, statistical reasoning in relation to specific practical problems, informal inference and interpretation of data. Few syllabuses contain much probability beyond simple definition and manipulation aspects (addition and multiplication of probabilities for mutually exclusive and independent events, respectively, but without the notions of mutual exclusion or independence being explicitly explained or illustrated). There is little discussion of distributions (and virtually no notion of their modelling role) or of statistical principles or techniques.

The syllabuses that include the most substantial probabilistic or statistical content are the three specific statistics syllabuses which include conditional probability, correlation and linear regression (but no study of probability distributions or sampling distributions, nor estimation or testing!), and those mathematics syllabuses where the board offers 'additional material' to that included in some other basic syllabus. Here we might encounter expectation, permutations and combinations, linear transformations of random variables, binomial, Poisson and normal distributions and, very occasionally, significance tests and confidence intervals, maximum likelihood estimation, bivariate distributions and the t- and χ^2-tests.

Thus the predominant (and often exclusive) 'statistical' elements of O-level mathematics syllabuses consist of a large component of (largely deterministic) descriptive statistics followed by some introductory material on probability. There is little reference to the mis-use of statistical data. Not surprisingly, the three full statistics syllabuses go furthest in the direction of application of statistical ideas and interpretation of data.

'It is noticeable that these syllabuses do refer more to the problems of collecting and classifying data, to the use of statistics in social contexts such as index numbers and birth- and death-rates, and to generally used statistical ideas such as regression and correlation, as well as to the usual statistical techniques involved in calculations. Although there is no specific reference to inference, phrases on question papers such as "Explain why the graph has this form", "When, approximately, do you think the employees wages were increased?" do show that pupils are intended to think critically about the data and make simple inferences. It is disappointing that such phrases don't occur more often.' Holmes (1976d).

There is only one mathematics/statistics O-level syllabus (in fact an AO-level syllabus) which contains assessed practical project work. Illustrations of O-level statistics syllabuses are given in Appendix I which presents the contents of the O-level syllabuses 063 (Statistics) and 185 (Additional Statistics) of the Associated Examining Board.

User Subjects. A large number of O-level syllabuses in non-mathematical subjects include mention of general aspects of data collection and interpretation, but often with little indication of just what material is to be taught to develop such expertise. At the other extreme there is sometimes mention of some specific statistical technique (such as correlation, regression, χ^2-tests) which is too sophisticated and lacking any supporting foundation in terms of prerequisite knowledge of statistical concepts and principles. The range of subjects in which statistical ideas are required, or are implicit in project work, is too large to detail, but it includes biology; commerce; economics; geography; general science; government, economics and commerce; history; religious studies; sociology; use of English.

A-level syllabuses.* (Fuller details are given in Chapter 2 of Barnett *et al*, 1979a).

Mathematics/Statistics syllabuses. At A-level we again encounter a wide range of variation in the amount of statistics material in different syllabuses, also now in the emphasis (more clearly directed at one extreme towards the

* It should be noted that most schools in Wales enter candidates for A-level examinations of Welsh examining boards, and statistics plays a much less prominent role in the syllabuses of those boards than it does in the corresponding syllabuses of the English examining boards.

mathematical, or at the other the statistical/data-interpretative aspects) and in the methods of examination and assessment. Three of the examining boards offer full syllabuses in statistics; three offer syllabuses in which statistics accounts for 50% of the material and another contains an optional 50% statistics component; all boards also have mathematics A-levels with statistical material which amounts to *less than* 50% of the syllabus content. Thus there are about 16 syllabuses which contain statistical material.

The content of A-level syllabuses obviously reflects the amount of space allowed for statistical topics – in the main, the lower this is, the more mathematical, less data interpretive, is the treatment of the statistics. Syllabuses where statistics is at least 50% of the material do not differ too widely in the formal statement of the statistical content. This is likely to include *data collection, representation, and summary; probability and simple probability manipulation; probability distributions and special cases (uniform, binomial, Poisson, normal); principles of estimation and testing with application to normal data; exact tests for normal means* and perhaps progressing to *bivariate distributions; correlation; regression; non-parametric methods; time-series calculations; simple analysis of variance, simple stochastic processes* or *finite population survey methods.*

There is a breed of supporting texts more-or-less designed for A-level studies (also differently styled books for O-level etc.). See Chapter 4 of Barnett *et al* (1979a). That none of these may be felt to fully meet the needs, reflects a complex array of factors. Syllabuses differ in literal quoted content, yet again in the interpretation of this content as reflected in examination questions. Teachers differ widely in their experience and acquired ability to teach statistics from any viewpoint and find great difficulty in identifying the prevailing viewpoint, harnessing it to their teaching and finding the *matched* book for class use.

It is common for syllabuses with less than 50% statistics content to represent this material as another aspect of mathematics or to use it to illustrate *mathematical* concepts. Syllabuses with at least 50% statistics may still tend to take an unduly formal view of the material with emphasis on algebraic and numerical manipulation, little contact with real-life problems, inadequate instruction and preparation on the modelling role of distributions, or on the essential *nondeterministic* aspects of data handling and interpretation. The methods sometimes manifest themselves as isolated recipes in a cookbook unconcerned with the meal the customer needs. And most serious of all, there is a tendency to grossly overload syllabuses in terms of concept, principle and detail due to failure to properly assess what the candidate can realistically cope with at this stage of his statistical studies.

However, this somewhat bleak view needs to be tempered by encouraging moves over the last five years. Examination questions are showing signs of greater concern for interpretation and judgement in handling data, and some newer syllabuses aim specifically at a more empirical approach by means of assessment procedures and detailed notes for guidance of teachers in implementing the syllabus. Two notable examples are the University of London Examination Board, *Pure Mathematics with Statistics*, and the JMB, *Statistics.* Most syllabuses require conventional examinations with two 3-hour papers

each containing a mixture of long (½ hour) and short (¼ hour) questions. The two syllabuses just mentioned also require practical project work. The JMB *Statistics* syllabus contains an outline syllabus with explanatory notes, detailed *notes for guidance* to teachers stressing the empirical aspects of the study of statistics and sets of worked specimen questions. A substantial project must be carried out and accounts for 15% of the total marks. The flavour of this syllabus is given by the following two examination questions,* and by the presentation in Appendix II of the basic syllabus and a short selected list of projects recently submitted.

6 In an investigation into the effectiveness of a particular course in speed reading a groups of 500 students was split into two groups, A and B, of sizes 300 and 200 respectively, thought to have been chosen at random.

Those in group A were given no special instruction; those in group B were given a course in speed reading.

Each student was asked to read the same passage and the time taken was measured. The results were

Group A: mean time 78.4 s, variance 14 s^2,
Group B: mean time 77.4 s, variance 15 s^2.

Carry out a significance test to see if there is evidence that the course has improved reading speed. State carefully your null hypothesis, alternative hypothesis and final conclusion.

You learn later that, of the original 500 students, 200 students had decided for themselves that they wanted to take the course in speed reading and that these students became group B. Discuss briefly how this might affect your previous conclusion.

7 A loom is used to produce cloth. Occasionally, at random, the machine falters and a flaw appears across the cloth. In the past it has been found that the distance between flaws in the cloth follows an exponential distribution with mean 1 metre. State briefly why an exponential distribution would be expected to serve as a reasonable model in this situation.

The 100 distances between successive flaws on a given day are shown in the following table.

Distance between flaws	Frequency
0 to 0.5 m	44
0.5 to 1.0 m	20
1.0 to 2.0 m	22
2.0 to 3.0 m	10
more than 3 m	4

(i) Plot these data as a histogram. On the same diagram draw an

* Reproduced by permission of the Joint Matriculation Board.

appropriately scaled graph of the density function of the exponential distribution with mean 1.

(ii) Calculate the corresponding expected frequencies for an exponential distribution with mean 1 and test whether this model gives a reasonable fit to the data. (Two significant figure accuracy for expected frequencies is accepted.)

These compare rather interestingly with a not un-typical, highly formal, A-level statistics question

A random variable X has probability density function

$$f(x) = k \sin(x/2) \qquad (0 < x < 2\pi).$$

Determine k, and the mean and variance of X.

For the London Board (*Pure Mathematics with Statistics*) candidates undertake 3 projects on specified parts of the syllabus (sampling, chance variations, experimentation) and two additional less constrained more substantial projects. Whilst not directly assessed, these projects are taken to the examinations and the candidate is expected to make use of them for illustrative purposes in answering questions on the examination papers.

User Subjects. At A-level the situation is similar to that described for O-level courses, except that more explicit and more (over) sophisticated topics may be included. See Chapter 2, Barnett *et al* (1979a), for details.

'In general the CSE, O-level and most of the A-level mathematics and statistics syllabuses are technique oriented and pay little regard to the statistics required in employment and adult life generally. They also tend to underestimate the difficulty of many of the statistical concepts being introduced and hence are too extensive (this is even more true of the other subject syllabuses that are introducing statistical techniques). They also fail to give an appreciation of the essential nature of statistics as an applied subject.' (Barnett, *et al*, 1979b).

1.3 CURRICULUM, PUBLICATIONS AND TEACHING

The development of a healthy programme of statistical education rests on the complex interaction of many different factors. It is necessary for there to be well-constructed syllabuses which

(i) are designed to meet the needs of different age ranges, and interest groups, of pupils

(ii) properly reflect, and in particular do not overestimate, the capacity of the pupils to absorb and assimilate statistical material

(iii) are sufficiently well defined, and documented, to enable teachers to unambiguously interpret their content, to assess the intended emphasis and to adopt the media of instruction appropriate to the target group of pupils.

These aims in turn depend on teachers being sufficiently well-trained and knowledgeable about the subject matter of the syllabuses, and on the availability of appropriate text-books, case-study material, equipment (e.g. for experiments) and visual aids. Finally, the social and educational climate has to be such that the teaching of statistics, at different levels and emphases and with acceptance of the concomitant investment in teacher training and curriculum development, is seen as an essential component of the educational system. It is necessary to take up these points in more detail, and to attempt to assess the extent to which the various criteria are satisfied.

1.3.1 *Curriculum development*

'The phrase "curriculum development" implies change which has been planned and coordinated. In the teaching of statistics . . . in England and Wales over the last 20 years we have seen much unplanned change. The most powerful instruments in these changes have been the . . . [CSE and GCE] examinations and syllabuses . . . [which] have affected [at the secondary level, dominated] the content, approach and nature of statistics teaching. Only recently has this effect . . . been officially recognised by the examining boards, and only for the most recent syllabuses have the aims and objectives . . . been published with the syllabuses'. (Barnett *et al*, 1979a).

Previously only bald statements of syllabus topics have been provided, leaving vast scope for arbitrary interpretation of depth and emphasis of treatment depending on the knowledge, experience or whim of individual teachers – reflecting in turn their own education and the availability of published resource material.

At the secondary level most syllabuses are drawn up by subject committees of the examinations boards, made up in the main of school teachers and representatives of the higher education sector. O-level and A-level syllabuses have to be registered, and authorised, respectively by the Schools Council, an autonomous body which receives its financial support from the DES and the LEAs and which also funds curriculum development projects outside the examination board sector.

The syllabuses produced by the examining boards (predominantly in the mathematics/statistics mould) have been felt by most statisticians to be grossly overloaded in content and highly formal and mathematical in style. Apart from isolated exceptions described above, they have made little attempt to represent statistics as a practical subject concerned with solving real problems, and have not encouraged an empirical approach to teaching statistics.

Influences on the curriculum from sources other than the examining boards have been spasmodic and fragmentary. Some sponsored curriculum development projects in Mathematics (financed by the Schools Council, Nuffield Foundation or other grant-giving foundations) have included rather limited components on statistics, and again statistics has often been presented in a very formal, mathematical, framework. A major project of the early 60's, pioneering 'new mathematics' was the School Mathematics Project.

Some of the statistical material, for sixth-form pupils, is presented in Durran (1970).

Other projects have developed some statistical material.

'The Continuing Mathematics Project recognised the lack of qualified statistics teachers and the growing use being made of statistics across the curriculum. Their materials propose that one solution to these problems is individualised learning. The booklets of this project are technique oriented and develop the facility in using techniques. They do not attempt to cover at all thoroughly the modelling aspect of statistics or show when or why particular techniques should be used.

'The Mathematics Applicable Project adopted the general philosophy that mathematics should be taught through problems. In fact this attitude does not show through particularly clearly in the book on probability, though the examples used do show imagination.

'Many other projects introduce statistical ideas and develop them as and when required. Amongst these are the Schools Council General Studies' Project (which uses data as a source of information), the Schools Council "Geography 14–19" Project and the Nuffield "Working with Science" Project (which, amongst other things, develops hypothesis testing).' (Barnett *et al*, 1979a).

There has been only one curriculum development project concerned specifically with statistics. This is the Schools Council Project on Statistical Education (POSE) which has been in operation at the University of Sheffield, UK, since 1975. It has produced, tested and evaluated, teaching units (for pupils and teachers) for 11–16 year olds. The aim is to introduce statistics for citizenship stressing the practical relevance of statistics by introducing concepts and techniques almost entirely through the medium of detailed real-life problems chosen from a wide range of applications areas. The concepts and techniques are absorbed by the pupil, rather than being taught to him, through empirical studies involving much pupil participation. The materials of the project have now been published. They include a teachers handbook entitled *Teaching Statistics 11–16* and Teachers and Pupils Notes have 28 units of teaching material each occupying about 5 hours in the classroom. (See Schools Council Project on Statistical Education (11–16), 1980, 1980a.) To illustrate the emphasis and style of this work, Appendix III presents a brief description of the project units and a section of one of the sets of pupils' notes. The material is not intended as a syllabus for examination, nor is it directed to the 'mathematics lesson' although in most of the trial schools it has been taught by mathematics teachers. The problems they have encountered highlight some of the points made in § 1.3.3 below about the difficulties of teaching statistics, in contrast to the way in which mathematics is taught.

Other influences on curriculum and on general aspects of the teaching of statistics have arisen from various professional bodies and national comittees; these influences have become more evident over the last few years.

The Royal Statistical Society has taken a general interest in the problem for some time; it has organised occasional national meetings and the yearly calendars of its local groups seldom fail to contain a meeting for teachers

on some aspect of teaching statistics. In conjunction with the Institute of Statisticians the RSS operates a Joint Education Committee (Chairman, Vic. Barnett) which monitors and initiates developments. It has submitted evidence (Barnett *et al*, 1979b) to the recently constituted government *Committee of Enquiry into the Teaching of Mathematics in Schools*, and has recently made proposals for the establishment of an Association for Teachers of Statistics.

A major new development is the introduction of the journal *Teaching Statistics* for 'teachers of pupils age 9–19' jointly sponsored by the RSS, the IoS, The International Statistical Institute and the Applied Probability Trust. It has about 1500 subscribers; about 75 per cent from the UK.

Mathematical committees with a passing interest in statistics teaching are the Joint Mathematical Council and the Joint Mathematical Education Committee of the Royal Society and the Institute of Mathematics and its Applications.

The embryonic Centre for Statistical Education at Sheffield University, UK, has limited funds (subscribed by industrial and commercial firms) and undertakes *ad hoc* studies, such as that which has yielded the report by Barnett *et al* (1979a) on statistical education and training of 16–19 year olds. The Centre for Statistical Education has been making detailed proposals to grant-giving bodies for the establishment of a curriculum development project for 16–19 year olds. The proposal suggests again an empirical, problem based, approach distinguishing the needs of three groups:

(i) continuation of the 'citizen arming' process for the non-technical/non-scientific pupils
(ii) development of sound and relevant statistical methods for biology, geography, economics etc. specialists
(iii) adding more of the formal background for the mathematically able and oriented pupils.

The Leverhulme Trust has recently agreed to fund a component of this work which will concentrate on identifying the needs of employers with regard to the statistical training of young employees and which will proceed to the development of teaching materials for applied subjects in the upper end of the secondary schools and in Further Education.

1.3.2 *Publications* (See Chapter 4 of Barnett *et al*, 1979a for further details.)

'[A member of Her Majesty's Inspectorate] said recently that the style of teaching in mathematics observed in surveying many schools was strikingly different to that in any other subject and could be characterised as "the syllabus being taught persistently and fast". This is a fair description of the approach to statistics found in a number of [secondary level] . . . texts [on statistics].' (Barnett *et al*, 1979a).

There is widespread dissatisfaction with the range of school-level books on statistics. The majority take too formal a view of the subject, stressing mathematical manipulation and virtually ignoring sound applications material and empirically-based development. A few texts contain serious misconceptions, or even downright errors on important topics. Some books do a sound job of presenting the mathematical-type material of most of the GCE syllabuses. This, of course, consolidates such an approach particularly for the many teachers who must depend on books to augment their own limited knowledge or experience of statistics. No good case-study, or wide-ranging applications-oriented, books exist to encourage teaching by sound practical example.

1.3.3 *Teaching and teacher training* (See Chapter 5 of Barnett *et al*, 1979a for further details.)

In spite of the presence of statistics in the school curriculum since the late 50's there still remains a serious crisis of confidence among teachers having to teach the subject. Earlier hostility to the subject and the tendencies to 'unload' it on the most junior teacher, or to regard it as a 'soft-option' for mathematically less able pupils, are now less in evidence. Many mathematics, and other subject, teachers are now convinced of the importance of statistics in the curriculum. But they do not seem to be becoming much more confident about teaching it.

The initial training of teachers seldom contains even passing contact with statistics. Current cuts in the teacher-training programme rule out hopes of a remedy to this. Graduate teachers have, in the past, had little exposure to statistics, although its wider role in university courses in a large range of subjects means that recent entrants have more chance of having encountered some statistics (if only the ubiquitous 10 lecture service course). The new optional course on statistical education in the postgraduate certificate of education course of the University of London Institute of Education is most welcome but probably unique.

If initial training is not the answer, in-service training might hold the key to producing a body of good teachers of statistics. Some universities, polytechnics and the DES organise short courses, but sadly if inevitably time and resources inhibit much of an impact on the problem. Courses of about 40 hours with plenty of practical work and student participation surely represent the minimum contact necessary to begin to bring the teacher to a position where confidence and ability may develop. Whilst falling school numbers may not rule out such prospects, economic factors are making them even less viable. But even a course of this scale cannot hope to teach the teacher statistics, how statistics is used in practical problems and then how to teach statistics! Three years as a practical statistician would be a better training, but is 'pie in the sky'.

One of the major difficulties resides in the rather special way in which statistics needs to be taught, compared say with mathematics.

'The mathematically trained teacher is traditionally used to teaching on a "brick on brick" principle; dealing with (and passing on from) each topic sequentially as if each was self-contained and isolated. This will not work with statistics (nor is it desirable in the more applications-oriented teaching of mathematics). The interrelationships are crucial throughout and need to be prominent; the subject must be presented as a practical one of ubiquitous relevance with copious illustration from real-life problems and opportunity

for students to develop intuitive understanding of basic concepts by much class cooperation in empirical work. It is not merely a body of techniques and recipes, each met and nodded-to before moving on. The techniques need justification and exemplification – their basic origins in the spectrum of statistical reasoning must be stressed, their relevance and limitations made clear by copious data investigation. It is worth pondering, in this context, the vast difficulty of adequately dealing with a topic such as *correlation* in a course on, say, Geography or Biology.

'For the mathematics teacher, the change of teaching emphasis to a broad-based, empirically-reinforced, look-and-learn, approach presents quite a challenge.' (Barnett *et al*, 1979a).

How is this challenge to be met? Surely not by casual one-off in-service courses. In the long-term the only hope seems to be in gradual self-training using a combination of in-service courses (the Open University televised and broadcast material can help) augmented by more detailed and sympathetically structured syllabuses and good supporting texts and practical case material. Perhaps the increasingly statistically-conscious social climate of the UK will provide the pressures for developing the required resources. The teacher needs to feel that he is not alone with his difficulties. The new journal *Teaching Statistics* and any Association for Teachers of Statistics can play an important role in dissemination of ideas and in uniting those who teach statistics.

1.4 CRITIQUE

How are we to judge the pattern of statistical education at school level in England and Wales? Essentially there are grounds for much satisfaction on the progress that has been made over the last 20 years or so. There is a substantial amount of statistical material in the school curriculum, possibly more than in any other country. Its growth has paralleled the ever-increasing use of statistical ideas and reasoning in society: in the media, in advertising, in official statements and discussions. There is growing interest and commitment among teachers. More and more effort is going into the construction of syllabuses with a view to defining their aims and emphases. Professional bodies and national organisations are beginning to take an interest and become involved. But of course many problems remain.

The recent uproar over the lack of numeracy of school leavers is leading to reappraisal of the teaching of mathematics. The citizen must above all, however, be *statistically* numerate to understand and critically assess the import of the statistical world he now inhabits. Much more needs to be done to identify needs here, to develop the appropriate syllabus and materials and to produce a teaching force which can handle this type of instruction. We must hope that POSE has impact on this problem. Next, we need to assess, distinguish and develop similar facilities for *other specific groups* of pupils (see end of § 1.3.1). The present attitude seems to say 'statistics is statistics is statistics'!

The employment sector is vaguely critical of the statistical education of young employees, but the criticisms provide virtually no substance to grasp.

Statisticians, teachers and employers must get together.

The training of teachers, and the provision of good books and materials, is of the utmost priority. See Cockcroft (1982).

Perhaps the one overriding obstacle to progress in the near future is the lack of any *coordination* of effort. Many well-meaning and informed individuals and organisations are working hard to improve the statistical education scene. But they do so in isolation. It is time that the statistical profession, government, teachers and employers had a forum for such action: some permanent national institute to assess, coordinate, initiate, produce and disseminate a sound structure for statistical education in England and Wales. There does not seem yet to be sufficient momentum or interest – but above all such a venture would be expensive and money is a very scarce commodity in the UK at present.

APPENDIX I *Associated Examining Board O-Level Syllabuses in Statistics*

The following syllabuses, which are reprinted with the permission of The Associated Examining Board, apply to the AEB Ordinary Level examinations for 1981. Students intending to enter for these O-Level examinations in any subsequent year should first check with the Board that details of the syllabuses are still applicable.

STATISTICS – 063

SYLLABUS – ORDINARY LEVEL
Two papers each of 1 hour and one paper of 2 hours

Candidates will be required to take 3 papers.
Paper 1 (1 hour) 25% of total marks
Paper 2 (1 hour) 25% of total marks
Paper 3 (2 hours) 50% of total marks

Paper 1 (Objective Test) will consist of a 25 item multiple-choice objective test and answers must be recorded on a separate answer sheet.

Paper 2 will consist of 20 short-answer questions and answers should be written on the question paper.

In Papers 1 and 2 candidates may attempt all questions.

There will be an interval of approximately ten minutes between these two papers.

Paper 3 will consist of longer traditional-type questions. Candidates will be required to attempt at most 4 questions out of 6.

Unless specific instructions to the contrary are given on the question paper the following will apply:

(a) Candidates may use any appropriate method.

(b) Mathematical tables may be used in Paper 3 but **not** *in Papers 1 and 2. Tables are to be provided by centres from the list given at the beginning of this Section. The* **final** *answer to questions requiring the use of four-figure tables should normally be given correct to three significant figures.*

(c) Slide rules may also be used where appropriate in Paper 3 but **not** *in Papers 1 and 2; candidates should indicate their use clearly by the letters (S.R.).*

(d) Calculators may also be used in Paper 3 but **not** *in Papers 1 and 2; candidates should indicate their use clearly by the letters (CALC). See the Regulations for Examinations in 1981 for the type of calculator allowed.*

(e) Set squares, graduated rules, protractors and compasses may be used.

SI units will be used in question papers as detailed in the British Standards Institution publication PD5686: 1972.

Collection of statistical data: census and sampling methods of enquiry;

pilot enquiries; simple ideas on the design of questionnaires and on dealing with bias.

Classification and tabulation.

Diagrammatic and pictorial presentation of statistical data.

Approximations; biased and unbiased. Absolute and relative errors in sums, differences, products and quotients.

Frequency distributions; histograms and frequency polygons; elementary descriptive properties of distributions. [The term 'frequency density' rather than 'frequency' will appear on the vertical axes of graphs of continuous random variables.]

Cumulative frequency distributions and graphs; estimation of medians, quartiles, deciles and percentiles.

Measures of average: arithmetic mean, geometric mean, median, mode and modal class.

Measures of dispersion: range, semi-interquartile range, mean deviation, variance and standard deviation.

[Unless otherwise stated, 'mean deviation' should be taken to imply 'mean deviation from the mean', and 'mean' should be taken to imply 'arithmetic mean'.]

Calculation of measures of average and dipersion from a set of numbers, a frequency distribution or a grouped frequency distribution.

Linear transformation of data to a given mean and standard deviation.

Scatter diagrams and a graphical treatment of regression and correlation; fitting a straight line to a scatter diagram using $(\bar{x}, \bar{y})$ and the calculation of its equation in the form $y = mx + c$.

Spearman's coefficient of correlation by ranks, $R = 1 - \frac{6\Sigma d^2}{N(N^2 - 1)}$.

[N.B. In the case of tied ranks the convention of giving the arithmetic mean rank to each equal item will be used; the correction for tied ranks will not be required.]

Elementary analysis of time series; secular trend, seasonal variation and random variation; moving averages.

Simple applications of weighted averages; index numbers as weighted averages of percentage relatives; birth, marriage and death rates; crude and standardised rates.

Probability; combination of probabilities of mutually exclusive events, dependent events and independent events.

ADDITIONAL STATISTICS – 185

SYLLABUS – ORDINARY (ALTERNATIVE) LEVEL
One paper of 3 hours

Candidates will be required to attempt at most 7 questions out of 9.

Unless specific instructions to the contrary are given the following will apply:

(a) Candidates may use any appropriate method.

(b) Mathematical tables may be used. Tables are to be provided by centres from the list given at the beginning of this Section. The **final** *answer to questions requiring the use of four-figure tables should normally be given correct to three significant figures.*

(c) Statistical tables may be used. The tables specified by the Board and to be provided by the centre, are Cambridge Elementary Statistical Tables.

(d) Slide rules may be used wherever appropriate. Candidates should indicate their use clearly by the letters (S.R.).

(e) Calculators may be used. Candidates should indicate their use clearly by the letters (CALC). See the Regulations for Examinations in 1981 for the type of calculator allowed.

(f) Set squares, graduated rules, protractors and compasses may be used.

SI units will be used in question papers as detailed in the British Standards Institution publication PD5686: 1972.

Candidates taking this paper will be examined on the subject matter of the Statistics Ordinary Level syllabus together with the following:

Time series: centred moving averages.

The Σ and functional notations.

Simple applications of ratio scales. Use of logarithmic graph papers.

Pearson's measure of skewness $\frac{\text{mean} - \text{mode}}{\text{standard deviation}}$.

Scatter diagrams. Simple ideas of regression; lines of regression using the method of least squares.

Correlation; calculation of the product-moment correlation coefficient.

Permutations and combinations.

Expectation. The binomial distribution. The use of np and npq for the mean and variance (no proof required).

Poisson and normal distributions. The fitting of a binomial, Poisson or normal distribution to experimental data.

The use of Poisson and arithmetical probability graph paper.

The use of the normal distribution as an error distribution.

Simple ideas of standard errors from continued experimentation; standard error of the mean $\sigma/\sqrt{n}$; null hypothesis; simple test of significance for the mean of a large sample.

Examples may be drawn from the field of Economics, Agriculture, Biology, Psychology, Education, Social Sciences.

Teaching Statistics in Schools throughout the World, 1982, Edited by Vic Barnett

APPENDIX II *Statistics A-Level Syllabus of the Joint Matriculation Board**

STATISTICS (ADVANCED) (two papers (I and II) each of three hours' duration and a project)
Statistics (Advanced) may not *be offered at the same sitting of the examination as Additional Mathematics (Pure Mathematics with Statistics) (Alternative Ordinary) or Pure Mathematics with Statistics (Advanced) or Further Mathematics (Advanced) Syllabus B.*

Notation

The notation indicated below may be used in question papers without further definition. Commonly used and unambiguous notation (e.g. +; –) is not included but may be used. Other notation not indicated below may also be used in question papers but such notation will be clearly defined in the context of the question. The numbers refer to the lists of notation given on pages 213 to 216 (all ranges are inclusive).

1 to 10, 14 to 26, 29, 31 to 36, 39 to 45, 47 to 54, 64, 65, 79, 80, 83 to 101.

Candidates will be provided with a booklet containing mathematical and statistical notation, definitions, formulae and statistical tables for use in the examination.

The aim of the syllabus

The aim of the syllabus is to promote an understanding of the basic concepts underlying statistical methods in such a way that the subject may be appreciated as an important component of modern applied mathematics and as a useful practical tool. In this respect the syllabus tries to avoid both extremes of a strictly mechanical approach to statistical methodology and of the highly formal (and superficial) use of probability or statistics as mere exercise material in the study and practice of pure mathematics.

The emphasis of the syllabus

The conceptual content of any course in statistics at the Advanced level is inevitably high. Sound application of statistical methods requires a proper understanding of basic concepts (both of specific concepts and of the whole framework of non-determinism in the modelling of practical problems). For this reason great care is needed in treating the concepts, in their appreciation, and in their application. It is regarded as important that the prime emphasis in an Advanced course in Statistics should be placed on thorough intuitive understanding (in relation to the level of the course) of concepts and methods, which may be encouraged by an empirical approach to the teaching of the material. Students might be led to an understanding through, for example, the use of classroom experiments and exposure to real situations and data as far as possible, with emphasis on the model building aspects of the problem. This approach may promote 'self discovery' and deeper understanding. A mathematical level of treatment beyond Ordinary level is essential, but additional mathematical material should be introduced as needed in the teaching of statistics, not as a separate examinable component. For this reason a statement of the mathematical knowledge required by candidates is given at the end of the syllabus.

* Reproduced by permission of the Joint Matriculation Board.

The type of student for whom the syllabus is intended

The syllabus is intended to meet the needs of those students who wish to embark on some study of statistics as a means of understanding and analysing problems involving an element of uncertainty ('chance') in the world around them. As such it should form a useful adjunct to subjects in the physical, biological or social sciences as well as serving as a second component for those students whose major studies are mathematical but who wish to keep open the prospect of studying mathematical statistics at the tertiary level.

Notes for guidance (Stats, I. May 1976) are published separately from the syllabus and are available from the Secretary on request. The *Notes* provide guidance on the requirements under each topic and, more important, the teaching approach to each topic likely to achieve the aim of the syllabus, and also suggest how sampling experiments may be used to demonstrate concepts such as probability distributions and properties of estimators.

The objectives of the examination

The following statement is intended to provide a general indication of the knowledge and abilities which the examination will be designed to test in conjunction with the subject matter listed in the later sections.

The detailed break-down shown under each heading is intended to amplify the type of ability included under the general heading. It is not suggested that clear distinctions can always be applied in constructing questions and a particular question may test more than one ability. It is also recognised that the categories which form the statement form an hierarchical structure, each category involving, to some extent, each of the categories listed before in the statement.

A. Knowledge and abilities to be tested

1. *Knowledge*
   - (a) Knowledge of statistical terminology.
   - (b) Knowledge of the basic concepts and ideas, e.g. probability theory.
   - (c) Knowledge of the general and specific methods and techniques used in statistical analysis.
   - (d) Knowledge of the necessary mathematical techniques and the ability to use a routine technique.
2. *Comprehension and application*
   - (a) The ability to understand and interpret statistical information including the translation of information from one form to another.
   - (b) The ability to relate basic concepts and ideas to specific problems.
   - (c) The ability to select and apply known techniques and principles to problems of a routine type.
   - (d) The ability to select and apply known techniques and principles to problems which are unfamiliar or are presented in a novel manner.

3. *Evaluation and analysis*
   (a) The ability to check that hypotheses or models are consistent with given information, to recognise unstated assumptions and to discriminate between alternative models.
   (b) The ability to assess the accuracy or appropriateness of statistical techniques.
   (c) The ability to construct statistical models of practical problems in an appropriate form for statistical analysis.

B. Weighting of the abilities

The marks allocated in the examination will be divided, as far as possible, as follows:

1. Knowledge	30 per cent
2. Comprehension and application	40 per cent
3. Evaluation and analysis	30 per cent

The form of the examination

The examination will consist of two written papers (I and II) each of three hours' duration. Eight questions will be set for each paper, of which candidates will be required to attempt six. In addition candidates will be required to submit a project.

Paper I (3 hours)

The questions will be designed to test the candidates' understanding of basic principles of probability theory and statistical inference in relation to practical problems.

Paper II (3 hours)

The questions will be orientated more towards the application of probability theory and statistical inference to the analysis of practical problems involving experimental data.

Questions which involve numerical information will usually be of the form where the relevant statistical data are provided in a summarised form.

Project work

Candidates will be required to submit a personal study of statistical (or probabilistic) methods applied to one specific area of practical enquiry. The length should be not more than 15 pages excluding the presentation of raw data. Projects will be assessed by the candidates' teacher and submitted to the Board together with the teacher's assessment report for use in moderating the centre's assessments. External candidates will be required to submit their projects to the Board for assessment by the Board's examiners, and, in addition, must attend for interview at a centre nominated by the Board.

Further information on project work is given in the *Notes for guidance* (Stats. 1, May 1976) which is available on request from the Secretary. External candidates are advised to read the *Notes* before planning their projects.

Allocation of marks

The allocation of marks will be as follows.

Paper I	42½ per cent
Paper II	42½ per cent
Project work	15 per cent

List of suggested books and references

For the benefit of centres and external candidates a list of books and suggestions for additional reading has been prepared and is available on request to the Secretary. The list is not to be regarded as prescribed reading nor is it exhaustive.

Subject matter to be tested in the examination

1. Probability theory
   (a) Relative frequencies.
   (b) Probabilities of events as limits of relative frequencies.
   (c) Sample spaces, elementary events, events, complementary events, mutually exclusive events, and exhaustive events.
   (d) Axioms of probability (motivated by properties of relative frequency). Addition law. Direct calculation of probabilities for experiments with equally likely outcomes.
   (e) Conditional probability and independence.
   (f) Illustrations from simple practical problems.
   (g) Random variables as mappings from the sample space to the real line.
   (h) Discrete and continuous random variables.
   (j) Discrete probability distributions.
   (k) Probability density functions.
   (l) Distribution functions.
   (m) The probability distribution as a model.

2. Some descriptive statistics and summary measures
   (a) Observed frequency and cumulative frequency distributions, their use in the construction of estimates of probability functions and distribution functions for discrete probability distributions.
   (b) Use of histograms to estimate probability density functions, and grouped cumulative frequency distributions in the estimation of distribution functions, for continuous distributions.
   (c) Measures of location and dispersion for observed frequency distributions (mean, median, mode; variance, standard deviation, range, interquartile range). Effect of change of scale and origin.
   (d) Measures of location and dispersion for discrete and continuous probability distributions. Effect of change of scale and origin.
   (e) Mean and variance of a linear combination of independent random variables (no proofs).

3. Standard distributions
   (a) Binomial, Poisson, exponential, normal, rectangular (uniform) distribution; parameters of these distributions; knowledge of means and variances. Derivation of means and variances only for Poisson and continuous rectangular distributions. Throughout, the model-building role of the different distributions should be stressed incorporating the ideas in section (2).
   (b) Poisson approximation to binomial, normal approximation to the binomial and Poisson distributions. (No proofs will be required).

(c) Use of tables of standardised normal distribution (with linear interpolation only).

4. Principles of estimation
   (a) Random sampling – simple sampling experiments exhibiting the distribution of means of samples.
   Use of sample means as estimators of population means.
   (b) Notion of the sampling distribution of an estimator; its standard deviation (standard error).
   (c) Sampling distribution of the mean of a normal sample (no proof).
   (d) Desirable properties of estimators: unbiasedness, relative efficiency. Intuitive notion of consistency.
   (e) Estimation of the mean, descriptive comparison of sampling distributions of sample mean and sample median, illustrated by sampling experiments. Approximate normality of the distribution of the mean of a large sample when the underlying probability distribution is not necessarily normal.
   Estimation of variance; construction of unbiased estimator using divisor $(n-1)$ (no proof).

5. Standard significance tests
   (a) Intuitive notion of a significance test illustrated by practical examples. Exact test based on binomial distribution with $p = \frac{1}{2}$.
   (b) Null hypothesis and alternative hypothesis. Critical region and significance level.
   (c) Brief discussion of one-sided and two-sided tests and the notion of the power of a test.
   (d) Two-sided test of normal mean, variance known.
   (e) Large sample tests based on the normal distribution:
      (i) test of mean, variance unknown.
      (ii) test of the difference between means of two independent samples.
      (iii) test of a proportion using normal approximation to Binomial.
   (f) Single sample t-test (no distribution theory).
   (g) Interval estimation in terms of 'confidence intervals' (introduced as the set of values of a parameter for which the corresponding two-sided significance test gives a non-significant result). Confidence interval for the mean of a normal sample.
   (h) Comparison of data and probability models. Use of Chi-square goodness of fit tests for fully specified discrete distributions.

6. Regression analysis
   Linear regression on one controlled variable (predetermined variable observed without error). Practical examples; graphical illustrations. Least squares estimation of regression coefficients.

Mathematical knowledge required by candidates

Mathematical knowledge will be required but mathematical topics will not be specifically examined. The topics listed below should be treated as an integral part of the syllabus and not as a separate course. It will be assumed

by the examiners that candidates have a knowledge of the relevant mathematical topics at the Ordinary level and, in addition, of the following mathematical topics.

1. Elementary set theory – union, intersection, complement, subset, null set, universal set.
2. Elementary ideas of mappings – functions as mappings. $f(x)$ notation.
3. Elementary calculus – including integration, maxima and minima.
4. Definite integrals in which the range may be infinite.
5. Σ notation, simple cases of sums of finite and infinite series (arithmetic and geometric progressions, sums of integers and their squares).
6. $n!$ and nCr or $\binom{n}{r}$ notation: binomial theorem for n an integer.
7. Exponential and logarithmic functions; exponential series.
8. Elementary notion of limits.
9. Manipulation of inequalities and moduli.
10. Equation of a straight line.
11. Minimisation of a quadratic function by completing the square.

Selected list of titles of Projects submitted by candidates for the 1978 A-level Statistics examination

SCHOOL

Administration & General

Bus lateness and pupil lateness.
Intelligence of arts and science students in the sixth form.
Effectiveness of two types of paperback cover on school library books.
Queueing for school dinners.
Relationship between 'O' and 'A' level results.
School meals compared with home meals produced for the same price.

Mathematics

The effect of 'rounding' on regression lines.
Are random numbers random?

Science

Effect of soil conditions on the distribution of wild plant.
Human reaction times and learning ability.
At study of winkles.
The effect of smoking on lung cancer.
Size of pebbles and position on storm beaches.
Quality control of blood testing.

Economics

Increasing the labour force.
Consumption of gas in UK.
Price rises in the local chemist's shop.

Languages

Sentence length in English and French novels.
Did Bacon write Shakespeare?

Geography
Do meander wave lengths vary with distance?
Population growth of Guisborough.
Rainfall and flow of the river Ribble.
History
Age group populations of Huddersfield 1901 to 1971.
LEISURE ACTIVITIES & SPORT
Effect of a sight on the accuracy of a rifle.
Predicting football results.
Effect of trap number on winning at greyhound racing.
Playing and watching sport.
A probability model for the game of tennis.
Comparison of performance at gliding in first and second year of tuition.
WORK
Commerce
Number of assistants required at a local newsagents.
Does a shop gain from moving to a shopping precinct?
Petrol sales at self-service and attended garages.
'Eight out of ten owners said their cats prefer "Whiskas Supermeat".'
The number of shillings in circulation.
Mistakes in newspapers.
Industry
Breakdown of lorries belonging to a particular haulage company.
Is 52 the average number of matches in a box?
Manual and automatic settings in the thickness of dough in a biscuit factory.
Lifetime of light bulbs compared with manufacturers' claims.
Transport
Arrival of traffic at a junction in 2 different time periods.
Delays in flight arrival times at Manchester Airport.
Are traffic lights biased?
Local road accidents 1974 and 1975.
How long do cars stay roadworthy?
Pupils modes of transport to school.
SOCIOLOGICAL PHENOMENA
Trends and changes in consumer spending.
Road accidents and casualties.
Distribution over the country of people with a particular surname.
Divorce in relation to other factors.
A statistical interpretation of telepathy and its possible existence.
Assessing the attractiveness of different towns in England and Wales.
Age distribution of male and female unemployment.

APPENDIX III *Schools Council Project on Statistical Education: brief description of teaching material*

(*text book* and *pupil's and teacher's notes*) under the title STATISTICS IN YOUR WORLD.
Published in 1980/81 by W. Foulsham & Co. Ltd., Yeovil Road, Slough, SL1 4JH.

Text Book
TEACHING STATISTICS ELEVEN TO SIXTEEN: a main Teacher's Book covering the subject of Statistics in general and combining a certain amount of theoretical discussion, with practical sections on the use of the Statistics in your World series not only in the Mathematics Department but for many other subject areas.

It is recommended as essential reading for teachers proposing to adopt the series, for it adds much background knowledge and advice in the teaching of this material across a broad spectrum in a school.

BRIEF DESCRIPTION OF TEACHING MATERIAL

Under 'Area of curriculum', an attempt has been made to indicate the main areas of application of the units. All are relevant to Statistics and Mathematics. Estimated lengths are for A4 pages, typed. Teachers' notes include test questions and all answers. Published versions will be in booklets of approximately A5 size. R pages will have copyright clearance for reproduction by schools.

LEVEL 1

Being fair to Ernie
Starting with the idea of fairness, this unit develops a method of random numbers which is then applied to the selection process behind premium bonds. The premium bond situation is modelled by a simulation game 'Ernie' in which some associated flaws and fallacies are studied.
Area of curriculum Mathematics
Length Pupil unit: 10 pages. Teachers' notes: 11 pages. 'R' sheets: 4 pages.

Leisure for pleasure
This investigates how pupils spend their leisure time. Most of the work is a summary of class activities. Pupils learn how to draw a pie chart and are introduced to a histogram. The final section gives a national perspective.
Area of curriculum Humanities, Social science, Mathematics
Length Pupil unit: 8 pages. Teachers' notes: 8 pages. 'R' sheets: 3 pages.

Tidy tables
This unit explains how to compose, read and clarify statistical tables. Simple data from the class are supplemented with real data on regional unemploy-

ment and accidents to pedestrians aged 16 and under.
Area of curriculum Mathematics, Social science
Length Pupil unit: 10 pages. Teachers' notes: 11 pages. 'R' sheets: 4 pages.

Wheels and meals
A framework and rationale for collecting data from members of the class is given. These include method of travel and distance travelled to school. The collected data gives pupils the opportunity to collate them into univariate and bivariate tables, display it on pictograms, bar charts and scatterdiagrams, and to discuss the implications. The aim is that pupils build up a picture of their own class.
Area of curriculum Humanities, Geography (local), Mathematics
Length Pupil unit: 8 pages. Teachers' notes: 8 pages. 'R' sheets: 1 page.

Shaking a six
This begins with the common belief amongst younger pupils that it is harder to get a six than any other number – an intuition based on contact with games such as ludo, which require a six to begin. Some experiments are described to enable pupils to test this hypothesis systematically. The data obtained are rather difficult to comprehend as many figures are involved. This suggests that pictorial representation and data reduction may be necessary. So pupils are encouraged to draw bar charts and also to calculate averages – the median and the mode. Results from an ordinary die are compared with those from a biased die.
Area of curriculum Mathematics, Social science
Length Pupil unit: 8 pages. Teachers' notes: 10 pages. 'R' sheets: 6 pages.

Practice makes perfect
Does one improve at certain tasks with practice and how can one tell? The first task used is catching a falling ruler, the second is estimating length. For the first task, simple comparisons are made between the left and right hand reaction time and between boys and girls. Pupils gain practice in collecting data and organising it to make simple inferences
Area of curriculum Science, Mathematics
Length Pupil unit: 8 pages. Teachers' notes: 10 pages. 'R' sheets: 3 pages.

Probability games
Pupils play a simple dice game to decide whether or not it is fair. Basic ideas of probability are introduced through games using dice and coins. The games develop in complexity and demand simple strategies. The unit concludes by encouraging pupils to compare relatively complicated probabilities in a game without carrying out detailed calculations.
Area of curriculum Mathematics
Length Pupil unit: 10 pages. Teachers' notes: 8 pages. 'R' sheets: 2 pages.

If at first . . .
Chance plays an important role in many real-life processes. The central theme of this unit is that of using simulation to model real-life situations. Children

often collect sets of cards. The first simulation investigates how long it would take to collect a set of four. Other simulations use dice and random numbers to model booking seats on a minibus, finding the right key for a door, the weather and being stopped at traffic lights.
Area of curriculum Mathematics
Length Pupil unit: 8 pages. Teachers' notes: 8 pages. 'R' sheets: 7 pages.

LEVEL 2

Authors anonymous
Statistical ideas even occur in English: literary styles are compared by analysing word lengths and sentence lengths in short passages. Various measures of central tendency and spread are used to compare and contrast the different distributions obtained.
Area of curriculum English, Humanities, Mathematics
Length Pupil unit: 11 pages. Teachers' notes: 10 pages. 'R' sheets: 7 pages.

First class football
Football results provide an excellent source for collecting information to estimate probabilities. Statistical techniques can be used to help answer questions about the probability of a home win and of a draw. Finally random prediction of a particular week's football results is shown to be bettered by only one of four newspapers.
Area of curriculum Social science, Mathematics
Length Pupil unit: 8 pages. Teachers' notes: 7 pages. 'R' sheets: 0 pages.

Seeing is believing
Pupils' reactions to a number of optical illusions give a rough measure of the power of each illusion using ordinal scales. Three of the illusions are followed up in detail. Optional sections include the magnitude of the deception measured on interval and ratio scales and the difference between nominal, ordinal, interval and ratio scales.
Area of curriculum Science, Mathematics
Length Pupil unit: 13 pages. Teachers' notes: 10 pages. 'R' sheets: 2 pages.

Fair Play
Pupils are encouraged to investigate stalls for a summer fair. After a discussion set on important design features, a few stalls are analysed probabilistically to help decide on entry fees and prizes. These range from push penny to fruit machines.
Area of curriculum Mathematics
Length Pupil unit: 11 pages. Teachers' notes: 11 pages. 'R' sheets: 1 page.

Opinion Matters
Many pupils have to use questionnaires in connection with projects. By using poor questionnaires and examining their inadequacies at first hand and by attempting to collect information using them, pupils are shown the impor-

tance of good questionnaire design. They learn to spot design errors as well as some general principles in writing good questions.
Area of curriculum English, Humanities, Social sciences
Length Pupil unit: 7 pages. Teachers' notes: 11 pages. 'R' sheets: 3 pages.

Getting it right
After a brief discussion on realistic levels of quoted accuracy, the unit moves on to a consideration of accuracy based on removing bias and reducing variability. The use of the mean of several readings to improve accuracy is considered specifically.
Area of curriculum Science, Mathematics
Length Pupil unit: 10 pages. Teachers' notes: 8 pages. 'R' sheets: 1 page.

LEVEL 3

Car careers
Pupils are encouraged to do a car survey in the local area and display their findings. The results are analysed and used to make predictions about the car fleet in the country, given the total number of licensed cars. In fact this leads to obviously incorrect results so pupils are forced to search for sources of bias. Finally some official published data are presented to enable pupils to estimate the growth and scrapping rates for cars.
Area of curriculum Social sciences, Mathematics
Length Pupil unit: 10 pages. Teachers' notes: 9 pages. 'R' sheets: 1 page.

Phoney figures
Abuses of statistics are almost a fine art especially in advertising. A few of the more common abuses are exposed to enable pupils to look at statistical data more carefully and critically.
Area of curriculum General knowledge, Humanities, Mathematics, Commerce
Length Pupil unit: 9 pages. Teachers' notes: 9 pages. 'R' sheets: 0 pages.

Net catch
The plight of blue whales forms the topical basis for this unit. The capture-recapture method is introduced via simulation and a practical example. The variability inherent in this method is illustrated and some real practical difficulties are brought out.
Area of curriculum Science, Mathematics
Length Pupil unit: 8 pages. Teachers' notes: 9 pages. 'R' sheets: 1 page.

Cutting it fine
Quality control and bread suggested the title for this work. After some examples, pupils are encouraged to try their hand by competing in a game involving making accurate piles in a limited time. The variability in weights is illustrated by actually weighing 50 bags of crisps. Pupils are encouraged to interpret simple quality control charts.
Area of curriculum Science, Mathematics, Commerce, Social Science
Length Pupil unit: 10 pages. Teachers' notes: 11 pages. 'R' sheets: 3 pages.

Multiplying people
The population explosion is of major importance in all decisions on future planning. Using the theme of populations, this unit starts with a simple projection of future world population and considers how it might be wrong. Subsequent sections illustrate such demographic techniques as population pyramids and the effect of birth and death rates on population growth.
Area of curriculum Humanities, Social science, Integrated science, Mathematics
Length Pupil unit: 12 pages. Teachers' notes: 9 pages. 'R' sheets: 3 pages.

Pupil Poll
This unit explains the basic techniques involved in opinion polls. It is centred around a poll of school children on favourite records but the wider implications for all polls are stressed throughout. Both random and stratified sampling are introduced and used.
Area of curriculum Humanities, General knowledge, Mathematics, Commerce
Length Pupil unit: 10 pages. Teachers' notes: 9 pages. 'R' sheets: 1 page.

LEVEL 4

Choice or chance
Eight out of ten cats prefer Whiskas – or do they? Pupils investigate the effect of chance in a variety of situations, are introduced to probability via tree diagrams and are asked to question the meaning and validity of such advertising claims.
Area of curriculum Mathematics
Length Pupil unit: 12 pages. Teachers' notes: 10 pages. 'R' sheets: 2 pages.

Sampling the census
A short classroom census is used to convey the rationale and basic method of census of population which is then looked at in more detail. The problems of question design and (briefly) confidentiality are considered and the census data are used to provide information and to make regional and time comparisons.
Area of curriculum Social sciences, Humanities, Economics, Commerce.
Length Pupil unit: 8 pages. Teachers' notes: 7 pages. 'R' sheets: 0 pages.

Testing testing
This unit discusses the use of the breathalyser and mass radiography, with the use of tree diagrams, the ideas of conditional probability and the occurrence of errors.
Area of curriculum Social science, Mathematics, Sciences
Length Pupil unit: 11 pages. Teachers' notes: 9 pages. 'R' sheets: 1 page.

Retail Price Index
What is the cost of living and how do we measure it? This unit introduces pupils to the concept of an index number. It starts with price relatives for individual items and builds up to calculating a weighted index number with a

fiscal section working out the retail price index and looking at the groups of items involved.
Area of curriculum Economics, Commerce, Social science, Humanities, Mathematics
Length Pupil unit: 8 pages. Teachers' notes: 6 pages. 'R' sheets: 2 pages.

Figuring the Future
The theme of the unit is to build up a statistical picture over the period 1965 to 1977 and to use a trend line to predict figures. The data used includes television licences, cinema attendances and admission charges, telegrams and telephones and there is scope for extension to other fields.
Area of curriculum Social sciences, General knowledge, Mathematics, Economics, Commerce.
Length Pupil unit: 14 pages. Teachers' notes: 10 pages. 'R' sheets: 1 page.

Smoking and Health
Starting with recent smoking habits in this country, this unit continues with an examination of some major diseases more common among smokers. A scattergram of smoking and birth weight is examined and pupils are asked to look at other evidence and consider whether smoking is harmful. Critical appraisal is invited of some conclusions of the Royal College of Physicians report 'Smoking and Health' (1977).
Area of curriculum Science, General knowledge, Health education, Mathematics
Length Pupil unit: 10 pages. Teachers' notes: 12 pages. 'R' sheets: 3 pages.

Equal pay
Whether the Equal Pay Act has worked is the main problem considered here. The wage rates of men and women are compared by using cumulative percentage curves. The median gives the central tendency while the spread is measured using the inter-quartile range. Comparison problems such as tax and the self-employed are raised.
Area of curriculum Social sciences, Humanities, Mathematics, Economics
Length Pupil unit: 10 pages. Teachers' notes: 12 pages. 'R' sheets: 0 pages.

Example of consecutive pages of one of the level 3 units (pupil's notes) (reproduced by permission of W. Foulsham and Co.).

B3 **Harder Numbers**

Sometimes the number of fish caught make the arithmetic difficult. It is not as easy to see the pattern. Table 3 shows the figures for **B2g** again.

Table 3 *Figures from* **B2g**

	In the pond	In 2nd sample
Marked fish	16	6
Fish altogether	N	9

We have written N for the number of fish in the pond. There are several ways of finding N.

Using the method described in **B2**, we get:

In the second sample the number of fish (9) is $\frac{9}{6}$ times the number of marked fish (6).

So in the pond, the number of fish *(N)* is about $\frac{9}{6}$ times the number of marked fish (16).

So N is about $\frac{9}{6} \times 16 = \frac{9 \times 16}{6} = 24$ fish.

We can use this method for any figures. Look carefully where the numbers came from. Can you see that we put:

$$N = \frac{(\text{No. of fish in 2nd sample}) \times (\text{No. of marked fish in pond})}{\text{Number of marked fish in second sample}}$$

We can use this formula to estimate the number of fish in the pond.

For example, suppose we catch 11 fish, mark them and put them back into the pond. In the second sample we catch eight fish and find that five of them are marked.

This gives: $N = \frac{8 \times 11}{5} = \frac{88}{5} = 17{\cdot}6$ fish

Since we cannot have part of a living fish, we estimate that there are about 18 fish in the pond.

Use the formula to estimate the number of fish in the following ponds:

	1st sample marked	2nd sample	
		Altogether	Marked
a	14	9	6
b	10	20	8
c	14	13	6
d	25	15	10
e	12	7	5

Suppose there are no marked fish in the second sample.

f When might this happen? What would your estimate be?

Suppose all the fish in the second sample are marked.

g When might this happen? What would your estimate be?

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SUPPLEMENT – *Statistical Education in Schools in Scotland*

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This brief review concentrates on the essential differences between the Scottish scene and that of England and Wales. It will be seen that despite these differences there are similar problems.

Sections are numbered to correspond with the main text of Chapter 1. If there is no comment on a given section it may be assumed that the situation in Scotland is more or less as described in that section.

1.1.3 *Public examinations*

There is a Scottish Certificate of Education at Ordinary grade (SCE O-grade) which is equivalent to GCE O-level. This is followed, after one year of additional study, by the SCE Higher grade. Thereafter for pupils remaining at secondary school for a sixth year there is available in various subjects a Certificate of Sixth-Year Studies (CSYS).

The SCE and the CSYS are administered centrally by the Scottish Certificate of Education Examination Board (SCEEB). Thus there is uniformity throughout Scotland in syllabuses and examinations. The Board operates through Panels in different subject areas. Statistics is under the aegis of the Mathematical Panel.

1.2.2 *Statistics in secondary schools*

There is an SCE O-grade in Statistics but no SCE Higher grade. There is also a Probability and Statistics component in the CSYS in Mathematics, this being quite independent of O-grade Statistics.

Generally statistical education in secondary schools is more closely associated with mathematics than appears to be the case in England and Wales.

O-grade syllabus. As a result of representations by Scottish academic statisticians the O-grade syllabus is at present under review. A Working Party consisting of schoolteachers, teachers in colleges of further education, academic statisticians, school inspectors and a non-academic professional statistician was established to consider Statistics syllabuses in schools. It proposed an O-grade syllabus whose main aim is 'to develop in pupils an ability to interpret, sometimes critically, much of the quantitative material presented by today's media. This interpretative ability is developed through the widespread employment of graphical methods and of simple descriptive statistics calculated for comparative purposes. Such techniques should always be presented in the context of specific problems; clearly ideas which are inaccessible in the abstract can become accessible when illustrated by familiar examples'.

The movement in emphasis appears to be in the same direction as advocated in Chapter 1 above, but again in Scotland 'so much will depend on how the teacher interprets the syllabus'.

CSYS syllabus. In the examination for the CSYS in Mathematics there is one optional paper on Probability and Statistics. The syllabus for this paper includes probability calculus and some statistical inference in the form of estimation and hypothesis testing. As in England and Wales at A-level there has been an tendency for this to be taught in a very formal way, but attempts are being made to set examination questions that will force some emphasis in teaching on interpretation and judgement in handling data.

1.3.1 *Curriculum development*

The method whereby curriculum development takes place in Scotland has already been partially indicated. A Working Party containing at least some members who are knowledgeable about the subject and have experience of teaching it is set up by the SCEEB to consider syllabus changes. This reports to the Mathematics Panel of the Board. If the Panel approves a syllabus it is sent to 'interested parties' for comment. The syllabus is revised in the light of these comments and finally established.

One of the defects of this system is that professional statisticians do not have enough influence in determining the final outcome. Interested parties tend to be mathematically oriented and of course the Mathematics Panel is dominated by mathematicians. The point is already made in the main report that there is a tendency for mathematicians to regard statistics as merely another branch of mathematics and to fail to appreciate that quite different teaching methods are required for it.

1.3.2 *Publications*

As in England and Wales there is dissatisfaction with school-level books on Statistics and any syllabus changes carry with them the necessity of producing suitable texts and teaching material. This tends to inhibit change.

1.3.3 *Teaching and teacher training*

The situation is broadly similar to that in England and Wales. There is a grave shortage of teachers adequately trained in Statistics and similar problems exist about in-service training. However there is now a trickle of graduates with an honours degree in Statistics entering the teaching profession and it is to be hoped that this trickle will increase in volume.

1.4 CRITIQUE

The organisation and control of education in Scotland are quite different from those of England and Wales and the mechanism allows changes to be made on a national scale. However as far as Statistics is concerned similar difficulties exist in achieving what the professional statistician might regard as adequate teaching of the subject in schools. The main ones are:

Teaching Statistics in Schools throughout the World, 1982, Edited by Vic Barnett

i) the lack of schoolteachers with sufficient knowledge of the subject to teach it properly;
ii) the lack of suitable teaching material;
iii) the inadequate influence of professional statisticians on syllabus content.

A remedy suggests itself: that a Statistics Panel be established by the SCEEB. In this way the coordination of effort advocated in the main report could be achieved in Scotland. But the establishment and success of such a Panel requires enthusiasm and effort on the part of professional statisticians as a whole. Unfortunately this has not been evident in the past.

CHAPTER 2

Report on Stochastics at High Schools in the Federal Republic of Germany

H. DINGES UNIVERSITY OF FRANKFURT, W. GERMANY

In the Federal Republic of Germany it is felt quite generally that some stochastics (or statistics) should be taught to high school pupils. Changes in this direction are in progress for the age-range 16—19 years.

2.1 TEACHER TRAINING AND STOCHASTICS AT GERMAN UNIVERSITIES

In the Federal Republic all matters of education are the responsibility of the *individual states*. Although the ministers of cultural affairs meet once in a while, their views on university organisation and on schooling are rather different. Thus there is no uniform picture. In the past most ideas for reform of the curricula have come from the school authorities (with no indication of who are their advisors) or from small groups of university professors taking an interest in school affairs. The teachers' associations have not shown much interest in the curriculum changes. How the idea to teach stochastics at high schools came to the attention of the *Kultusministerkonferenz* (assembly of the ministers of cultural affairs) is not clear. The reform is supported now by many institutions.

The *universities* have always been in charge of the first (the so-called 'scientific') phase of the education of high school ('gymnasium') teachers which consists of 8 semesters of tuition in two school subjects, as for example mathematics and physics, plus some basic studies of educational methods. In addition there are *teacher colleges*, which have in some states been incorporated into the universities. They offer 6 semesters of tuition, with more emphasis on educational than on scientific study. Only teachers who have passed 'first examination' after 8 semesters study can teach above the 10th grade level (that is, above age 15). Some types of teachers at practical professional schools might also have been educated at universities, but we shall restrict attention in this report to the situation in the high schools.

Following the scientific university examination there is practical teaching instruction in institutions called *'Studienseminare'* spread over the country. This second phase of teacher training ends after two years with a 'second examination'. The teaching staff of the 'Studienseminare' clearly have more influence on what is actually taught by the young teachers than the universities. Most of them recognize the importance of stochastics, but their views of what stochastics should be vary widely.