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i) the lack of schoolteachers with sufficient knowledge of the subject to teach it properly;
ii) the lack of suitable teaching material;
iii) the inadequate influence of professional statisticians on syllabus content.

A remedy suggests itself: that a Statistics Panel be established by the SCEEB. In this way the coordination of effort advocated in the main report could be achieved in Scotland. But the establishment and success of such a Panel requires enthusiasm and effort on the part of professional statisticians as a whole. Unfortunately this has not been evident in the past.

CHAPTER 2

Report on Stochastics at High Schools in the Federal Republic of Germany

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In the Federal Republic of Germany it is felt quite generally that some stochastics (or statistics) should be taught to high school pupils. Changes in this direction are in progress for the age-range 16–19 years.

2.1 TEACHER TRAINING AND STOCHASTICS AT GERMAN UNIVERSITIES

In the Federal Republic all matters of education are the responsibility of the *individual states*. Although the ministers of cultural affairs meet once in a while, their views on university organisation and on schooling are rather different. Thus there is no uniform picture. In the past most ideas for reform of the curricula have come from the school authorities (with no indication of who are their advisors) or from small groups of university professors taking an interest in school affairs. The teachers' associations have not shown much interest in the curriculum changes. How the idea to teach stochastics at high schools came to the attention of the *Kultusministerkonferenz* (assembly of the ministers of cultural affairs) is not clear. The reform is supported now by many institutions.

The *universities* have always been in charge of the first (the so-called 'scientific') phase of the education of high school ('gymnasium') teachers which consists of 8 semesters of tuition in two school subjects, as for example mathematics and physics, plus some basic studies of educational methods. In addition there are *teacher colleges*, which have in some states been incorporated into the universities. They offer 6 semesters of tuition, with more emphasis on educational than on scientific study. Only teachers who have passed 'first examination' after 8 semesters study can teach above the 10th grade level (that is, above age 15). Some types of teachers at practical professional schools might also have been educated at universities, but we shall restrict attention in this report to the situation in the high schools.

Following the scientific university examination there is practical teaching instruction in institutions called *'Studienseminare'* spread over the country. This second phase of teacher training ends after two years with a 'second examination'. The teaching staff of the 'Studienseminare' clearly have more influence on what is actually taught by the young teachers than the universities. Most of them recognize the importance of stochastics, but their views of what stochastics should be vary widely.

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The *gymnasium* is the traditional form of high school, comprising grades 5–13 (ages 11 to 18 years). We distinguish: *primary school* (grades 1–4), *secondary level I* (grades 5–10), *secondary level II* (grades 11–13). There is a natural break after grade 10, when pupils definitely not intending to go to a university, leave. About two thirds of those who complete 13 years of schooling (a total of almost 20 per cent of the relevant age group) go on to university or some other form of tertiary education. In some places the gymnasia have been put under one roof with some other types of schools (e.g. *'Realschulen'*) finishing after the 10th grade. The final examination at a gymnasium is at the same time the entrance examination for the university. In some states (e.g. Bavaria) the written final examinations are strictly centrally controlled, in others (e.g. Hessen) individual schools and teachers have much more freedom. In all states there is rather strict control on the school books which are selected and bought by the individual schools.

Professors of didactics are in charge of the training of teachers for schools other than high schools, e.g. for the Haupt- und Realschulen. A professor of didactics of mathematics may be a member of the mathematics department at some universities; at others there are separate units for didactics of the sciences (and there still exist some teacher colleges, which do not have the rank of a university). Among professors of didactics of mathematics there is great interest in the prospect of introducing stochastics into the high schools. Unfortunately only a few of them have had any substantial training in statistics or probability. Yet they are required to teach some stochastics to the students training to be teachers. In the past, geometry, analysis and above all structural mathematics were considered the main subjects. The instruction for future mathematics teachers is very much the same as that for students going for the Diplom (corresponding perhaps to a master's degree with thesis).

At German universities there are no *statistics departments* (with the exception of Munich University and the rather new university of Dortmund). About 15 years ago most universities started to establish chairs for stochastics ('Wahrscheinlichkeitstheorie und mathematische Statistik') within the mathematics departments. Probability and statistics are now well represented almost everywhere. Stochastics is an accepted field of specialization for the Diplom-thesis and the final examinations, but it is not compulsory. Its popularity varies from university to university; among the students likely to become school teachers it is generally less popular because of the pressure of other subjects. Other departments also offer courses in statistics, which are usually compulsory (statistics for psychologists, medical statistics, etc.) Statistical methods are taught which are not meant to be of interest for students outside the department, nor for teacher students. The professors who are in charge of this type of statistical education usually show little interest in high school curricula; but they do tend to complain about insufficient preparation on the part of the high school graduates who enter the universities.

Outside the universities there seem to be no influential groups active in developing ideas about statistical education in the schools. Some few prominent statisticians take part in discussions on that issue within the ISI (International Statistical Institute) and within the DMV (Deutsche Mathematiker Vereinigung) or at other international meetings.

There seem to be no movements to establish statistics as a school subject separate from mathematics, as some groups have demanded for 'Informatik' (that is, Computer Science). Arguments analogous to theirs could easily be put forward in favour of statistics as a school subject, if the style of teaching mathematics is not changed.

2.2 THE PRESENT STATE OF STOCHASTICS IN THE REFORMED GYMNASIUM (FOR 16–19 YEAR OLDS)

As already mentioned education is the responsibility of the individual states. However, in 1972 the Kultusministerkonferenz agreed to drastic changes in high school education. The old system, where all pupils took the same subjects, was abolished and the *'course system'* was introduced with each student choosing his own subjects in specified areas (social studies, languages, mathematics, sciences). There are two types of courses; more intense and less intense (basic) ones. This system has produced a demand for interesting subjects to attract the talented students into the more intense courses.

Within the mathematics syllabus, courses in *calculus* hold the most prominent place. Besides *linear algebra*, however, *'Stochastik'* is the only discipline, which is acknowledged in all states for both the basic and more intense courses. In a few states probability or statistics is compulsory at the basic level. (All this is rather new; and a fundamental question arises. Should the basic course take a different approach from the more intense one, or should it merely be a scaled down version? Equivalently, can the more intense course be an inflated version of the basic course, or must it be entirely different in conception?)

Before the recent reforms there were many voluntary school courses on various subjects, offered by enthusiastic teachers for especially interested pupils. Apparently stochastics became quite popular at that time with some high school teachers; courses on boolean algebra or group theory seemed less stimulating. (Less flexible teachers, however, seem to like these latter subjects better, since they are more clear-cut and supposedly allow more 'objective' examinations te be set).

Looking through the tables of content for *basic courses* on stochastics one gets the impression that there is some agreement about the *minimal requirements*. Everywhere we find the same probabilistic concepts mentioned: event, space of events, (relative) frequency, probability space, independence of events, distribution, random experiment. The basic formula of combinatorics and the calculus of probability based on Kolmogoroff's axioms are required or at least recommended. The binomial distribution is treated, and as a culmination Bernoulli experiments and binomial distributions are taken as the basis for the discussion of the ideas of hypothesis testing and confidence intervals.

In spite of the similarity in the selection of topics there are basic differences in the goals of the various courses. The intention of some courses is to clarify intuitive concepts expressed through such terms as 'probable', 'random', 'frequent', etc. Others try to make clear the differences and similar-

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ities in causal and indeterministic descriptions of natural phenomena. Others praise the lucidity of Kolmogoroff's axioms and aim to exemplify the axiomatic method of mathematics. All take a reductionist point of view. They are anxious to make clear, for example, that random variables are nothing but functions. A crucial question has not yet been adequately thought about: In what mixture and in what relation should the components of proving theorems, interpreting results and discussing real data be taken up? (Dr. R. Sträßer from the IDM* in Bielefeld has given a very valuable survey of the situation in the 11 states.)

All courses on stochastics seem to be rather *isolated* from other courses in mathematics and from the courses in the natural and the social sciences. This may partly be caused by the course system as such, where the teacher cannot rely on a homogeneous preparation and on a unified outlook of the participants. A more basic reason for the isolation of these courses may be that the authors of the school books do not know which lines of stochastical thought are relevant in other fields. They stress the similarities with 'pure' mathematics. A. Engel's ideas have been influential in working against this tendency, but most teachers find his presentation too 'informal' and 'unmathematical'. So far, steps to introduce stochastics into schools have only been taken for the upper grades. It seems important, however, that certain lines of stochastical reasoning should be introduced earlier, for example in grades 5–10 (ages 11–16), in conjunction with more traditional school material. For example, relative frequencies could be discussed when fractions are introduced, mean values with addition and division, and mean square deviation with quadratic functions.

2.3 MATHEMATICS AND STOCHASTICS

Mathematics is clearly a systematic discipline. Its coherence depends essentially upon the uniformity of its methods. Many of the present generation of high school mathematics teachers are anxious to represent and teach this outlook to all their pupils. But a growing number of mathematicians have accepted the view that teaching mathematics should not be restricted to teaching part of the systematics so convincingly put forward by Bourbaki. They know that students must develop sound ideas about the use and the relevance of mathematical thinking in our culture. It seems inevitable that mathematics students and teachers of mathematics will in the future study more applied mathematics, in particular more stochastics.

Many professors of stochastics, although well integrated into the mathematical world, favour introducing some ideas of model-building in mathematical education at the university and in high school. Some consider stochastics as the 'art of reasonable guessing' (following J. Bernoulli), and believe that schools should help students develop 'stochastical thinking': the ability

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to cope with unprecise figures and varying data. They see stochastical intuition as analogous to geometrical intuition: the ability to perceive orientation in space. They believe that mathematics can contribute as much to stochastical thinking as to geometrical thinking – on many levels of formalisation. They consider it important to develop stochastical thinking, starting at a very early age.

In December 1978 a meeting in Oberwolfach entitled 'Stochastik auf der Schule' (statistics in schools) was attended by many university statisticians, enthusiastic teachers, authors of school books and advisors of school authorities. The assembly agreed on a resolution which expressed their view on the situation of stochastics in schools:

'We welcome the emphasis on stochastics as a central topic in the new syllabuses for mathematics at high schools, and we welcome the increasing readiness of the mathematics teachers to teach stochastics. We are confident that teachers can overcome the difficulties which arise from the intimate relations of stochastics to the fields of application. The special situation of statistics in the German academic world should not have negative effects. Valuable work has been done by different bodies (school authorities, teacher seminars, universities, international organisations like OECD, UNESCO and ISI) to promote stochastics as a school subject which opens opportunities to pursue important educational goals. These goals include:

- increasing students' activities in class
- improving their attitude to mathematics in general
- giving the pupils experience of how mathematical models may help to structure complex situations
- fostering cooperation with other school subjects
- overcoming the rather sterile thinking within narrow mathematical systems.

Discussing the phenomenon of randomness and the idea of correlation in school should enable the young generation to read imprecise figures critically and react to statistically supported assertions more adequately. The pupils should develop understanding of the possibilities and of the limitations of conclusions drawn from statistical data.

'The educational authorities should not try to solve the problems of teaching stochastics by rigid administrative prescriptions. They should accept the fundamental point that it is of little value to build up a terminology for elementary probability theory, unless at the same time the tool-character of the concepts becomes explicit. It is equally inappropriate for schools aiming at general education to teach statistical machinery, if adequate concepts are not developed simultaneously. . . .

'The time available for stochastics teaching for pupils aged 10–16 should be used to give them experience with imprecise data and fluctuating figures. Not too much time should be wasted collecting data; instead one should discuss how data can be organized and what type of pragmatic conclusions are commonly drawn from it. Special attention should be paid to systematically chosen examples of stochastical thinking which can be related to traditional subject matter (e.g. fractional numbers or quadratic functions).

'For pupils aged 16–19 years, thinking in probabilistic terms can be made

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more precise, building on former experience. Some probability computations should be done. However, we consider it harmful to the development of stochastical thinking to stress the formal mathematical components at this level. Building up the well-known axiomatic-deductive system, under the title of the theory of normed Boolean algebras rather than elementary probability theory, can inhibit the development of stochastic thinking at this level.

'In the teaching of stochastics, applications should not be mere illustrations suitable for motivation. Rather, they should provide an opportunity to use mathematics to understand more precisely aspects of realistic situations. In stochastics teaching it should become clear that mathematical competence is not confined to situations where definitive answers can be given to well-posed problems.'

This is the resolution, with one minor omission, which criticises certain current bureaucratic attitudes. It has been published (in German) in various journals, e.g. Stochastik auf der Schule (1979).

2.4 PROSPECTS

What are the deficiencies in the present state of stochastical education? What must now be done? Who can help? Can the chances to learn stochastical thinking, presently given to the 16–19 year olds, be extended to include the 10–16 year olds.

There are at present no administration-independent curriculum development projects on teaching stochastics in the Federal Republic.

There is no agreement, not even a serious discussion, on which parts of the syllabuses (for the 10–16 year olds) should be cut out in favour of material from stochastics.

There is little stochastical culture among the general public; even educated people use a rather sloppy language when it comes to statistical arguments. We lack a German equivalent of the book by Tanur, J. *et al* (Eds.) (1972) *Statistics: A Guide to the Unknown.* San Francisco, Holden-Day.

The teachers of mathematics are not aware of the wealth of instructive elementary problems. We need a German version of a book like Mosteller, F., *et al* (Eds.) (1973) *Statistics by Example.* New York, Addison-Wesley.

There is insecurity about the degree of rigour appropriate for statistical argumentation in school books and in examinations. The problem of the right level of mathematical rigour in university courses for teachers is also unsolved.

There are no established groups of mathematics teachers which take more than marginal interest in teaching stochastics. (There are such groups for geometry or calculus, but not for stochastics). The international Conference on the Teaching of Statistics planned by ISI may help. (This is to be held in Sheffield, U.K., in August 1982).

The new Journal 'Teaching Statistics' could probably be developed into a forum for discussion among such groups in various countries. The very different traditions of statistics in the various countries should have a stimulating effect. (Professor F. Eicher, of the University of Dortmund, has translated the

first two issues into German and has circulated them among high schools.)

It is not clear what type of statistical training the student teachers really need. Can the present university courses be adjusted to serve both the students going for the Diplom and for the teacher examination? How can we ensure that high school teachers of geography, economics, biology, etc. get some background knowledge of statistical matters?

Too little is done for teachers who have not studied stochastics during their university education. The school authorities should encourage acting teachers to participate in stochastics courses given by professors who really understand the nature of stochastical thinking.

Professional statisticians and professors of stochastics and of didactics must not lose contact with the views of stochastics being presented at the (above-mentioned) 'Studienseminare'. These views must be openly discussed before they are taught to high school students.

The school authorities must be warned not to impose rigid stochastic syllabuses on the teachers. They should be reminded of the disaster with 'New Math', which was probably caused by the fact that the intentions of the reform were not made clear either to the teachers or to the public. The authorities should rather be asked to support communication about the goals of stochastics teaching and to assist all serious endeavours to guide the teachers in statistical thinking.

REFERENCES

Stochastik auf der Schule. (1979) Zusammenfassung des Diskussionsstandes bei der Oberwolfacher Tagung (10.–16.12.1978). Mitteilungen der Deutschen Mathematiker-Vereinigung Februar 1979.

Engel, A. (1973) *Wahrscheinlichkeitsrechnung und Statistik*, (two volumes) Stuttgart, Klett Studienbücher.

Sträßer, R. (1977) Stochastik in der neugestalteten gymnasialen Oberstufe, *Schriftenreihe des IDM* 8/1977, 312–343 (Bielefeld).

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