Teaching Statistics in Schools throughout the World, 1982, Edited by Vic Barnett

CHAPTER 4

Pre-University Stochastics Teaching in Hungary

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This chapter surveys stochastics teaching at different ages and in different schools in Hungary, including discussion of future plans, and of teaching experiments. The word 'stochastics' is used here in a restricted sense; it means only probability theory, and mathematical and descriptive statistics. (In primary schools it is called simply 'probability', at secondary level it is termed 'calculus of probability', in schools of economics both mathematical and descriptive statistics are referred to as statistics).

Differences in national education systems force us to provide information on the Hungarian school system (§4.2) having the main characteristic of being centrally governed down to almost the last detail.

§4.3 is devoted to teacher training, since the teacher's attitude and knowledge is a dominant factor. Then we offer a short historical view showing a permanent struggle for including stochastics in the school subject-matter. §4.4 shows that the struggle resulted in an up-to-date curriculum including considerable stochastics in primary schools. At secondary level, however, there is retrogression rather than progression. For the time being stochastics is taught only in specialised classes. It seems that only the Bolyai János Mathematical Society cares for the future of the subject. The Society initiated experiments aiming to convince both the officials and the teachers that stochastics can be 'embedded' into the subject-matter as an organic part thereof. A brief account of some of these experiments is given in §4.5.

4.1 ON THE HUNGARIAN SCHOOL SYSTEM

4.1.1 *Generalities*

For children of ages between 6 and 14 it is compulsory for them to attend the so-called *general school* (*primary school*), which consists of grades 1–8. After finishing the general school, about 80 per cent of the children continue their studies in *secondary schools*. There are 3 types of secondary schools in Hungary:

a) secondary school of the general kind (the so-called '*gymnasium*'), with 4 grades,
b) secondary schools which give specialised training for certain professions (e.g. the textile industry, agriculture, commerce, economics, traffic, postal services, fine arts, etc.), with 4 grades,
c) *training colleges* with 3 grades.

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The three types of secondary schools are attended by about 20 per cent, 23 per cent, 40 per cent of the children, respectively.

Type a) gives a general education which enables the students to continue their studies at higher level (for example, at University). Type b) also provides a general education, but, at the same time, an expressed goal of the school is to enable the students to work for the particular line chosen. They, too, might enter universities. The goal of the schools of type c) is to train skilled workers while enlarging their general knowledge. Some characteristic data on the school system is given below in Table 4.1.

	No. of schools	No. of classes	No. of students	% of students	No. of teachers	Passed the final exam
a) Gymnasium*	290	3233	102079	48.7	6700	97%
b) Specialized schools**	238	3407	107567	51.3	7049	95%
Total	528	6640	209646	100.0	13749	96%

* including 10 denominational schools
** including 8 art schools

Table 4.1 Some statistical data from the school year 1974/75.

The evaluation is done mainly by traditional methods. In almost every case the teachers give marks for certain tasks. There are 3 kinds of tasks: 1) Oral questioning; 2) Short written tests (of 5–15 minutes), 3) Longer written tests (usually of one hour, and 4 times during the school-year). We can best characterise the general stand-point by saying that it does not distinguish between the method of evaluation or marking, and it puts a special emphasis on the knowledge instead of the problem-solving ability. National examinations are organised for the secondary school of types a) and b), only. These examinations (called *matriculation*) are at present not compulsory, nevertheless almost every student takes them. Both the examination papers and the questions for oral examinations are set by the Ministry of Education.

4.1.2 *Mathematics in the primary schools*

Starting with the school year 1974/75 a new syllabus is being gradually introduced in the primary schools. This new syllabus is a nation-wide realisation of a successful experiment known as the OPI Mathematics Project and described as a 'composite method for the mathematical education of young children'. The trial of this project went on under the direction of Professor Tamás Varga. The new syllabus gives the following main directives. Detailed information is provided by Nemenyi, Szendrei-Radnai and Varga (1974).

a) The subject-matter is unified in the sense that the subject-matters in each grade are built on those of the previous grades, and, at the same time, pave the way for those in the following grades, including possibly higher schools.
b) The subject-matters were prepared for the average pupils, and contain only a few hints for progression for the more talented ones. The basic requirements are determined in such a way that the less talented pupils could comply with them.
c) Although the children are expected to become good at sums, less time is spent on rote learning and greater emphasis is put on self-checking, and on the solving of simple problems.
d) As to the higher grades, it is emphasised that the goal is not learning rules by heart, but it is motivating, constructing, understanding, developing an ability to make correct decisions and modelling simple situations.
e) Methods used in the learning process include many games, and discussions of problems of everyday life are involved. Having accumulated quite a bit of information, the children are led to formulate definitions and to draw conclusions by themselves. The concepts become gradually richer, deeper and more precise. The curriculum is, in this sense, of spiral type.

4.1.3 *Secondary schools of general kind (so-called 'gymnasiums')*

In the early sixties specialised classes were organised in most of the subjects. Figures on specialised classes are provided in Table 4.2.

Specialisation	No. of classes	No. of students
Language	685	21218
Biology	123	4279
Mathematics	216	6933
Chemistry	136	4429
Physics	188	6054
Drawing	12	374
Singing, music	24	731
Sport	26	872
Other	9	297
Total	1419	45187
Ratio within gymnasium	43.9%	44.3%

Table 4.2 Specialised classes in gymnasiums in 1974/75.

The increasing number of specialised classes led to a kind of 'facultative' or group-based school system, introduced in 1979/80. An 'educational plan', published in 1978, prescribes the numbers of lessons in each subject; see Table 4.3.

Grade:	1st	2nd	3rd	4th	Total
Hungarian language (mainly grammar)	2	2	1	1	6
Hungarian literature	2	3	3	3	11
History	2	2	3	4	11
Ideology	–	–	–	2	2
Russian	4	3	3	2	12
2nd foreign language	3	2	–	–	5
Mathematics	5	4	3	3	15
Physics	2	2	3	3	10
Chemistry	2	4	–	–	6
Biology	–	–	4	2	6
Geography	3	2	–	–	5
Singing, music	1	1	1	–	3
Drawing	1	1	1	–	3
Physical training	3	3	3	3	12
Technology	2	2	–	–	4
General education by the class-master (approx.)	1	1	1	1	4
Profession orientation	–	1	–	–	1
Facultative subject group	–	–	7	9	16
No. of compulsory lessons	33	33	33	33	132
Optional subject	–	2	2	2	6

Table 4.3 Numbers of lessons per week in the new educational system introduced in the gymnasiums in the school-year 1979/80.

This *facultative* system is understood in the following sense. In grades 1 and 2 (age 15, 16) the subject-matter is unified. In grade 2 a special seminar aims to give an orientation to the children in order to decide on a certain special line. After grade 2 the students have to choose one of the groups of 2–3 subjects which the school offers them. The subjects belonging to the chosen groups will be taught in more detail and in more lessons during the last two years of their education. Stochastics, as an independent subject, cannot belong to any group. It is dealt with, however, to a very limited extent, within mathematics, provided mathematics belongs to the chosen group.

It is to be mentioned that the mathematics curriculum shows no sign of being adjusted to the present syllabus of the primary schools. Therefore it must presumably be a transitory one.

Specialisations in mathematics

There are just 7 classes per grade throughout the whole country, in which the syllabus goes fairly deeply into mathematics. These are the only classes at secondary level where an entrance examination is officially approved, and the only specialisation in science, that the new system maintains.

These classes were organised for the particularly bright children who might be expected to become creative mathematicians. For these classes the syllabus prescribes only the framework of the subject-matter, and the teachers have greater freedom both in choosing additional topics and in the methods of teaching. There are 8 lessons per week during all the four years. Common additional topics are: Number theory (prime numbers, number systems etc.); Determinants: Polynomials (and their roots); Combinatorics; Set-theory; Logic, Complex numbers; Probability theory; Methods of numerical approximation; Graph theory. In grades 3 and 4 some 30 lessons are spent on one of the following three areas of application: Computer technology; Mathematical programming; Mathematical Statistics.

Most of the teachers teaching in these classes are well versed in higher mathematics and experts in methodological questions. Most of them exercise some personal preferences in the choice of subject-matter; therefore the classes (even in the same school) have different character.

Mathematical contests

The 'mathematical contests' play an important role in the education of the more talented students. There is a periodical edited for secondary school students: 'Középiskolai Matematikai Lapok' (Mathematical Journal for Secondary Schools) and published by the Bolyai Mathematical Society. It has 10 numbers a year. This periodical publishes 6 elementary problems, 6 more advanced problems, and 4 difficult ones. The students have got one month to send in their solutions to the editor. The solutions are evaluated according to a certain point-system, separately for the different grades; the points are accumulated all the school year round. The periodical judges the entries and publishes the best solutions.

There are two other nation-wide contests for secondary schools, both of them organised jointly by the Ministry of Education and the Bolyai Society: Arany Contests for grade 1 (age 15) and separately for grade 2 (age 16), and National Contests for grades 3 and 4 (age 17–18). These competitions are of two rounds. The winners are awarded 3 prizes and a few honourable mentions. For the first ten top students in the National Contest the University entrance examinations are waived.

4.1.4 *Secondary schools, specialised in certain professions*

There are 75 different specialisations in the secondary schools of this kind. They have a double goal:

a) to give as much knowledge about the line chosen as possible, from both the theoretical and practical point of view,
b) to provide the students with a subject-matter which enables them to continue their studies at a higher level.

The second goal makes it necessary for the subject-matter (at least in the most important topics) to be adjusted to that of the secondary schools of general type. On the other hand they have to spend a lot of time on the special line chosen. In particular, concerning mathematics, this means that the need for mathematics teaching has a wide range. Most of these specialisations have less time to spend on mathematics, and the other subjects need more skill in computation. Therefore, the students here do not have the opportunity to go as deeply into the different topics as the students of general secondary schools. In contrast to this, a lot of time is spent on practising certain computational rules, like the use of trigonometric and logarithmic tables and the slide-rule.

It would seem likely that the material actually covered is much less than that of the gymnasiums, and there would seem to be no way to introduce stochastics material into the subject-matter. This does not apply to a few specialisations, like the one in economics, and the one in computer-technology. Here the syllabus does contain some stochastics even now.

4.1.5 *Training colleges*

As their name indicates the primary task of these schools is to train skilled workers, while attention is also paid to the enlarging of the general culture of the students. During the 3 years of the education in these schools, mathematics is taught in 2,1,1 lessons (respectively) a week. The number of lessons indicates that not too much mathematics can be done. The subject-matter goes hardly beyond the material of the primary schools. These schools are attended mainly by the less talented children, so that the students are already weak in arithmetic. In addition, most of the professions need rather more than simple arithmetic, e.g. there is a need for solving equations, for some geometrical knowledge, etc. The class room activity consists mainly of solving problems that are considered of practical use. For this reason special exercise-books had been compiled, containing problems of practical use in the given profession.

4.2 TEACHER TRAINING

In Hungary, teachers are trained at three different levels, after a succesful secondary school graduation.

1) Teachers for grades 1–4 in general (primary) schools are trained in the so-called Teacher Training Colleges (3 years). Here the present programme enables them to teach the new project in mathematics; they deal mainly with methodological questions, and hardly go beyond the secondary school subject-matter in mathematics.
2) Teachers for grades 5–8 in general schools are trained in the 'Institutes of Higher Pedagogics' (4 years). Here the students are specialised in 2 or 3 subjects. Those who choose mathematics get acquainted with the rudiments of higher mathematics.

3) Teachers for secondary schools are trained in the universities. They study two subjects. Those choosing mathematics must prepare themselves for teaching physics, too. During their 5 years of education the students, specialising in mathematics, are supposed to become acquainted with higher mathematics.

   It must be noted that would-be teachers never meet statistics during their studies in any of these 3 types of institute.

   There are also continuation and refresher courses organised by the National Pedagogical Institute (OPI), the Bolyai Society and by all teacher training institutes. These courses deal with the new methods of teaching, but mainly different aspects of the subject-matter are discussed here.

   Methodological questions are also dealt with in the periodical 'A Matematika Tanitása' (Mathematics Teaching).

4.3 HISTORY OF THE TEACHING OF STOCHASTICS IN HUNGARY

In 1849, probability theory first appeared in the secondary school (gymnasium) curriculum in Hungary.

From that time on, 'inclusion' and 'getting-rid-of' periods alternated. Whenever it was included, however, it was almost completely restricted to combinatorial reasoning. Statistics was hardly mentioned. Some important chronological dates are as follows.

1757: Prof. I. Hatvani of Debrecen published the first Hungarian work on probability. It treated the 'classical' probability theory, but also contained statistical data on mortality and a relating discussion. He had led his students through his book.

1849: The first syllabus which contained probability (as part of combinatorics) was introduced in secondary schools.

1861: The number of mathematics lessons was officially decreased, and probability was omitted.

1871, 1874, 1879: Three consecutive syllabuses included combinatorics and more and more probability in the last year of the gymnasium.

By 1903 only 10 combinatorial problems remained in the text-book as the last traces of probability theory; its teaching had stopped.

1919: The curriculum introduced probability in the 5th grade (age group: 15 years). This curriculum, however, was repealed within one year (because of political reasons).

1924: Probability re-entered the curriculum.

1931: Probability preceded combinatorics and was used to introduce it.

1938: There was no probability (and combinatorics) at all.

1948–1960: The curriculum required the teaching of probability but no text-book contained any section on it. Consequently it is not taught (at first unofficially, later with official approval).

1970: A new text-book was published for final year students, which devoted some 50 pages to combinatorics and probability. Unfortunately, different reasons prevented most of the teachers from teaching it.

1973: The Ministry of Education decided that the children were 'over-loaded'. This was yet another occasion to get rid of probability.

From 1975: A new syllabus is being gradually introduced in the elementary schools, which contains more stochastics than any previous one has done (even for the gymnasium).

1979: There started to be a switch over to a 'facultative' educational system in the gymnasium. Some probability is planned for the final two years, but only for those who have chosen a subject group containing mathematics.

4.4 STOCHASTICS IN THE PRESENT-DAY CURRICULA

4.4.1 *Primary schools*

As mentioned above, in the primary schools a new mathematical curriculum is being introduced. It contains considerable stochastics, starting with the 1st grade (age group 6). The syllabus summarises the areas of combinatorics, probability and statistics, as follows:

> Empirical study of combinatorial problems, searching for laws and rules.
>
> Comparison of the probabilities of different events by observing their frequencies ('more likely', 'less likely', corresponding with greater and smaller probability respectively). Relative frequency. Trend of relative frequencies: probability as a number which is near to the relative frequency in the case of many trials. Probability as a number between 0 and 1. Hypotheses of equal probabilities based on symmetry. Classical probability calculus, connection with combinatorics. Constructing and applying tables of random numbers. Expectation. Sample mean, median, mode. Simple characteristics of (sample) dispersion.
>
> Elements of descriptive statistics.

We have already mentioned that the curriculum is of the spiral type. This means that the different topics turn up repeatedly in the higher grades while getting richer, deeper and more precise. As a general guideline it is stressed that the children should have enough time to spend on getting acquainted with the new material. The subject-matter (*SM*) and the requirements (*R*) in the different grades are detailed as follows (stochastics only):

Grade 1: SM: Games and experiments for distinguishing among 'certain', 'possible, but not certain', 'impossible' events. Recording the results of the experiments.
Collecting, recording and graphically representing data from everyday life.
R: Registering collected data and the outcomes of experiments.

Grade 2: SM: See *SM* for grade 1. Additionally: formulating (defining) different events in simple games, and counting their frequencies.
R: Determining the frequencies of given events in experiments.

Grade 3: SM: Tree-diagram (flow-chart) description of simple experiments.

Listing all possible outcomes of simple experiments. Experimenting with 'more likely' and 'less likely' events. Conjecturing and checking the conjectures empirically. Arithmetic mean, sample mean. First steps toward the concept of median.
R: Correct use of the expressions 'certain event', 'impossible event', 'possible, but not certain event'. Listing all outcomes of simple experiments.

Grade 4: SM: Frequency and relative frequency of random events. Their graphical representation. Constructing examples for 'certain', 'possible but not certain' and 'impossible' events. Ordering events according to their probabilities. Representing and ordering statistical data. Most frequent data value (mode), 'middle number' (median), arithmetic mean (sample mean).
R: Ordering, representing and grouping statistical figures and outcomes of experiments.

Grade 5: SM: Discussing the common properties of earlier random experiments. Relative frequency. The stability of the relative frequency: empirical study only. Probability of certain or impossible events. Assigning equal probabilities to events by symmetry. Deducing probabilities in such cases. Comparison of relative frequencies and assumed probabilities. Characteristics of means and dispersion of statistical figures.
R: The graph of the empirical distribution: histogram. Expressing relative frequencies as decimal fractions and percentages.

Grade 6: SM: Random experiments. Hypotheses and their testing (both statistically and by reasoning) Assigning probabilities by symmetry considerations. Expectation. Empirical comparison of the expectation and the sample mean in a number of cases.
First steps in introducing the notion of independence.
R: Calculating probabilities in simple cases. Comparing the results with observations.

Grade 7: SM: Dependent and independent random events. Real-life examples. Demonstration by random experiments.
Product- and sum-rule. Using tree diagram for assigning probability in cases where symmetry considerations alone would not help.
Optional: Investigating random events by using tables of random numbers. First steps in forming the concept of correlation. The usefulness of the correlation concept.
R: Solving problems of the considered types.

Grade 8: SM: Summing up the student's knowledge about probability. The basic space of events, elementary events. Events as subsets of the basic space. Relative frequency and probability. Independent events. Calculating probabilities if the probabilities of elementary events are given.
R: Solving problems of the considered types. (As stated earlier, the requirements are set to match the less-talented pupils)

The pupils use text-books and work-sheets in their work. *Appendix I* contains some typical material illustrating them. We have chosen two pages from the text-book for grade 3 and some work-sheets for grades 3, 4 and 5.

(Text-books and work-sheets have been issued up to grade 7, only, so far.)

Experiences and reactions are rather mixed, as to the teaching of the subject. The younger teachers were already trained for teaching the 'new math' as it is called. They have no problem. Others obtained some special instruction by attending some special courses organised by the National Pedagogical Institute, which proved, however, to be somewhat inadequate in duration and content. Despite the efforts made by the Institute, the older generation seems to be reluctant. They do not feel a 'steady base' to stand on and are afraid of being 'tricked by the random'. However, by the binding force of the Educational Act, they *are required to* teach probability and cannot refuse to do so. And now we can witness the better side of this binding force. The teachers, who had never picked up any knowledge of probability theory, must start learning it via its teaching. It is to be hoped that within a few years the majority of the teachers will be 'converted' and overcome the present difficulties.

4.4.2 *Gymnasiums (Secondary schools of general type)*

We have mentioned in § 4.3 that there has been no probability material in the syllabus for the gymnasium from 1973 on. The only exceptions are 7 classes, which are highly specialised in mathematics. These include only about 200 students per grade in the whole country. Below, we will detail the relevant subject matter in stochastics. Efforts had been taken to introduce stochastics in the new curriculum. Here are some obstacles in the way of its general introduction.

- Teachers attitudes: They get a formal education in axiomatic probability theory without a real comprehension. No statistics courses are offered. They do not see any real or potential application or applicability.
- There are no acceptable plans to include stochastics problems in future test-papers for matriculation or entrance-examinations.
- Time-factors: Conducting experiments is rather time-consuming. Stochastics is considered as part of mathematics and the number of mathematics lessons are strongly limited.
- Traditional areas defend their positions. (E.g., more than one third of the lessons are devoted to geometry.)
- Combinatorics ('finite mathematics') first: Many reformers share the view that statistics should be preceded by probability and probability by combinatorics. Besides, combinatorics provides a 'steady base' to teachers; therefore its introduction meets less resistance.
- Lack of popular books on the utility of statistics.

The present situation can be termed as a 'compromise'. Stochastics is available only for those who have chosen a subject group containing mathematics. For them, mathematics teaching goes in two versions. In Version A mathematics is taught in 6 or 7 lessons per week, while in Version B the number of lessons is 7 or 8, respectively. Text-books are available for grade 3 in manuscript. Probability theory (and no statistics) was officially planned for

both versions and in both grades. The text-book for version A has, however, no probability at all. The other one contains a chapter on probability theory, intended for some 20 lessons. A survey of this chapter is given in Nemetz (1981).

About the future. In 1983 a considerable proportion of the students coming to gymnasiums will be brought up on 'new-math', and by 1987 all of them will. It is difficult to believe that the officials would disregard and abandon their knowledge and education in stochastics. It is promising in this respect that we have got official permission to carry out experiments in two classes, both of them attended by children who learned the 'new-math' in the primary school. We had to plan the subject-matter for 15 lessons for all the four grades. An account is given on our experiments in § 4.5 below.

4.4.3 *Stochastics in the mathematics classes*

These are the only classes where the teachers have great freedom in choosing and arranging the subject-matter; therefore the syllabus is just an *informative* one:

Grade I (Age 15): Statistics in 15 lessons. *SM:* Grouping and representing statistical data. Frequency, relative frequency. Sample mean. Empirical distribution, histogram. Rules for simplifying calculations.

Grade II: Combinatorics and probability theory in 34 lessons. *SM:* Random events. The algebra of events. Axioms of probability (discrete case only). Combinatorial problems in calculating probability. Mixed problems.

Grade III: Probability in 20 lessons. *SM:* Conditional probability and independence. Binomial and negative binomial distributions. Expectation and other characteristics of probability distributions. Bounds on the difference between the probability and the relative frequency. Simple applications.

Grade IV: No compulsory lessons in stochastics. In this class it is possible to choose among three additional specific areas. These areas are: Computer Technology, Operation Research and *Mathematical Statistics*. The choice is up to the teacher and not the children. Even the subject-matter is fixed by the teachers and it is therefore impossible to provide details. Usually it covers the material of a two-semester university course.

4.4.4 *Secondary schools specialising in certain professions*

It is somewhat surprising that stochastics is neglected in specialisations like telecommunications, transportation, postal communications, hydrology. On the other hand, it is perhaps understandable that most of the specialisations feel that no time can be spared for the subject: they can utilise much better the efforts spent on technology. Of the 75 specialisations only 3 have stochastics in the curriculum. We will discuss them separately.

Computer technology specification (100–150 students)
There are 6 mathematics lessons a week during the 4 years of their education.

Stochastics comes up in the last two grades, preceded by the rudiments of combinatorics. The main components are as follows:

Combinatorics and Probability (about 15 lessons in grade 3 and 30 lessons in grade 4):

- Algebra of events. The notion of event, operations with events. Elementary and complex events. The basic space of events.
- Frequency, relative frequency and its trend. Probability as a measure. 'Classical' probability calculus (Finite case). Assigning probability by combinatorial and geometrical methods. Conditional probability; independence.

Statistics and mathematical statistics (about 25 lessons):
Means and methods of the statistical analyses (grouping, tables, time-series, sample mean, indices 'local sample means', momentums, moving averages). Rudiments of higher-dimensional distributions.
Goals: Skill in solving simple problems in probability. Understanding the basic concepts. Skill in applying statistical methods.

Specialisation in financial management and administration (400–450 students)
No stochastics is included in the mathematics subject-matter. Instead, *descriptive statistics* appears as an independent subject in the last grade, with two lessons per week, making a total of 66 lessons. Main components are as quoted from the syllabus:

- Discussion on the nature of statistics. Statistical populations and criteria. Data collection, sampling units. Questionnaires.
- Presenting statistical data. Grouping. Frequency tables, graphical representation of frequency tables.
- Location parameters for quantitative data, classes of averages, arithmetic mean, mode, median. Average rates of change. Standardisation. Use of averages.
- Statistical index numbers: General concepts, construction of indices. Consumer price index, production index. Chain indices, link relative indices. Uses of index numbers.

Specialisation in accountancy and economics (100–150 students)
Here also, descriptive statistics is an independent subject. It is dealt with at a general level in grade III, with 3 lessons per week (99 lessons a year). In the last grade it has 120 lessons and offers a highly special subject-matter in one of the following 4 sub-areas:

A) Industrial statistics
B) Agricultural statistics
C) Internal-trade statistics
D) Transportation statistics

In grade III the subject-matter is somewhat more detailed than that described above for financial management and administration. Additional topics are:

- Measures of variability for quantitative data (range, mean derivation, standard derivation). Analysing time series by averages, moving averages. Use of statistical tables.
- Organising statistical work. Types of data collection. Laws on supplying statistical data. Official data collecting institutions and their work.

The 4 subspecifications offered in grade IV van be best characterised by quoting the 'Teachers guide and instructions':

These specifications discuss the vital problems and the activities of the chosen area by making use of and specifying the general means and methods of statistics learned in grade III. They have to learn new notions and statistical index numbers according to the practice and demands of the given area. The students have to acquire an ability and skill to use these indexes. They have to know their economic relevance and be able to compare them quantitatively. They have to be skilled also in formulating the results, conclusions and deductions of the statistical analysis, in both oral and written form.

4.5 EXPERIMENTS ON TEACHING STOCHASTICS IN GYMNASIUMS

In 1970 the Bolyai János Mathematical Society initiated a project on making the mathematical curriculum up-to-date for secondary schools. As part of this project we had carried out experiments on teaching stochastics in several secondary schools of Budapest. Different experimental materials had been developed for this purpose by K. Bognár, G. Tusnády and the author. In this section we give a short account on our goals and experiences. Earlier partial accounts were given in Barabás and Nemetz (1978), Bognár and Nemetz (1977), Bognár, Nemetz and Tusnády, G. (1978), Halmos (1975), Herczeg (1977), Nemetz (1970, 1977), Tusnády, G. (1973, 1976) and Tusnády, K. (1973). A fuller account of the recent state of this work is given (in English) in Nemetz (1978) and the material of this section is based on this paper. Some unpublished experimental material providing an empirical background is presented in Appendix II. All the material is gathered together in the book by Bognár, Nemetz and Tusnády, G. (1980). Many seminal ideas and suggestions were obtained from Engel (1970), Feller (1968), Glayman and Varga (1973), Harrison (1966) and Mosteller (1965).

4.5.1 *Goals of teaching stochastics at secondary level*

We had started our experiments by formulating the goals of teaching stochastics. These were as follows:

a) Make the pupils acquainted with the notion of 'randomness'. Make them able to view stochastically the notion of random events.

b) Teach then to evaluate statistical data. They have to be able to understand the nature of a statistical decision, and the limitations of stochastical arguments.

c) The stability of relative frequencies should be made as clear as possible.

d) Get the children acquainted with the basic laws of probability theory and the main ideas of mathematical statistics.

e) Show them how to construct stochastic models in real-life problems. At least, do the first steps in this direction.

As it can be seen, these goals were not (and are not even now) fully specified. This applies especially to point d) where, for example, time limitation may not allow us to proceed as far as we wish.

4.5.2 *General matters concerning experimental materials*

We had agreed that the experimental materials had to match the following general principles:

1) Activity approach is preferred to formal lecturing.
2) Statistics and probability should proceed simultaneously.
3) Considerable time has to be devoted to providing empirical background. (This aspect must be prepared in a way as to permit parts of it to be transferred to primary schools as soon as possible.)
4) The subject-matter has to achieve a balance between problems and situations where
- only purely statistical arguments can be applied
- easy combinatorial reasoning can help
- there are 'exact' solutions which are, however, hard to compute

5) We have to prepare both separate units on stochastics and 'dispersed' parts which can be dealt with in connection with other mathematical disciplines.
6) Exactness is desired wherever it is possible.

We were prepared to meet or encounter the following problems:

- There is a limited range of statistical 'populations' which can be easily inspected and processed by the students. Which kind of statistical data could be collected without sacrificing too much time?
- In our teacher training system there is no place for statistics. Secondary school teachers had never met statistics and their knowledge in probability is also poor. Not surprisingly, most of them are reluctant to teach stochastics.
- Evaluating the knowledge and progress of the students is a difficult problem, especially during the preparatory phase, i.e. when providing them with the empirical background.
- There are three kinds of students, namely the ones who
  - follow their studies in science and technical fields
  - go to study arts (literature, psychology, etc.)
  - do not intend to go to universities.

  They have needs which are obviously different; nevertheless more or less the same subject-matter is desirable.
- The number of lessons in each year is likely to be kept as low as possible, by official policy.
- Conducting statistical experiments is very time-consuming and tiring. Repeating the same experiment many times soon gets very boring.

4.5.3 *Some teaching experiments*

There is a handful of enthusiastic teachers who are keen to contribute to the

education of their pupils by providing them with some extra material. They are willing to sacrifice some of the other lessons to let the pupils become acquainted with mathematical ideas and methods not included in the subject-matter. Our first aim was to get their attention turned towards stochastics. The best way to do this seemingly was to provide them with some material which can be covered during 4–5 lessons and which does not require deeper knowledge. At the same time it should be consistent with our general principles. Of course, our goals were kept to a minimum in this case. As we had put it, we wanted to organise only a short guided tour into the world of RANDOMNESS. We wanted to provide patterns for sequences of independent and dependent random variables, for uniform (discrete) and non-uniform distributions, and to show the character of statistical decision-making. We had looked for problems where the strength of statistical reasoning could be demonstrated. We discovered that Shannon's guessing game was very suitable; it was as if it was invented for this very reason.

Shannon invented his guessing game for estimating the entropy of printed English text, but it can be applied for estimating the entropy of any source with a given finite alphabet. It goes as follows.

The player is told what the source is. He might observe the random variables emitted by the source up to a given time. Then he has to guess the subsequent symbol (random variable). He has to guess repeatedly for the same symbol whenever he fails. If his guess is correct, he is told so, and the number of his guesses is recorded. Then he proceeds to the next symbol. One can get the different patterns mentioned before by suitably specifying the source.

We have choosen the following three specifications:

i) Every subsequent letter of a meaningful Hungarian text.
ii) Every 10th letter of a meaningful Hungarian text.
iii) 'Random' numbers produced by a fair die.

We have added to the material the problem of breaking a secret code consisting of an unknown permutation of the alphabet.

Our conceptions and experiences were published in the Hungarian semi-official periodical on teaching mathematics (Nemetz, 1970; Tusnády, K., 1973). Reflections and experiences were favourable. Especially, written languages proved to be very useful sources for describing the method of stochastical reasoning.

Obviously, real comprehension of the main notions of probability theory and statistics need a more complex introduction. The children have to meet quite a number of practical situations: games, etc. having simple random structure. We have prepared experimental material for this purpose, and tested it during the years 1973–75. Both the essential part of the experimental material and an account on the teaching experiences thereof were published in the periodical mentioned above; see Barabás and Nemetz (1978) and Bognár, Nemetz and Tusnády, G. (1978). A brief account was also given in Bognár and Nemetz (1977). The material consists of 18 games played by the children in pairs. Their observations are collected and discussed.

We have drawn the following conclusions:

– Adequate empirical background can be provided during some 20 lessons.

– The work of recording the observations can be considerably speeded up by purposefully designed protocol-forms given to the children. We can test their understanding by letting them design the protocols on the last lessons.

– The children are delighted to invent strategies to beat the RANDOMNESS. This, in turn, leads them to discover the 'customs of RANDOMNESS', and the need for recording and analysing data.

– The games were regularly followed by classroom discussions. The children eagerly argued with and tried to convince one another. The most complicated issue was the question of independence. This was the only matter where they needed their teacher's guidance.

– Feature extraction is a very important aspect of this preparatory phase. We had found it very useful to help the children in this respect by adding a few questions to each experiment. Any acceptable answer to the questions anabled them to choose good strategies in the games.

– The time needed to carry out the experiments varies a lot from pair to pair. We tried to overcome this difficulty by dealing with more than one game at the same time and by including some problems to think over in relation to the material.

– The most talented children require much less time for introduction, and they would need the 'exact treatment' sooner.

– Even the enthusiastic teachers need a carefully designed teachers' manual which calls their attention to some possible false reasoning.

– There is a definite danger that the passion for gambling takes possession of some pupils.

We were also experimenting with a more direct introduction. It aimed at the introduction of probability as a numerical measure of chance and was intended to show the strong connections between the probability and its statistical estimate, i.e. the relative frequency. We had presented an account thereof at the 9th Österreichischer Mathematikerkongress, Salzburg, September 26–30, 1977.

It is also our goal to get the children acquainted with the basic laws of probability theory. In this respect one must insist on teaching the weak law of the large number, as a minimum. Since our time is limited, we need to seek the shortest way there. This is why we consider only discrete random variables, where there are no measure-theoretic problems. Axioms of probability as measure are straightforward ones in this case for the children having learned the 'customs of RANDOMNESS' from their own experience. Just as another motivation, we discuss the properties of length, area and volume. Then we build on the intuitive notion of independence, and proceed as follows: to the development of the weak law and the central limit theorem as described in detail in Tusnády (1976).

Experimental results using this approach are also promising. We are strongly convinced that implementable, realistic, subject-matter could be built up along these lines. Of course, the teachers have to get accustomed nation-wide to this idea. This led us to concentrate our ideas into a four-lesson radio

course intended both for teachers and students. We then asked them to extend their studies in locally organised 'study circles'. This work was helped by some written material published in the mathematical periodical for secondary school students; see Herczeg (1977).

As mentioned before, a new syllabus containing considerable stochastics is being introduced in the Hungarian primary schools. In the school year 1977/78 we had the opportunity to carry out experiments in the first grade of a secondary school, where all the pupils were taught according to this project during the previous 8 years of their education. The experiment was also extended to a new first-grade class of the same type during the school year 1978/79.

We planned experimental material according the general goals outlined above. The essential part of it is given as Appendix II. It is centred around five contests. The first of them contains just a recapitulation of the knowledge they are supposed (and expected) to have learned earlier. The second contest aims at the introduction of probability as a numerical measure, and, at the same time, paves the way for the concept of maximum-likelihood estimation of the probability of an event. The last three contests allow us to introduce the formal description of observations and random variables. They were intended to show how to concentrate a random variable (and observations) into a few characteristics, like mode-median-expectation, and deviation-variance (and the corresponding sample characteristics).

As a methodological novelty we had chosen 1–3 responsible 'umpires' for all experiments. They were asked to perform and analyse the problems at home. During the lessons they had to do all the administration and answer any questions of the others concerning the experiment. This speeded up the manual work and proved to be very effective in every respect.

Questions used in the contests and experiments were in accordance with the children's subject-matter in other areas of mathematics. Typical questions were:

- How should the umpire choose his 'prediction' in the full knowledge of all observations?
- Give the gain (loss) of the players as a function of their wagers and the relative frequency of the event A (in the 2nd contest).
- Is event A or C the more likely in the first experiment?
- Compare the probabilities of the events A in the 3rd and 8th experiments!

The only aim in presenting Appendix II is to give a more documented picture on the work we are doing. Therefore, we do not wish to analyse it in more detail. We are aware that from just two classes we cannot draw conclusions of general validity. Thus we conclude by presenting some information on the time needed. These figures had been confirmed in 5 more classes during the last two years. All experiments had been done 10 times by all students and the results were accumulated within two 45 minute lessons. Solutions to the minimum problems connected to contests 2, 4, and 5 each required one lesson. The identical algebraic transformation, which helped an easy calculation of the average loss in the second contest, took one lesson, as

Teaching Statistics in Schools throughout the World, 1982, Edited by Vic Barnett

well. Comparisons of the likelihood of the events A, B and C in the individual experiments varied a lot. For this we had used all the 10 further lessons placed at our disposal. Meanwhile, wherever it was straightforward to do, the description of the corresponding random variables was also given. Contests 3, 4 and 5 were treated in grade 2. We had managed to discuss them during 14 lessons. At the end of Appendix II, we give a set of problems introducing Contest 4, illustrating how can one smuggle this subject into traditional areas.

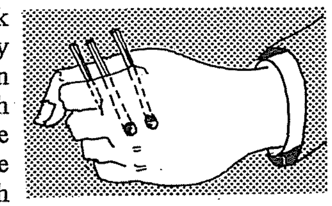
APPENDIX I *Illustrations of Pupils' Material in Primary Schools*

This appendix illustrates the pupils' material in primary schools. We bring two pages from the text-book for grade 3, and some work-sheets for grades 3, 4 and 5.

A. *Excerpt from the text-book for grade 3*

Three girls Mary, Eve and Betty wanted to decide randomly who was going to set the table. But how to choose randomly? They had 5 different ideas of to do it:

1) They would take 3 match-sticks, and break off the head of one of them. Then Mary would take the 3 sticks into her hand in such a way that no one could see which match was headless. Eve and Betty were to pull a stick each, leaving the third one to Mary. The girl with the headless match would set the table (we will say that she is the loser).

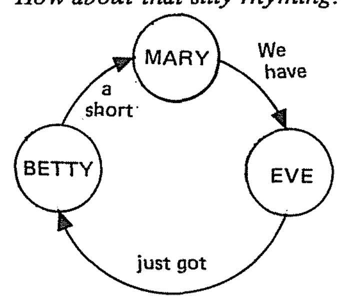


2) They would compose a silly rhyming like this:

   We have / just got / a short / cable /
   Praising / you as / pretty / able /
   Therefore / hurry / set the / table /

   and 'count out' one of the three of them.
3) They would throw a die. If it was 1 then Mary, if it was 2 then Eve, if 3 then Betty was going to set the table. In case of 4, 5, 6 they would throw again.
4) They would open a thick book randomly, and check which of the three initials M, E, B was the first on the chosen page. The girl with this initial would be the loser.
5) They would drop two coins. In the case of two heads Mary, for two tails Eve, otherwise Betty, would be the loser.
   Which methods are fair? Which is the least fair? Why?

How about that silly rhyming?

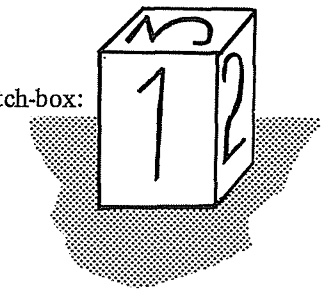


Mary started counting with herself, and Betty was the last. Then she tried in the opposite direction and was not the loser either. The other two began to suspect.
Is there anything wrong? Shall Mary start with Eve?
Try it! What can you see?

And the letters?
What is your opinion on which of these three initials shows up first? Why?
What would be the case with other initials, say A, B, C?
A little statistics:
Count the number of the letters, a, b, c on a randomly choosen page:

	Strokes	Total
a		
b		
c		

B. *Work-sheet for grade 3 (age: 9 years)*



Write numbers onto each side of an empty match-box:
Write 1 on the top
6 on the bottom
2 on one of the longer sides
5 on the other
3 on one of the shortest sides
4 on the other

Throw it up 20 times! First guess how many throws of the match-box will result in 1, 2, 3, 4, 5 and 6.

	1	2	3	4	5	6
Write here your guess						

Try it! Register the results!

Tabulate! Number of

1	2	3	4	5	6
___	___	___	___	___	___

Repeat it! Results:

Number of

1	2	3	4	5	6
___	___	___	___	___	___

C. *Work-sheet for grade 4*

There are 10 pearls in a box: 8 yellow and 2 blue. Shake it, then draw a pearl. Put it back, shake it again, then draw. And so on.

What do you think? Is it possible that
the first pearl is blue? . . . .
the first one is yellow and the 2nd is blue? . . . .
the first two are yellow and the 3rd is blue? . . . .
the 10th draw will be the first blue pearl? . . . .
among the first 10 there will be no blue at all? . . . .
What do you think, which is the most likely? . . . .

Try it! Draw till you obtain the first blue pearl! Repeat it 20 times.
Write here how many drawings were needed to get a blue pearl!

Tabulate these 20 numbers!

The number	1	2	3	4	5	. . .				
was observed so many times										

The most frequent number was . . . .
List the numbers in the order of their frequencies, starting with the most frequent one. . . . .
. . . .

Try it again in a slightly different way: do not put back the pearls till you get a blue one. Then put back all of them, and repeat 10 times.
Tabulate these numbers, too. Is there any difference?

D. *Work-sheet for grade 5 (age: 11 years)*

Follow the commands of the flow-chart:

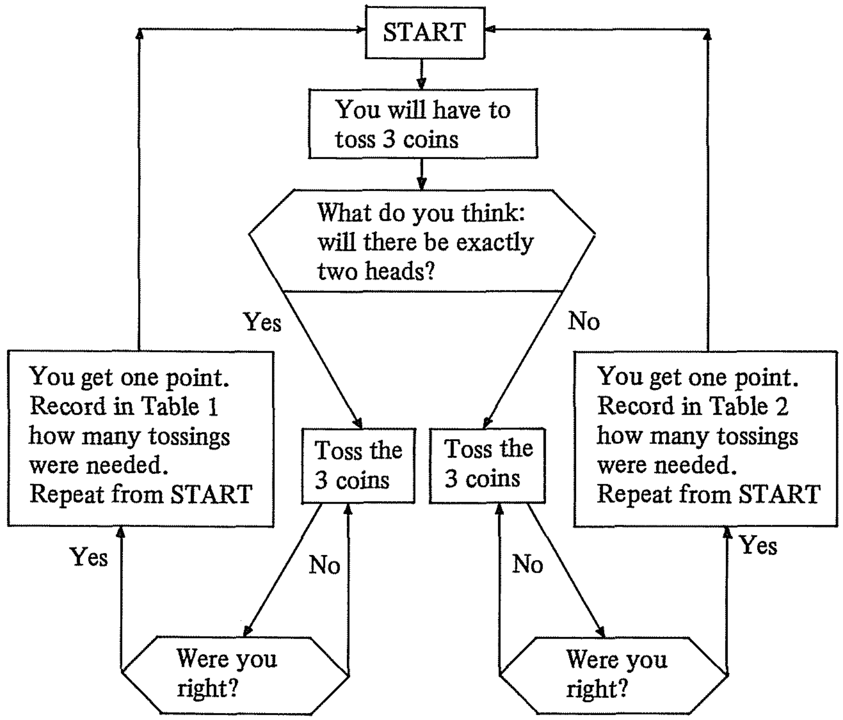


Table 1. No. of tossings				
I thought that there would be exactly two heads.				

There were .... tossings
and I got .... points.
In what ratio of the tossings did
you get points? ....

Table 2. No. of tossings				
I thought that the number of heads would not be two.				

There were .... tossings
and I got .... points.
In what ratio of the tossings did
you get points? ....

APPENDIX II *Part of the Experimental Material for the 'Five Contests'*

You can find 13 experiments described below. All students have to carry out all experiments 10 times. We will choose or draw one (or more) 'umpires' for all experiments. They will be responsible for the tools (or toys) needed. The task of each will be to give any information concerning 'his' experiment, record all outcomes and decide on or lead a discussion on all disputed questions in this respect.

Only integers can arise as the outcome of any experiment. The students have to predict these integers, and 5 different contests will be evaluated according to the 'goodness' of their predictions. Of course, different predictions may be given for the different contests. Everyone has to record his own outcomes, predictions and the goodness of the predictions on a work-sheet, which is enclosed. The five contests are the following:

1st Contest: TOTO

Three events are specified in the case of all experiments (A, B, and C). You are supposed to suggest which of these three events will be observed most often during all performances of the given experiment (i.e. when all students have carried it out 10 times). A correct suggestion is called a hit. Whoever collects the most hits will be the winner of this contest.

(1st column of the work-sheet: your tip, 2—11 columns: your observations, i.e. outcomes of the experiment, 12—13—14 columns: frequencies of the events A, B, C in your own observations.)

2nd Contest: BETTING

You are supposed to offer a wager (denoted by W_{own}) on the occurrence of the event A. The umpire, however, acting as a croupier, may accept or may 'reverse' your offer. He can do so by specifying his wager, called the wager of the bank (denoted by W_{bank}). Only integers between 0 and 100 (inclusive) can be chosen as wagers.

Your prize is counted according to the following rule in the case of *all* observations:

Your prize =

$$\begin{array}{rl} W_{own}, & \text{if } W_{own} < W_{bank} \text{ and A had occured,} \\ W_{own} - 100, & \text{if } W_{own} < W_{bank} \text{ and A had failed to occur} \\ -W_{own}, & \text{if } W_{own} > W_{bank} \text{ and A had occured} \\ 100 - W_{own}, & \text{if } W_{own} > W_{bank} \text{ and A had not occured} \\ 0, & \text{if } W_{own} = W_{bank} \end{array}$$

Fix W_{own} before starting the contest and write it into the 15^{th} column of the work-sheet. Record your prize collected after *all* observations of the given experiment in the 16^{th} column. The player with the largest overall prize will win. Evaluate the contest also in the case when the umpire chooses W_{bank} with full knowledge of the observations!

1. What is the chance of the occurrence of the event A?
2. What wagers would you reverse as Player II? Why?

3. Would you prefer laying a wager on the complementary event?
4. Will you change your wager from game to game? Does the 'history' of the game influence your offers?
5. Can reasoning help you in finding a 'good' offer? If not, what else could help you?

3rd Contest: HIT or MISS
Guess the outcome of the experiment, and write your guess into the 17th column of the work-sheet. You will get a point whenever an observation coincides with your guess. Record your points in the 18th column. In order to win you need to collect the most points.

4th Contest; APPROXIMATE
In this and in the following contest you will not loose everything if you miss. Nevertheless you have to try to give as accurate a prediction as you can, since you have to pay a penalty according to the rule

$$\text{Penalty} = |\,\text{prediction} - \text{observation}\,|\,.$$

Write your prediction before the contest into the 19th column; the sum of penalties for *all* observations on the given experiment into the 20th column. In this contest the winner will be the player with the smallest overall penalty.

5th Contest: LEAST SQUARES
The only difference between this and the previous contest is that now the loss incurred by an incorrect prediction is quadratic:

$$\text{Penalty} = (\text{prediction} - \text{observation})^2.$$

Obviously, you may choose a different prediction for this contest. Write it into the 21st column, and record the penalty collected in the 22nd column. (Note: the prediction need not be an integer.)

Experiments:

1) Throw 20 coins, and count the number of heads.
Event A: there are at most 6 heads
Event B: the number of heads is greater than 6 and smaller than 14
Event C: there are at least 14 heads
Umpire : -
2) Drop 10 drawing pins from a height of 1 metre to the floor, and count how many of them point upwards. Call this number the outcome of the experiment.
Event A: the outcome is smaller than 4
Event B: $4 \leqslant$ outcome $\leqslant 6$
Event C: the outcome is larger than 6
Umpire : -
3) Cast 5 dice, and count the number of sixes
Event A: There is no six
Event B: There are one or two sixes
Event C: There are at least 3 sixes
Umpire : -

4) Cast 5 dice as before and count the number of sixes *and* threes. This will be the outcome of the experiment.
Event A: outcome < 2
Event B: $2 \leqslant$ outcome $\leqslant 3$
Event C: outcome $\geqslant 4$
Umpire : -
5) Choose 'randomly' a letter in an old newspaper, and count the number of vowels in the segment of the following 40 consecutive letters. Use adjacent segments for the 10 experiments!
Event A: There are at most 15 vowels
Event B: $16 \leqslant$ the number of the vowels $\leqslant 20$
Event C: There are at least 21 vowels
Umpire : -
6) Choose 'randomly' a page in a thick book. The number of letters in the 10th word on this page will be the outcome of the experiment
Event A: outcome $\leqslant 3$
Event B: $4 \leqslant$ outcome $\leqslant 6$
Event C: there are at least 7 letters in the word.
Umpire : -
7) Throw 5 dice and add the numbers obtained. This will be the outcome of the experiment
Event A: sum $\leqslant 9$
Event B: $10 \leqslant$ sum $\leqslant 17$
Event C: sum > 17
Umpire : -
8) Throw 5 dice. The outcome of the experiment will be the smallest number obtained.
Event A: the outcome is one.
Event B: the outcome is two or three.
Event C: the smallest number is not smaller than 4.
Umpire : -
9) Choose a page 'randomly' in a 'thick' book, and consider the 10th letter on that page. Starting with this letter count the letters preceding the following first vowel. This is the outcome of the experiment
Event A: Outcome = 1.
Event B: Outcome = 2.
Event C: Outcome $\geqslant 3$.
Umpire : -
10) Find the vowel following the 10th letter of a 'random' page of a book as in the 9th experiment. Starting with this vowel, count the letters preceding the following (first) vowel. This number is the outcome of the experiment. The events A, B, C are specified as in the 9th experiment.
Umpire: -
11) Throw a die till you hit the first six, and count how many throws were needed. This is the outcome.
Event A: Outcome < 4
Event B: $4 \leqslant$ outcome $\leqslant 6$
Event C: Outcome > 6
Umpire : -

12) Throw a die till one of the numbers shows up for the second time. The number of throws gives the outcome.
Event A: there were at most 3 throws.
Event B: there were 4 or 5 throws.
Event C: you could not stop before the 6[th] throw.
Umpire : -

13) Shuffle a pack of cards, then check them one by one till you find the first ace. The number of cards checked is the outcome of the experiment.
Event A: there is an ace among the first five cards.
Event B: $6 \leqslant$ outcome $\leqslant 13$.
Event C: there is no ace among the first 13 cards.
Umpire : -

Set of problems preceding contest 4

1) Divide your birthday by 6. Let a denote the remainder ($0 \leqslant a \leqslant 5$). Throw a die till a number different from a shows up. Denote it by b. Draw the graph of the function

$$f_1(x) = |x-a| + |x-b|$$

Find the set of the minimum points!

2) Draw the graph of the function

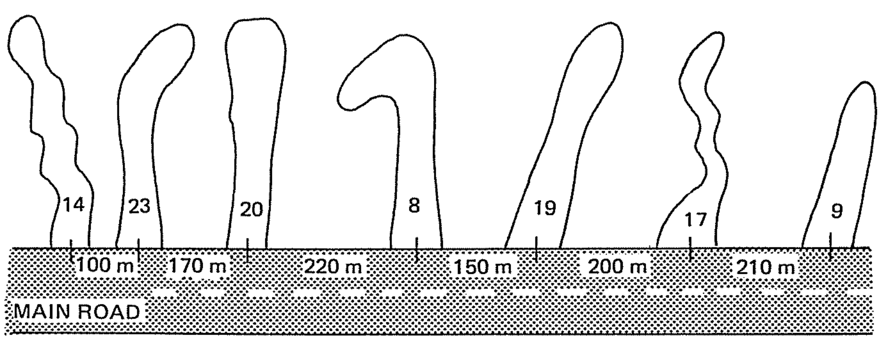
$$f_2(x) = |x+1| + |x-7|,$$

and find the set of the minimum points!

3) Use problems 1 and 2 to deduce the minimum points of the functions

a) $y = f_1(x) + f_2(x)$
b) $y = f_1(x) + 3 \cdot |x-3|$
c) $y = f_2(x) + 3 \cdot |x-2|$

4) All inhabitants of a town take the bus twice every day. The bus runs on the main road and there can be just one stop. The main road can be reached only on the streets depicted in the Figure. The numbers of inhabitants in each of the streets, as well as the distance between streets, are shown. Where should the bus-stop be placed in order to minimize the total distance walked by all inhabitants from home to the bus-stop?



5) Let $a_1, a_2, \ldots, a_{13}$ denote the heights of the first 13 students (in alphabetic order) of the class. You can compare any two of them to find the taller one, but are allowed to measure the height of just one of the 13 students.
Explain how you would find the value minimising the function

$$|a_1 - x| + |a_2 - x| + \ldots + |a_{12} - x| + |a_{13} - x|.$$

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CHAPTER 5

The Teaching of Stochastics in Italian Upper Secondary Schools

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5.1 INTRODUCTION

The structure of the Italian Upper Secondary School (*scuola secondaria superiore*) goes back many years. Practically all curricula were established before 1961 and in the course of the last twenty years have undergone only marginal changes.

Compulsory education begins at the age of six and continues until the completion of the fourteenth year. These eight years (for all students who do not have to repeat a year) cover two cycles of study: Elementary School (*scuola elementare*, a five-year course; ages 6–10) and the Lower Secondary School (*scuola media*, a three-year course; ages 11–13).

Up to 1979, stochastics was absent from the curricula. Only limited experimental teaching had been carried out, involving an insignificant fraction of the total number of pupils.

In 1979 the syllabuses of the Lower Secondary School were reformed. Among the new objectives for natural, chemical and mathematical sciences are those of 'ordering and correlating data' and 'verifying the correspondence between hypotheses formulated and experimental results achieved'. Among the methodological proposals, there is that of 'drawing attention to the differences between the certain and the probable, between mathematical and empirical laws'. Finally, of the six themes into which the syllabus is organized over the three-year course, one concerns stochastics.

5.2 THE TEACHING OF STOCHASTICS IN THE UPPER SECONDARY SCHOOL: THE PRESENT SITUATION

At present 700,000 students, representing 72.3 per cent of the appropriate age of the population, enroll in the first year in the Upper Secondary School (9th grade). Of these, 141,000 (20.2 per cent) choose one of the 'academic' types of school (*liceo classico* and *liceo scientifico*) and 559,000 (79.8 per cent) choose vocational schools (*istituti magistrali*, teacher training colleges for elementary school teachers; *scuole magistrali*, teacher training colleges for pre-elementary school teachers; *istituti tecnici*, technical institutes; *istituti professionali*, training schools for skilled labour; *licei artistici* and *istituti d'arte*, art colleges). Courses in these schools vary in numbers of years from five down (rarely) to two.