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CHAPTER 5

The Teaching of Stochastics in Italian Upper Secondary Schools

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5.1 INTRODUCTION

The structure of the Italian Upper Secondary School (*scuola secondaria superiore*) goes back many years. Practically all curricula were established before 1961 and in the course of the last twenty years have undergone only marginal changes.

Compulsory education begins at the age of six and continues until the completion of the fourteenth year. These eight years (for all students who do not have to repeat a year) cover two cycles of study: Elementary School (*scuola elementare*, a five-year course; ages 6–10) and the Lower Secondary School (*scuola media*, a three-year course; ages 11–13).

Up to 1979, stochastics was absent from the curricula. Only limited experimental teaching had been carried out, involving an insignificant fraction of the total number of pupils.

In 1979 the syllabuses of the Lower Secondary School were reformed. Among the new objectives for natural, chemical and mathematical sciences are those of 'ordering and correlating data' and 'verifying the correspondence between hypotheses formulated and experimental results achieved'. Among the methodological proposals, there is that of 'drawing attention to the differences between the certain and the probable, between mathematical and empirical laws'. Finally, of the six themes into which the syllabus is organized over the three-year course, one concerns stochastics.

5.2 THE TEACHING OF STOCHASTICS IN THE UPPER SECONDARY SCHOOL: THE PRESENT SITUATION

At present 700,000 students, representing 72.3 per cent of the appropriate age of the population, enroll in the first year in the Upper Secondary School (9th grade). Of these, 141,000 (20.2 per cent) choose one of the 'academic' types of school (*liceo classico* and *liceo scientifico*) and 559,000 (79.8 per cent) choose vocational schools (*istituti magistrali*, teacher training colleges for elementary school teachers; *scuole magistrali*, teacher training colleges for pre-elementary school teachers; *istituti tecnici*, technical institutes; *istituti professionali*, training schools for skilled labour; *licei artistici* and *istituti d'arte*, art colleges). Courses in these schools vary in numbers of years from five down (rarely) to two.

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The situation regarding the teaching of probability and statistics in Upper Secondary Schools is indicated in Table 5.1. These are subjects which are taught, if at all, during the last years of the course (11th, 12th, and rarely 13th grade). At these levels the enrollments amounted, in 1979, to some 400,000 pupils: representing 45 per cent of the population over the age of seventeen.

Certain tendencies are emerging. Some room is being found in the vocational schools for the teaching of stochastics, while this work is totally absent from the academic type of schools. In the vocational schools the curriculum of 32 per cent of the pupils in their 12th grade (some 88,000 pupils) entails, at various depths of study, the teaching of stochastics. In the *liceo scientifico* no more than an introduction to the study of combinatorics is envisaged.

Within the vocational school, the teaching of probability and statistics is carried out principally within the 'service' and 'administrative' streams. In the technical- industrial streams, the two disciplines are entirely absent, except in the case of the specialization in informatics (a total of 2,000 students in the 12th grade; 0.5 per cent of the total).

It is a particularly serious matter that in the scientific sector of the academic schools (19.8 per cent of the total student population in the twelfth grade in the Upper Secondary School) stochastics is completely ignored. Equally serious is the fact that this discipline does not figure in the curricula of technical-industrial streams (15.3 per cent of the total school population). In practice, the future technical and scientific cadres carry on their school studies, following an approach that is almost exclusively deterministic, both in mathematics and in the other scientific and technological disciplines.

Not even teacher training colleges for elementary school-teachers make provision in their curriculum for stochastics (in such schools only 12.5 per cent of the timetable is dedicated to the study of mathematics and physics). This is a rather worrying circumstance, because it has not proved at all easy to insert the teaching of stochastics at the crucial level of the elementary school, nor is it likely to prove easy in the future.

The period in which the syllabuses were launched has had a considerable influence on the inclusion in the curricula of stochastics. Probability and statistics are to be found more frequently (either singly or together) as the period in which the curricula were defined approaches the present day. This indicates that a need has emerged, which is confirmed by the experimentation which is going on (see § 5.4 below) and this gives some reason for hope for the future.

The syllabus for stochastics is strictly related to the period in which it was formulated. For probability, this goes from relative frequency, the stability of relative frequency and the basic concepts of probability (in many cases), to a much better articulated syllabus (in two cases) which includes the space of events, the space of probability, conditional probability and independence, random variables, binomial, Poisson and normal distributions and stochastic processes. For statistics, the syllabuses are even more diversified. In a few cases, the course contents are limited solely to frequency distributions and graphical representation. Often the simpler forms of descriptive statistics are thrown in, arithmetic mean and other averages and measures of dispersion. Sometimes, concepts such as index numbers, smoothing, linear regression and correlation are added. Only rarely do the syllabuses envisage the testing of hypotheses and sampling techniques.

For certain streams, in addition to the subjects already indicated, information (and utilization) relative to statistical documentation in a particular sector is given (economic statistics, statistics of tourism, etc.).

The above picture represents the norm for the teaching of probability and statistics in Italy. In reality, nothing has ever been done to check the effective quality of the teaching carried out in schools. It is to be presumed that the standard does not come up to that required, essentially on account of the scant preparation of the teachers.

5.3 TEACHER TRAINING

The teachers in the Upper Secondary School in Italy are almost all in possession of a University degree. Only in very few degree courses (amongst these Mathematics and Physics) is provision made for teacher-training.

Since 1969 a legislative provision allows students on any of the degree courses to decide their own curriculum, by choosing from the various courses offered in each University (throughout the various Faculties). The Board of each Faculty approves these curricula after considering their cultural and professional content. Thus, no subject can be considered obligatory *a priori*. Even so, experience acquired so far allows us to make some assessments.

As far as our disciplines are concerned, statistics is obligatory in the Faculties of Political Science, Statistics, Economics and Business Administration and Sociology. Students in the Faculty of Informatics have an obligatory course, lasting one semester, of stochastics.

In certain other Faculties, statistics and probability are optional subjects; in many cases such courses are totally lacking.

The degree in Mathematics envisages optional courses of probability and mathematical statistics; the degree in Physics provides an optional course in probability.

As the majority of teachers of the subjects listed in Tables 5.1 and 5.2 come from the two last-named courses, it is possible (indeed extremely probable) that these teachers will never have had to deal with stochastics during their own degree courses.

In Table 5.3 a summary is given of some information regarding the qualifications of teachers whose disciplines include probability and/or statistics in the Upper Secondary School.

At the time when staff are recruited the assessment of their preparation in stochastics is rather restricted. At times, during the last twenty years, recruitment has occurred without any assessment at all, on the basis of the University certificates held by the candidates. Requirements for competitive examinations envisage notions of probability and elementary descriptive statistics (frequency distributions and graphical representation, arithmetic mean and other averages, measures of dispersion). Some information is summarized in the last columns of Tables 5.1 and 5.2.

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Refresher courses for the subjects under consideration are not very widespread. Generally they are carried out by associations of teachers of Mathematics (the Italian Mathematical Society – UMI, Mathesis, etc.). Often they are connected with experimental work.

5.4 STOCHASTICS: A SUBJECT COMING INTO ITS OWN

Despite everything, stochastics seems to be gradually gaining elbow room within the Upper Secondary School. The most recent syllabuses give a certain amount of importance to the discipline, both in terms of the number of hours per week dedicated to it and with respect to the contents (nos. 3 and 8 of Table 5.1).

Since 1970, the shorter cycle of Upper Secondary Education in training schools for skilled labour has been experimentally extended, for a part of the relevant student population, up to the thirteenth grade. Almost all of these experimental courses (93.0 per cent) envisage the teaching of the basic concepts of probability and notions of descriptive statistics (Table 5.2). In one case, a fairly wide-ranging teaching programme is envisaged, going as far as the testing of hypotheses (no. 8 of Table 5.2).

Finally, another 8.8 per cent of the total student population in the 12th grade also come to grips with problems of stochastics.

Since 1971, awaiting the total reform of the entire Upper Secondary School system, individual experimental programmes have been started in some schools. In 1979 these involved some 200 Upper Secondary Institutes, that is, 3 per cent of the total number. The percentage in terms of classes and pupils involved was very much smaller (under 1 per cent). In some 70 institutes the experiments have taken an extremely innovative direction compared with traditional school practices. Thus, in 31 centres, probability and more often statistics (and in a few cases demography) appear in the programmes. On average 3–4 hours' teaching per week were dedicated to these subjects. In eight cases the subjects were included among the disciplines taken by all students; in 28 cases among the specialist subjects. (The total exceeds 31 because the subjects could be taken at the same time, at different depths of study, in both groups.)

Another experimental programme worth noting, for its quality rather than extent, is that undertaken by the Italian Commission for the Teaching of Mathematics of the Italian Mathematical Society (UMI) within the framework of certain research contracts stipulated by the National Research Council (CNR). At the end of 1978, the teaching of mathematics at Upper Secondary School level was being carried out experimentally by four teams of teachers in Pavia, Pisa, Trieste and Parma. Other teams were working at Lower Secondary School level in Bari, Cagliari, Genoa and Rome. Of particular interest is experimental work being carried out within the terms of a Mathesis/National Research Council (CNR) contract. This is the RICME project (Renewal of the Curriculum for the Training in Elementary School Mathematics) which involved, at the end of 1977, 276 elementary school pupils in Rome.

A particularly novel and worthwhile aspect of this experimental work is precisely the introduction into the programmes of stochastics.

5.5 STOCHASTICS WITHIN THE FRAMEWORK OF THE REFORM OF THE UPPER SECONDARY SCHOOL

Ever since the mid-sixties and with increasing intensity since 1971, a lively debate has been going on in Italy regarding the reform of the Upper Secondary School. The new norms have yet to be fully worked out, but it is extremely probable that by the mid-eighties a reform will be brought about.

It will then be necessary not to waste the wealth of experience gained from the pupils in the Lower Secondary School (by that time the new school curricula will have been in operation for six years; cf. § 5.1) and to establish the lines for further development in the teaching of stochastics. It is in fact a widely held opinion that room must be found in the reformed Upper Secondary School for probability and statistics. The main obstacle to their insertion in the new programmes is probably the specific training of present-day teachers. In addition, the rigid division of class-subjects makes it difficult to fit them in.

Two lines of development seem to emerge, which could be complementary: that of the mathematicians, for whom emphasis would be placed on probability, to be developed within the framework of mathematics, with the aim, amongst others, of bringing mathematics closer to the realities of everyday life. The other line would be that taken by the statisticians, who aim at statistics, essentially incorporated in disciplines requiring statistical methods (such as economics, technology, chemistry, physics, etc.). The grounding in statistics would be achieved by way of certain unifying methodological concepts, possibly within the framework of the teaching of mathematics. These two lines of development can easily be reconciled and combined effort will probably lead to the introduction of stochastics in the reformed Upper Secondary School.

No.	Type of school	Stream	Subject included	Year in which the programme was established	Subjects taught (a)	Grade in which teaching is undertaken (g)	Average no. of hours per week	Time dedicated to matter of probability and statistics (% with respect to time dedicated to the subject)	No. of students involved on average in the 11th-12th grades	Competence of teachers in probability and statistics at the time of recruitment of teachers (i))
1	Training Schools for Skilled Labour (Accountancy)	Vocational	Applied mathematics	1966	B_1, C_1	11	2	30–50	1.8	B_2, C_1, C_2, C_4
2	Technical Institute for Commerce (except no. 3)	Vocational	Mathematics, Financial mathematics, Mathematical statistics	1961	B_1	12	2	40–60 (h)	19.0	B_2, C_1, C_2, C_4
3	Technical Institute for Commerce (computer programming)	Vocational	Mathematics, probability, statistics	1979	B_2, C_1, C_2 (f)	11–12	5	20–30 (h)	0.3	B_2, C_1, C_2, C_4
4	Technical Institute for Tourism	Vocational	Economics, Statistics, Public finance	1966	C_1, C_2	11	3	40–60	0.5	D_1
5	Technical Institute for Girls (community organization)	Vocational	Accountancy and statistics	1967	C_1, C_2	13	2	100	0.1	none
6	Technical Institute for Girls (dietician)	Vocational	Accountancy, Statistics, Financial mathematics	1967	C_1	13	2	10–20	0.2	none
7	Technical Institute for Accountancy and Foreign Correspondence	Vocational	Mathematics, Applied mathematics, Statistics	1966	B_1	11–12–13	3	20–30 (i)	0.8	B_2, C_1, C_2, C_4
8	Technical Industrial Institute (cibernetics)	Vocational	Probability, Statistics, Operational research	1979	B_2, C_1, C_2, C_3, C_4 (f)	11–12–13	3	80–90	0.5	B_2, C_1, C_2, C_4
	Other vocational schools (a) – whose students are introduced to probability and/or statistics in experimental courses which extend short cycles to the 13th grade (b)	Vocational	–	1961–69	–	–	–	–	47.3	–
9	Liceo scientifico	Academic	Various Mathematics	1970 1961	various A	various 13	various 3	various 0–5	(8.8) 20.0	various B_2
	Other academic schools (c)	Academic	–	1961	–	–	–	–	9.5	–
	Total								100.0	
	Total enrollments in 12th grade								400,000	
	Retentivity (%) (d)								45.0	

(a) This includes the other Technical Institutes, the other Training Schools for Skilled Labour, Teacher-Training Colleges and Art Colleges.
(b) For details, see Table 5.2.
(c) That is, the *Liceo classico*.
(d) With respect to the student population who have completed their 17th year.
(e) A: an outline of combinatorics; B_1: basic concepts of probability; B_2: concepts and simple applications of probability; C_1: frequency distributions and graphical representation; C_2: arithmetic mean and other averages, measures of dispersion; C_3: index numbers, smoothing, linear regression and correlation; C_4: testing of hypotheses. Outline of combinatorics has certaintly been dealt with as the programme envisages probability only.
(f) Half the students follow a slightly reduced programme which is less up-to-date in its contents.
(g) The 11th grade corresponds to age 16, the 12th to age 17 and so on.
(h) In the course of studies, information about statistical documentation is also provided, with particular regard to economics.
(i) The subjects are classified according to the scheme described in note (c); to these are added D_1: elements of economic statistics.

Table 5.1 The Teaching of probability and statistics in Italian Upper Secondary Schools – 1979.

No.	Type of school	Subject included	Year in which the programme was defined	Subjects taught (b)	Grade in which the course is held (c)	Average no. of hours per week	Time dedicated to probability and statistics (% of total time dedicated to the subject)	Students involved in the 11th and 12th grades (%)	Competence of teachers in probability and statistics at the time of recruitment (e)
1	Training School for Agriculture	Mathematics	1970	C_1	11	4	0–5	10.3	B_2, C_1, C_2
2	Training School for Commerce (1st group)	Mathematics	1970	A_2, B_2, C_1	13	2	60–80	35.3	B_2, C_1, C_2
3	Training School for Commerce (2nd group)	Informatics, business statistics	1970	A_2, C_1, C_2	13	3	30–50	9.1	B_2, C_1, C_2
4	Training School for Handicraft (a)	Mathematics	1970	A_2, B_2, C_1	12–13	3	0–5	0.1	B_2, C_1, C_2
5	Training School for Industry (1st group)	Mathematics	1970	A_2, B_2, C_1	12–13	4	0–5	39.9	B_2, C_1, C_2
6	Training School for Industry (2nd group)	Mathematics	1970	A_2, B_2, C_1	12	3	10–20	0.6	B_2, C_1, C_2
7	Training School for Industry (3rd group)	Mathematics	1970	C_1	13	3	10–20	13.5	none
8	Training School for Tourism	Business organization and statistics	1970	C_1, C_2, C_3, C_4	13	4	20–30	4.6	none
	Total							114.3 (d)	
	Students involved in the above-mentioned schools at the 12th grade, as a % of the total enrollment in the 12th grade							35,000 8.8.	
	Students of other experimental courses in the Training School for Skilled Labour in which the teaching of stochastics is not provided for							2,500	

(a) Including some courses in the Training Schools for Girls.
(b) See Note (e) of Table 5.1.
(c) See Note (g) of Table 5.1.
(d) The total exceeds 100 because some students take probability and/or statistics in more than one class-subject.
(e) The subjects are classified according to the scheme described in Note (e) of Table 5.1.

Table 5.2 The teaching of probability and statistics in experimental courses which extend short cycles to the 13th grade (Training Schools for Skilled Labour) – 1979.

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Degree Course	The existence in the University curriculum of probability and statistics (as a % of all disciplines)	Incidence of degrees among secondary school teachers teaching probability and statistics	Graduates in 1977
– Mathematics and Physics	0–2	very high	3186
– Informatics	0–2	low	324
– Engineering	0–2	very low	7107
– Law	0–2	low	6554
– Political Science	4–8	low	2861
– Economics and Business Administration	4–8	low	4209
– Statistics	40–60	very low	179

Table 5.3 The provision for probability and statistics in University curricula followed by teachers; the existence of different University degrees among secondary school teachers of probability and statistics.

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CHAPTER 6

Statistical Education in Schools in Sweden

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In the middle of the sixties when 'new maths' started in Swedish schools and abroad, statistics and probability became part of the general mathematics course. Certain topics were included in the mathematics curriculum whichever subject one took. Demands from universities and other institutes receiving students from the senior high school (*gymnasium*) (10–12th grade i.e. 17–19 year olds) also contributed to the decision to study these topics at school.

The work of the Scandinavian Committee for Modernizing School Mathematics was of great importance in the introduction of statistics and probability. The committee published experimental texts which were used not only in Sweden but also in other Scandinavian countries.

As there was no teaching tradition for these topics, at least not in the schools, great difficulties arose and many questions were posed: for example,

- which topics are realistic to teach to students and at what age,
- how should teaching methods and teaching aids be transferred from university to school,
- how should we train teachers for statistics? (Most of the teachers had not studied statistics or probability at the university at that time.)

The use of the experimental texts mentioned above was received with enthusiasm by the students. However, statistics and probability, as a part of the mathematics syllabus for everybody, encountered severe criticism. There are many reasons why interest subsided. A few of them are:

- probability theory seemed to be too theoretical for many students,
- growing demands from universities for other topics of mathematics left less time for statistics and probability,
- the standardized achievement tests in the 12th grade had not included any problems in statistics since the middle of the seventies.

The last point is of course very important and more about central tests appears later in this chapter. Although there have not been any principal changes made in the curriculum since the middle of the sixties, the general aim and direction have changed. We will try to interpret what is going on in statistics and probability education by giving examples from tests of different types. They will give a better indication of the standard and extent of contemporary teaching in statistics and probability than can be obtained from studying the curriculum.

It is appropriate, however, to first give some information about the Swedish school system before describing the teaching of statistics and probability.