PART 2

North America

Teaching Statistics in Schools throughout the World, 1982, Edited by Vic Barnett

CHAPTER 7

Statistical Education at the School Level in the United States and Canada

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7.1 INTRODUCTION AND SUMMARY

While Canada is somewhat ahead of the United States in the development of high school statistics courses, there really are relatively few teachers in these two countries who feel comfortable teaching statistics. Moreover, there is a dearth of suitable material at that level. Thus, to improve substantially statistical education in the schools in North America, it is clear that we need some appropriate textbooks and additional ways to help teachers learn about statistical thinking and methods.

To understand one attempt that has been made to help correct this deficiency in statistical thinking at the school level, let us briefly review one aspect of the situation. A more extensive exposition on this can be found in the excellent article 'Statistics in the High School Curriculum' by Pieters (1976). After some earlier attempts to start something around the 'Sputnik period' (the late 1950's), Fred Mosteller, as President of the American Statistical Association (*ASA*), addressed the Annual Meeting of the National Council of Teachers of Mathematics (*NCTM*) in April of 1967 and called for the creation of a committee to attack the problem of improving statistical education in the schools. This resulted in a Joint *ASA/NCTM* Committee on the Curriculum in Statistics and Probability, and Mosteller served as the first chairman. Among other things, this committee created two significant publications: *Statistics: A Guide to the Unknown* [Tanur, *et al*, (Eds.) 1972] and *Statistics by Example* [Mosteller, *et al*, (Eds.) 1973]. The former is a collection of essays about statistical applications in a number of fields, written for a general audience. The latter is a set of examples of uses of statistics that could be used to introduce statistical concepts into existing high school mathematics courses.

After these two books were published, the members of the joint committee, then under the chairmanship of Richard Pieters, spoke at many *NCTM* meetings during the years 1973–1976. But the teachers who attended these sessions consistently remarked on the lack of text materials and, of course, on the insufficient education of many of them in statistical methods, particularly of the 'hands on' type of statistics (projects, etc.).

Since 1977, the joint committee, now under the chairmanship of the authors (Hogg, 1977–79, and Swift, 1980–82), has tried to secure substantial funds from various sources to help correct these two problems. So far these efforts have failed! The committee has also tried to move forward

a little without funding, but the progress has been slow. It has done the following.

1. Some preliminary writing has been accomplished, particularly in the areas of 'stem and leaf' displays (and other good descriptive methods) and nonparametric methods.
2. A listing, along with a brief description of each, of almost 100 possible statistical projects has been made.
3. A bibliography, including a few brief remarks about each entry, of appropriate textbooks and handbooks is now in the creative stage.

It should be carefully noted that the joint *ASA/NCTM* committee's efforts are not nearly of the level of those of the *Schools Council Project on Statistical Education* at Sheffield University in the UK. The funding has been, at most, very minimal; and the committee has not accomplished nearly as much as the English group (Peter Holmes, director) has in convincing one of our governments' agencies to finance the total effort. Accordingly, cooperation among these sorts of groups in various countries is highly desirable, and that is the reason why the *ISI Taskforce on Teaching Statistics at the School Level* is so important.

From the joint *ASA/NCTM* committee's limited royalties from the two publications, there have been occasional meetings of the committee and there are plans to help sponsor a few short workshops (each about 2 days in length) for high school teachers in the two countries. Thus we hope to make some progress in improving the situation. However, to have a massive impact on statistical education in our countries, it seems as if substantial support must be obtained from some agency of one of our national governments. The evolutionary process, beginning with the introduction of more statistics in the mathematical education of teachers, will probably be a slow one. Those of us, who feel strongly about the importance of statistical education throughout the school system, believe that we cannot afford to wait that long. However, it almost seems as if another sputnik-type event is needed to provide statisticians and teachers with an appropriate lever to obtain necessary funding from one of our governments.

As the teaching of statistics in the schools has been developed a little more in Canada than in the United States, *a description of the situation in Canada is given first*.

7.2 SCHOOLS IN CANADA

7.2.1 *Organisation*

The educational system in Canada comes under the jurisdiction of the Provincial Governments. There is, therefore, considerable variation from Province to Province as regards the detail of the organisation of the curriculum. In such a brief report as this, it is not possible to give the details for all the provinces that would be necessary for a comprehensive picture. An attempt has been made, however, to present an overview of current trends insofar as this is known to the authors; and we apologize for any omissions.

Education is compulsory for all children aged 6 to 16. The school system spans Grades 1 through 12 in all provinces except Ontario which has a Grade 13 for students who plan to attend college or university. Students from other provinces enter university after Grade 12. Approximately 70 per cent of students would complete Grade 12. Graduation from high school requires the completion of a number of courses, some of which are mandatory. The courses that are required for graduation vary somewhat from province to province, but mathematics is usually compulsory up to Grade 10. A full course of one year typically involves about 110 to 130 hours of instruction, so a normal timetable would include 7 or 8 courses in the secondary grades (8–12) and 6 courses in Grade 13. The timetable becomes course-oriented in the secondary school around Grades 7 and 8. Before that time there is usually a greater integration of subject matter.

A school normally accepts all students from a given geographical area. However, not all schools would offer all courses. For example, a 'Collegiate High School' would probably not offer a full range of technical courses, like drafting and auto-mechanics, and students needing such courses would travel to another school.

The organisation of the grades into schools comes under the jurisdiction of the School Districts (or Boards of Education) into which a province is divided. There is considerable variation and almost every conceivable pattern is to be found. Common patterns (with grades in parentheses) are:

Elementary (1–6), Junior High School (7–9), Senior High School (10–13);
Elementary (1–8), Secondary (9–13);
Elementary (1–7), Junior Secondary (8–10), Senior Secondary (11–12).

7.2.2 *Syllabus and curriculum*

Little can be said about the curriculum that would be true for all provinces. Provinces publish course outlines for the major courses, but the degree to which these outlines are considered mandatory varies. Some parts are considered to be prescriptive. For example, British Columbia has a core curriculum which specifies those items which must be taught. But, for courses or material outside the core, the guidelines are frameworks around which the teacher is encouraged to develop his course. Examples of this would be, perhaps, the Probability and Statistics courses in Nova Scotia and British Columbia.

In some provinces, the guidelines that are issued by the Ministry of Education are not intended to be courses of study. For example, the following paragraph appears in a circular, dealing with the Elementary school curriculum in Ontario.

> ... thus, while the Ministry articulates the board goals, it is the responsibility of the local school boards – through their supervisory officials – to formulate local programs that are within the rationale of provincial policy and at the same time reflect local needs and priorities.

Some provinces, such as British Columbia, have regular assessments in the major areas, leading to an updating or revision of the curriculum. It is possible

that these reviews will mean a greater uniformity from school to school than would be so without such provincial assessment. But, in any case, in a province like British Columbia that has a policy of prescribing course textbooks, it is difficult, on the grounds of expense, for a school to choose the textbooks that it would like to use in a course. Thus, in British Columbia for example, the guidelines for Mathematics in Grades 11 and 12 are able to refer to chapter and page of the prescribed textbooks. In such cases, the guidelines can become somewhat prescriptive. This would not be the case in Ontario where a relatively wide range of textbooks is permitted.

Mathematics is required in most provinces up to Grade 10. There is a single 'course of study' for Mathematics in the earlier grades, but by the time a student reaches Grades 8 and 9, there is usually a choice of levels. These range from enriched or 'honors' (academic) courses to the general-level courses such as consumer and business mathematics. Typically, perhaps 50 per cent of students in Grade 10 might be enrolled in academic courses. But this figure would vary greatly from school to school, and might be higher in a collegiate high school.

Statistics is widely found in the textbooks for general-level courses, but it is largely confined to descriptive material, calculations, and possibly the misuse of statistics. In addition, all mathematics courses at the intermediate level include similar material. It is apparent that many teachers omit these sections on the grounds that they are dull and cannot be made interesting. There does, however, seem to be a trend away from this; a growing number of intermediate teachers are introducing activities of a more experimental nature. Examples of these activities may be found in the *STATISTICS AND INFORMATION ORGANISATION* (Hoffer, *et al*, 1978) package of the *MATHEMATICS RESOURCE PROJECT* at the university of Oregon, USA, and now published by Creative Publications. There is also a move in many provinces towards including a greater emphasis on statistics and probability in the compulsory mathematics courses. See, for example, the later notes on the Ontario Intermediate Math curriculum.

7.2.3 *Teacher training*

Although a degree is mandatory for most teaching certificates, it is still quite rare to find an elementary teacher who has taken any university-level mathematics courses. Even among those teaching mathematics in the Grades 8, 9, and 10, there are not many teachers having more than 3 Math courses on their transcripts. But there is a strong financial incentive, at the very least, to upgrade ones qualifications. Many teachers at all levels are taking further training in mathematics. For example, the University of Waterloo in Ontario offers, using taped lectures, a Master of Math degree by correspondence. This is aimed specifically at mathematics teachers, and it does bring them abreast of current developments. Three of the five courses offered include combinatorics, probability and statistics.

In-service training for teachers of Statistics is important. One illustration of this is the annual *NANAIMO* statistics workshops (*NANSTAT*) on the teaching of Statistics held in British Columbia each February. (See § 7.2.6)

About 15 teachers, most of whom have attended these workshops, are now offering the Grade 12 course in Statistics.

7.2.4 *The process of curriculum revision*

Again, there is no model that would apply to all Provinces. The variation can be seen in the following outline of curriculum revision in British Columbia and Ontario.

In British Columbia the process has been to establish a revision committee for Mathematics. This committee was appointed by the Ministry of Education and consisted of about 12 people from schools, universities, and the Ministry. The committee drew up course outlines and selected the textbooks after consulting on an informal basis with groups of teachers. The outlines were then adopted for use, alongside existing courses for about a year, after which the older courses were phased out. Locally developed courses are permitted, but these may not replace provincial courses. So, for example, a Grade 12 Calculus course may be developed locally, but this is not true of a Grade 12 Algebra course since this latter course is prescribed at the provincial level. This process has recently been modified with the advent of the provincial assessment program. A mathematics advisory committee has been appointed by the Ministry with the advice of the Universities and the Teachers Federation (including the British Columbia Association of Math Teachers). This committee will plan the assessment of the mathematics curriculum and suggest changes in the light of the results and other information. A revision committee will then be given terms of reference to implement a curriculum revision.

In Ontario, a similar committee develops course guidelines which are then circulated among teachers for comment and reaction. The latest revision, which was to be introduced in Grade 7 in 1979, ran into some opposition. The implementation was therefore delayed. As was mentioned earlier, since the policy in Ontario is to provide for local development of specific courses, a suggested revision is a much less prescriptive document than in British Columbia. Thus a High School in Toronto has been able to include two statistics units in each of its Math courses in Grades 10–12 and even more in some Grade 13 courses. This has been achieved within the context of existing guidelines. Such a situation would be less easy to achieve in British Columbia. Variations on this theme can be found in the remaining provinces.

7.2.5 *Statistics in the curriculum*

There seem to be two major trends:

(1) the development of courses devoted to statistics, usually in the terminal year of the secondary schools; such courses presently exist in British Columbia, Manitoba, Ontario and Nova Scotia, and one is under serious consideration in Alberta,

(2) the inclusion of a greater emphasis on Statistics in the regular Mathematics courses. The stimulus for this activity often comes from

teachers who have taught the Statistics courses mentioned above, and who can see the possibilities for the regular courses. Some teachers in Ontario have successfully pursued this approach.

To enable the reader to put some detail into this outline, a brief description will be given of Statistics courses in British Columbia, Manitoba, Ontario and in Nova Scotia. This is followed by a description of the Ontario effort to include more Statistics in the intermediate level mathematics curriculum.

7.2.6 *Existing statistics courses*

(i) British Columbia

A course in Probability and Statistics in Grade 12 was introduced in 1977. The course outline (see Appendix I) is quite bare and is based on the prescribed textbook, *Probability and Statistics* by Willoughby (1977). Only one teacher offered this course in 1977, 7 in 1978, 15 in 1979 and 25 in 1980. The *NANSAT* workshops were started to help teachers with this course. During these workshops, teachers are introduced to an approach that is very heavily biased toward the collection and interpretation of data. The Willoughby book is supplemented by a preliminary draft of the handbook of experiments, *STATISTICS BY EXPERIENCE*, which was compiled for the joint *ASA/NCTM* committee (1978) and by units written in Ontario (Del Grande, *et al*, 1980). Topics covered in the course include the following.

Information from samples, – an intuitive approach to the binomial distribution, practical experience of surveys.
Introduction to probability – tree diagrams, conditional probability, games of chance.
The chi-squared distribution – an experimental approach.
Sampling procedures – Binomial and Normal distributions.
Nonparametric methods.

A great deal of importance is given to projects, especially the final one during which students are taught any further material necessary for successful completion. Project topics have included:

style analysis and identification of Canadian authors; a comparison of emergency ward admissions with rainfall and atmospheric pressure; a comparison of the time taken to sell a house with the selling price and (asking-selling price).

(ii) Manitoba

The course is a half credit course for students in their final year of high school. See Appendix II. It includes the following topics.

Descriptive statistics; Measures of central tendency and variability; Index numbers – weighted means; Probability with emphasis on an experimental approach; Binomial and Normal distributions; Regression lines.

The course is seen as a combination of descriptive and experimental (data

gathering) statistics. A balance between these two components is urged.

The primary reference texts are:

Probability and Statistics – an introduction through experiments by Berkeley (1972).

Statistics and Probability in Modern Life by Newmark (1975).

(iii) Ontario

It can be seen, from what has already been said, that the local development of courses is encouraged in Ontario. In addition, there is no provincially developed statistics course. So it is not surprising that there is a large number of experimental schemes in existence. It would be far too lengthly to describe all of these, so one outline will be given of a course developed by the Scarborough Board of Education for students in Grade 13. The course content (see Appendix III) is standard for an introductory course at this level and includes such topics as the following.

Samples and Populations; Graphical representation; Averages; Measures of variability and skewness; Binomial and Normal distributions; Correlation and prediction; Sampling methods and estimation; Chi-squared; Rank correlation; Transformations.

Experimental methods are encouraged, and emphasis is placed on projects and field trips to industrial statistical departments. Use is also made of the BBC film series on statistical inference. (*MATHEMATICS IN ACTION-STATISTICS*. British Broadcasting Corporation, 1967).

Other experimental courses have been given, and a number of them have extended the provincial course in the mathematics of investment by putting a greater emphasis on statistics than is given in the provincial guidelines.

(iv) Nova Scotia

The course, given in Grade 11 or 12, covers much the same material as previously described ((i), (ii) and (iii)). But there are some unique elements which are worthy of mention, and the following extracts from the course outline (see Appendix IV) are emphasised for the way in which they reflect the most recent trends in teaching Statistics.

'At present there is no textbook, particularly with Canadian content, available for this course. In fact, the nature of the course might be destroyed if a specific text was chosen, and used year after year. The course depends on fresh realistic input. A number of books and publications are listed in the Reference Section. These books offer a well rounded selection of problems and resource material. It is important that a wide variety of problems be used including some from social sciences, biology, medicine, sports, economics, etc.

Teachers are urged to develop their course, using:
the topic outline which follows;
recent material from the media and Government;
local data that can be collected from the school or community;
and problems from the books in the reference section.'

This course is offered in about 15 per cent of the schools in Nova Scotia. A revision of the Grades 9–12 mathematics curriculum is under way, and it is expected that statistics will be a part of the mathematics programme for all students at each grade level. Nova Scotia seems to be a bright spot in the Canadian statistics scene!

7.2.7 *Statistics in regular mathematics courses*

All provinces include some descriptive statistics, as well as some elementary probability, in their regular mathematics courses, usually in the intermediate grades. The evidence seems to suggest that these topics are ignored by a majority of teachers, who see them as optional items.

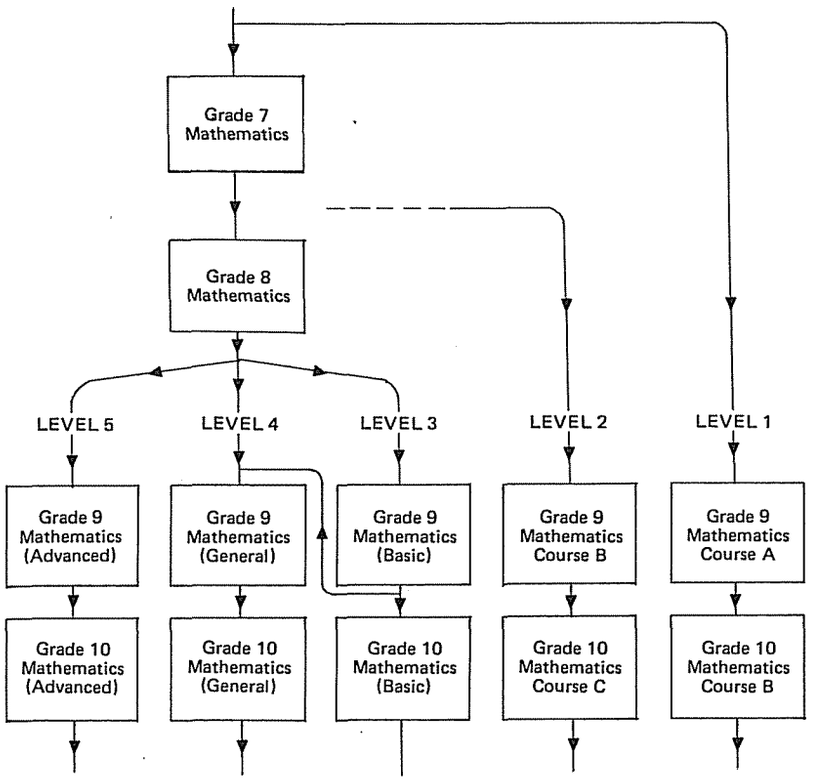
Several teachers in Ontario have, however, been successful in going beyond this level and their efforts have, to some extent, been reflected in the latest revision of the Intermediate Math Curriculum. Prior to this revision, some teachers from the Sudbury Board of Education wrote a Grade 9 statistics unit. This extended the descriptive side of statistics into sampling techniques, statistical tests with samples, and the consideration of possible bias in a sample and in a sampling procedure. At the same time some teachers at A.Y. Jackson Secondary School in North York (Toronto) were writing the units mentioned earlier for Grades 10, 11, and 12, using experimental, intuitive methods to introduce sampling and inference with binomial and Chi-squared models. These units later achieved a wide circulation, including Nova Scotia and British Columbia.

With this activity, and with the presence of one of the Sudbury teachers on the revision committee, it was not too surprising that the statistics content of the intermediate mathematics curriculum was increased. The intermediate curriculum recognises a single level at Grades 7 and 8 but 5 levels at Grade 9 and 10; see Figure 7.1. Statistics or probability appears in every grade and in all 5 levels. The emphases are seen in the extract that follows from the draft copy of the mathematics curriculum guide (1977).

> 'Statistics is becoming increasingly important in our complex world. However, this subject is still new in the elementary and secondary programs and must be treated with care in the mathematics program. When developing programs from these outlines, it should be kept in mind that statistics should be seen by the students as being useful.
> Initially the emphasis should be on the representation of statistical data using various methods. These visual representations should be seen as useful techniques for compressing large amounts of data into comprehensible forms. The second major emphasis should be on inferential statistics; that is, deductions and inferences which can be made from samples. The ideas of sample size, reliability of inference, and so on should be approached intuitively and experimentally at this stage. Measures of central tendency – mean, mode, median – are only of significance in relationship to the previous comments and should not be computed for their own sake.'

Some of the topics in the revised guide are:

OVERVIEW OF THE MATHEMATICS PROGRAM



The chart illustrates the sequence of courses in the Intermediate Division for which the *Outlines of Topics* have been planned. The topics develop sequentially through Grades 7 and 8 and then within each of the five levels in Grades 9 and 10.

Figure 7.1 Mathematics courses in Ontario. (Draft curriculum guide – mathematics, 1977; see § 7.2.7).

Grade 9 Collecting data using questionnaires; random, clustered and stratified samples; practical experience in data surveying; organising data; appropriateness of the mean, median and mode as measures of central tendency; elementary probability using tree diagrams; counting problems in real life situations; probability applied to games of chance.

Grade 10 Concept of a sample space; probability of occurrence and non-occurrence; dependent and independent events; experiments leading to the concept of data forming a distribution; experiments in which statistical measures of samples are compared to corresponding measures of the population; simple inferences; predictions based on sample polls and the reliability of such predictions.

This sample of the topics illustrates the flavour of the new material.

The process of revision in Ontario depends upon feedback and reaction before a new guide is introduced. In this particular case, the reaction was not always positive; and this forced a delay in the implementation which had been scheduled to begin in Grade 7 in the Autumn of 1979. To anyone familiar with developments in England in the 60s, the arguments that followed the publication of this guide must seem very familiar. The guide included much transformation geometry, and this was not well received. In addition, many teachers felt uncomfortable with the increased amount of statistics. So it was clear that better public relations for statistics were necessary. For this, John Del Grande of the North York Board of Education organised a committee to produce statistical materials in an effort to show teachers what is possible in Grades 7–13; moreover, this committee provided reproducible material for use by teachers who wish to try them in the classroom. The committee includes teachers from Sudbury and from A.Y. Jackson Secondary School as well as University representatives from the Universities of Waterloo, Hamilton, Western Ontario and Queen's University. Statistics Canada (the statistics department of the Canadian Government) and practicing statisticians are also represented. The first units were made available early in 1980, under the title *Pre-college Statistics for the 80's* (Del Grande *et al*, 1980) and are being used as supplementary material in courses in British Columbia and Ontario.

The problems that are evident in Ontario are, perhaps, those associated with a large system. This may be one reason why smaller provinces such as Nova Scotia might have greater success in giving statistics its appropriate place in the Curriculum of the 80's.

7.3 SCHOOLS IN THE UNITED STATES

7.3.1 *The normal pattern*

The education system in the United States in such that most of the jurisdiction rests with local school boards, of which there are thousands. Hence

there is a wide variety of patterns, but they are somewhat similar to those in Canada. Most children are in school systems that have 13 grades: Kindergarten (*K*) plus Grades 1–12. In most instances, these are organised into three groupings: elementary school (Grades *K*-6 for ages about 5–12), junior high school (Grades 7–9 for ages about 12–15), and senior high school (Grades 10–12 for ages about 15–18). Sometimes senior high school will include Grade 9 and occasionally the 'middle school' will consist of Grades 6–8 or maybe only Grades 7 and 8.

Due to local control, there are no national nor state tests. However, for the purpose of educational assessment, many schools, and some state systems, do use private testing services. For example, possibly 25 per cent of the school systems throughout the country use the Iowa Tests of Basic Skills for Grades *K*-8 and/or Iowa Tests of Educational Development for Grades 9–12. There are very few questions concerning statistics on these examinations. But, interestingly enough, the results on the few questions that do involve statistics in some way are usually quite poor. After studying some of these mathematics assessment examinations and the corresponding results, it is most clear to the authors that statistics could greatly help improve the performance in many mathematical areas, particularly percentages, fractions, and 'thinking with and about numbers, in general.'

It is true that only an extremely small percentage of mathematics teachers in the US feel comfortable teaching statistics. A few that do have made, with the expenditure of much effort and energy, substantial progress in constructing their own statistics courses. However, due to the fact that these teachers do not have a great deal of time, these efforts are in rough forms that are not suitable for publication. Hence each such course is one designed by and for a particular teacher.

Of course, most high schools use one of several successful series of texts in their mathematical programmes. Each of these includes, at each of the grade levels, some materials on statistics or probability. However, these are viewed by the teachers as optional topics and are frequently not covered in the high school courses. One reason for these omissions is that most teachers have had little in the way of statistical education.

7.3.2 *Other efforts*

In addition to some of the joint *ASA/NCTM* committee's work, there have been other efforts to improve introductory statistical education.

(i) *NCTM*'s programme *PR*iorities *I*n *S*chool *M*athematics (*PRISM*) is considering 10 major strands of mathematics, one of which concerns statistics and probability. There were to be major discussions on each of these ten strands at the annual *NCTM* in the spring of 1980 on the themes 'What is' and 'What Ought to be in the 1980's.'

(ii) Statisticians and high school teachers interested in statistics have continued to make presentations at national and regional *NCTM* meetings.

(iii) The 1981 *NCTM* Yearbook is devoted to Statistics and Prob-

ability. Al Shulte, a former member of the joint *ASA/NCTM* committee, is the editor.

(iv) There have been many strong individual efforts by high school teachers and exciting courses in Statistics and Probability have been given, primarily during Grade 12. Most of these are in rather rough form and the resulting notes are not polished enough for publication. Possibly the greatest effort has been by Cox (1977) but even his *STAT-TREK* notes are not suitable for publication without substantial revision.

(v) *ASA* will attempt to sponsor a few Workshops and 'Statistics Fairs, Contests, or Days' in various areas. A subcommittee of the joint *ASA/NCTM* committee is working on these possibilities.

(vi) A number of in-service courses have been funded by the National Science Foundation and other bodies. These have all been devoted to the teaching of statistics in elementary and secondary schools. Two formats are common. The more intensive format is that of the summer institute, which might last between 2 and 4 weeks for several hours a day. Examples of this kind of workshop are those directed by Dr. Gottfreid Noether at the University of Connecticutt and by Peter Holmes, from Sheffield (UK), at the National Institute of Education. The second format is the 1 year or 1 semester course that would meet for about 3 hours a week for several months. Such courses have been directed by Dr. Maurice Bryson in Colorado, using Cox's *STAT-TREK*, by Dr. Bryant Chow at the University of Southwestern Louisiana and by Dr. Claire Newman at Queen's College, for elementary teachers in New York. The latter course illustrates the great demand for these courses – over 150 teachers applied for the 35 places that were available. In-service courses will continue to be an important part of the efforts to improve the teaching of statistics in schools.

(vii) The Association of Computing Machinery (*ACM*) has a major subcommittee ES^3 (*E*lementary and *S*econdary *S*chool *S*ubcommittee) that has been charged to investigate the use of computers in the schools. Fortunately this group does recognise the usefulness of statistics and its interface with computing, and there is hope of cooperation between ES^3 and the joint *ASA/NCTM* committee.

(viii) While at the college level, many of the modules produced by *UMAP* (*U*ndergraduate *M*athematics and its *A*pplication *P*roject) are appropriate for high school students. Douglas Zahn of Florida State University is in charge of the modules in statistics.

Needless to say there are many activities in this area of which the authors are not aware. The publication of the 1981 *NCTM* Yearbook records many of these and gives a summary of the situation as we enter into the 1980's. With this, along with the recommendations of *PRISM*, interested statisticians should be able to assess the needs of teachers who want to teach statistics at the school level. Hopefully, they, in combination with the teachers, will be able to present a reasonable programme that will have a major impact on statistical education by the year 1990.

7.4 CONCLUSION

The picture presented is essentially one of few materials that use the most modern trends, as well as a reluctance, on the part of teachers, to expose themselves to an area in which they lack confidence. As the reader moves through this report, these will not be unfamiliar words. But, on the other hand, a comparison of this report with those of the ISI round tables of 1973 and 1975 (see Råde, 1975, and Breny, 1976), may give us grounds to feel encouraged.

APPENDICES (Description of some full- or half-year courses in statistics and probability)

APPENDIX I *The Province of British Columbia*

PROBABILITY AND STATISTICS 12 (PS 12)

COURSE OUTLINE

This course is designed for students interested in the application of mathematics to areas such as probability and statistics. Topics may be chosen from the list below to meet the needs, abilities and interests of various groups of students. For example teachers might emphasize applications to biology, business, social sciences. It is suggested that an experimental approach should be used.
Time Allotment: Approximately 100 hours.

Learning outcomes
The student is able to develop an understanding of, and an ability to perform, the required operations for each topic.

1. Algebra of sets
   - $A \cup B$; $A \cap B$; A'
   - distributive law for sets
   - $n(A)$ – number of elements in a set
   - tree diagrams.
2. Permutations and combinations
   - factorial notation
   - binomial theorem
   - counting problems – combinations, permutations
3. Elementary probability
   - sample spaces
   - simple examples
   - conditional probability
   - independent events, mutually exclusive events
   - repeated trials (Bernoulli trials)
   - games of chance
   - expected value
   - mathematical and experimental probability.
4. Statistics (a descriptive approach)
   - representation of data by various types of graphs: mean, median, mode – calculation and use – grouped and ungrouped data
   - standard deviation
   - 'grading on the curve'
   - sampling theory
   - – random samples
   - – variance of samples

   - – small samples
   - – large samples
   - correlation
   - testing hypotheses.
5. Optional topics
   a) Logic
      - $a \wedge b$; $a \vee b$; $a \rightarrow b$; $\sim a$
      - truth tables
      - converse, contrapositive
      - algebra of statements
      - arguments
   b) Review of exponential and logarithmic functions and graphs.
   c) Binomial and normal distributions.

Reference materials for Probability and Statistics 12

Blakeslee, Chinn
Introductory Statistics and Probability (Houghton Mifflin)

Goodman and Ratti
Finite Mathematics with Applications (Macmillan)

Benice
Finite Mathematics with Algebra (Saunders)
– Instructor's Manual

Mizrahi/Sullivan
Finite Mathematics with Applications (Wiley)

Mosteller, *et al.*
Exploring Data, Statistics by Example (Addison-Wesley)
Weighing Chances
Detecting Patterns
Finding Models
– Teacher's Manual and Solution Key

Lipschutz
Finite Mathematics (Schaum Publishing Co.)

M.J. Moroney
Facts from Figures (A Pelican Book)

Kaber and Rynyon
General Statistics, 2nd edition (Addison-Wesley)
– Teacher's Manual
Student Workbook

Mosteller, Rourke, Thomas
Probability with Statistical Applications, 2nd edition (Addison-Wesley)
– Teacher's Manual

Meserve
Introduction to Finite Mathematics (Addison-Wesley)

Ingram
Introductory Statistics (Cummings)

Mosteller, Rourke, Thomas
Probability a First Course (Addison-Wesley)
– Instructor's Manual

Freund
Mathematical Statistics (Prentice-Hall)
Zuwaylif
General Applied Statistics (Addison-Wesley)
Malik and Mulklen
A First Course in Probability and Statistics
Y.E. Bates
Probability (Addison-Wesley)
Morris H. De Groot
Probability and Statistics (Addison-Wesley)
Sets, Probability and Statistics – The Mathematics of Life Insurance
The Canadian Life Assurance Assn.
44 King Street West
Toronto 1, Ontario
Stephen S. Willoughby
Probability and Statistics (Silver-Burdett) (GLC, Canada)
Joan Gary Taylor, *et al.*
Finite Mathematics, 1973 (Harper and Row) (Fitzhenry and Whiteside)
Meyer
Introductory Probability and Statistical Applications
2nd edition (Addison-Wesley)
Wisner
Elements of Availability (Scott, Foresman)

APPENDIX II *The Province of Manitoba*

STATISTICS AND PROBABILITY 305

COURSE OUTLINE
(Half Credit)

Rationale for Studying Statistics

1. Statistics is a branch of mathematics which has applications in many fields – economics, psychology, sociology, agriculture, medicine, business – in fact, any field in which research is involved.
2. Statistics is involved with the collection, organisation, analysis, and interpretation of data. There is a need for citizens to be able to interpret tables, graphs, and data found in newspapers, magazines or on radio and television.
3. Statistical experiments are interesting and are the basis for games theory, decision theory, and quality control in industry.

Calculators

Access to calculators is essential for students. Electronic calculators with the following features are recommended: +, –, ×, ÷, chain operations, memory. (√ is desirable but adds considerably to the cost). Where students have access to the Computer Services Network, PLUM Programs may prove useful.

Introduction

The course is intended as a combination of descriptive and experimental (data gathering) statistics. Teachers should attempt to strike a balance between the two components of the course.

In order to cover the outlined material both student reference texts are necessary. It should not, however, be necessary for each student to have a copy of each text.

The books suggested as teacher references contain additional theory and applications from which teachers should choose according to the interests and needs of their students. *Statistics by Example* (Mosteller *et al*) and other publications contain illustrations of how statistics are used in many fields.

References:

1. *Primary References:*
   a) *Student Reference:*
      Berkeley, E. *Probability & Statistics; an Introduction Through Experiments*. Newtonville, Mass.: Berkeley Enterprises, 1972.
      Newmark, J. *Statistics and Probability in Modern Life.* Toronto: Holt, Rinehart and Winston, 1975.
   b) *Teacher Reference:*
      Alder and Roessler. *Introduction to Probability and Statistics*. San Francisco, California: Freeman & Co., 1972.

Mosteller, F. *et al. Statistics by Example* (set of four). Don Mills, Ontario: Addison-Wesley, 1973.
Naiman *et al. Understanding Statistics*. Toronto: McGraw-Hill Ryerson Ltd. 1972.
Spiegel, M.R. *Theory & Problems of Statistics*; Schaum's Outline Series. Toronto: McGraw-Hill, 1961.

2. *Supplementary References:*

Auslander *et al. Mathematics Through Statistics.* Baltimore, Md.: Williams & Wilkins Co., 1973.
Bartholomew and Bassett. *Let's Look at the Figures*. Markham, Ontario: Penguin Books, 1971.
Buckeye, D. *Experiments in Probability and Statistics*. Troy, Michigan: Midwest Publications, 1970.
Malik and Mullen. *A First Course in Probability and Statistics*. Don Mills, Ontario: Addison-Wesley, 1973.
Mosteller *et al. Probability With Statistical Applications*. Don Mills, Ontario: Addison-Wesley, 1972.
Tanur *et al. Statistics: A Guide to the Unknown.* Toronto, Ontario: Holden-Day, 1972. (available through *NCTM*).
Willoughby, S.S. *Probability & Statistics*. Morristown, N.J.: Silver Burdett, 1968.

TOPIC OUTLINES

Unit I Organisation of Data

1. *Description of data*
   a) frequency distributions
   b) graphical representations
2. *Measures of central tendency*
   a) summation notation
   b) mean, median, mode
3. *Measures of variability*
   a) range
   b) mean deviation
   c) variance, standard deviation
   d) percentiles, z-scores

Unit II Index Numbers

1. Price relatives
2. Simple aggregate method
3. Weighted aggregate method
4. Changing the base period of index numbers

Unit III Probability

Note: An experimental approach is intended here.

1. Introduction – selected experiments
2. Calculation of probability

   a) definition of probability
   b) permutations and combinations
   c) rules of probability
      – $P(A \text{ or } B) = P(A) + P(B)$ (mutually exclusive events)
      – $P(A \text{ and } B) = P(A) \cdot P(B)$ (independent events)
      – $P(A \text{ and } B) = P(A) \cdot P(B/A)$ (conditional probability)
3. Applications of probability

Unit IV Binomial Distribution

1. Binomial expansion
2. Properties
3. Applications – distribution and variability

Unit V Normal Distribution

1. Properties of normal curve
2. Applications – distribution and variability

Unit VI Regression Lines

Teaching Notes

Note: Bracketed references refer to teacher rather than student materials.

Unit I Organisation of Data (Suggested time: 16 hours)

1. *Description of data*
   a) Frequency distributions – class boundaries, class interval, class marks, frequency, cumulative frequency.
      – Newmark; Chapter 2, 3
      – (Schaum's; Chapter 4)
      – (Alder; Chapter 4)
   b) Graphical representations or display of data – distinguish, between use of histograms, frequency polygons, circle graphs, and pictographs or ideographs.
      – Newmark; Chapter 2
      – (Schaum's; Chapter 2)
      – (Alder; Chapter 2)
2. *Measures of central tendency*
   a) Summation notation
      – Newmark; Chapter 3
      – (Schaum's; Pages 45, 50–51, 64)
      – (Alder; Chapter 3)
   b) Mean, median, mode
      – Newmark; Chapter 3
      – (Schaum's; Chapter 3)
      – (Alder; Chapter 4)
3. *Measures of variability* – include range, variance, mean deviation, standard deviation, percentiles, z-scores.
      – Newmark; Chapter 3
      – (Schaum's; Chapter 4)
      – (Alder; Chapter 4)

Unit II Index Numbers (Suggested time: 4 hours)
NOTE: Unit II can be taught at any time during the course.
1. Price relatives
2. Simple aggregate method
3. Weighted aggregate method
4. Changing the base period of index numbers
   – (Schaum's; Pages 313–314, 317–318, 319–320, Chapter 17)
   – (Alder; Chapter 14)

Unit III Probability (Suggested time: 16 hours)
NOTE: An experimental approach is intended here.
1. *Introduction*
   Experiments 1, 2, 5 and 6 from *Probability and Statistics – An Introduction Through Experiments* by Edmund C. Berkeley would be suitable as an introduction to probability.

   Experiment 1 – flipping a coin 50 times
   Experiment 2 – flipping coins fairly and unfairly
   Experiment 5 – rolling one die
   Experiment 6 – repetitions of rolling one die – shows that in the long-run the probability of rolling, say a 4, approaches the fraction one-sixth.

2. *Calculation of Probability*
   It is strongly recommended that experiments 8–11 in *Probability and Statistics – An Introduction Through Experiments* by Berkeley be done by all students.
   Experiments 6, 15, 22, 29, 32, 35, 40, 42, 44, 48, 52, 57, 60, 61, 62, and 64 in *Experiments in Probability and Statistics* by Donald A. Buckeye involve calculating simple probabilities and may be used where time permits.
   Problems involving calculations through definition are found in Newmark; Chapter 5.

   Use permutations only to show the development of combinations. Define combinations and do calculations using the formula

$$C(n,r)=\frac{n!}{(n-r)!r!}$$

   Sections on permutations and combinations are found in Newmark; Chapter 5.
   Experiments 3, 59, 67 in *Experiments in Probability and Statistics* by Buckeye involve permutations and combinations.
   Sample problems involving only the basic laws of probability
   [$P(A \text{ or } B) = P(A) + P(B)$; $P(A \text{ or } B \text{ or both}) = P(A) + P(B) - P(A \text{ and } B)$; $P(A \text{ and } B) = P(A) \cdot P(B/A)$, independent events $P(A \text{ and } B) = P(A) \cdot P(B)$] should be done.
   – Newmark; Chapter 6
   – (Schaum's Outline – Chapter 6)

3. *Other Applications* – examples based on real-life problems.

   Teachers or students may select readings from the following:
   Statistics by Example – Finding Models
   Statistics by Example – Weighing Chances
   Statistics by Example – Detecting Patterns
   edited by Mosteller *et al* (Addison-Wesley)
   Statistics: A Guide to the Unknown – Tanur, Mosteller *et al* (*NCTM*)

   Statistics Canada and Statistics Manitoba publish reports which show applications of statistics.

Unit IV Binomial Distribution (Suggested time: 10 hours)
Binomial Expansion
Properties
Applications – distribution and variability
– Berkeley-Experiments 15, 16: Pages 48–56; and their supplements: Pages 58–63
– Newmark; Chapter 7
– (Schaum's; solved problems Pages 125–128, exercises Chapter 7)
– (Alder; Chapter 6)

Unit V Normal Distribution (Suggested time: 6 hours)
Properties of Normal Curve
Applications – distribution and variability
– Berkeley-Experiment 19; Pages 77–84; other experiments Page 85
– (Schaum's; Chapter 7); Newmark: Chapter 8
– (Alder; Chapter 7)

Unit VI Regression Lines (Suggested time: 6 hours)
– Newmark; Chapter 9
– (Schaum's; Pages 220–221 for formula(e))
– (Murdoch and Barnes; Chapter 9)
Data from biology, chemistry, etc., may be used for application of regression lines.

NOTE: Some students may be interested in extending their study of regression lines to include the computation of correlations. Formula(e) available Schaum's, Pages 244–245, Newmark, Chapter 9, or Alder, Chapter 12.

ANNOTATIONS
STATISTICS AND PROBABILITY 305

PRIMARY REFERENCES
a) *Student Reference:*
Berkeley, E. *Probability & Statistics; an Introduction Through Experiments.* Newtonville, Mass.: Berkeley Enterprises, 1972,
The purpose of this book is to provide an introduction to probability

and statistics by means of actual physical experiments. Considerable detail is provided for each experiment.

Newmark, J. *Statistics and Probability in Modern Life*. Toronto: Holt, Rinehart and Winston, 1975.
This book presents statistical concepts in a complete yet understandable way. Examples are plentiful and are chosen from a variety of subject areas. Appropriate explanations of statistical theory are given and the 'whys' of statistical methods are explained. The text contains an abundance of exercises.

b) *Teacher Reference:*

Alder and Roessler. *Introduction to Probability and Statistics*. San Francisco, California: Freeman & Co., 1972.
This book is suitable for a basic statistics course serving students from social, physical, and biological sciences. Two years of high school algebra are required as mathematical background for this text. Graded exercises chosen from various fields of specialization are given at the end of each chapter.

Mosteller, F. *et al. Statistics by Example* (set of four). Don Mills, Ontario: Addison-Wesley, 1973.
This series presents a set of readings on projects carried out by the authors. Examples are drawn from many fields – social science, biology, ecology, etc. The series may be used as supplementary readings or as a source of project ideas.

Naiman *et al. Understanding Statistics*. Toronto: McGraw-Hill Ryerson Ltd., 1972.
This textbook illustrates statistical principles through a series of examples and questions which are unusual and modern. Material is presented at an intermediate to high reading level.

Spiegel, M.R. *Theory & Problems of Statistics*; Schaum's Outline Series. Toronto: McGraw-Hill, 1961.
Schaum's outlines are designed to supplement courses. Each topic is introduced by a brief presentation of material, followed by a collection of completely solved representative problems and a section of supplementary problems for which answers but not solutions are given.

II. *SUPPLEMENTARY REFERENCES*

Auslander *et al. Mathematics Through Statistics*. Baltimore, Md.: Williams & Wilkins Co., 1973.
This book presents mathematics at an elementary reading level and is suitable for students of limited mathematical background.

Bartholomew and Bassett. *Let's Look at the Figures*. Markham, Ontario: Penguin Books, 1971.
This book presents a series of readings to illustrate the ways in which statistics may be used in real-life situations.

Buckeye, D. *Experiments in Probability and Statistics*. Troy, Michigan: Midwest Publications, 1970.
A series of 81 short experiments. Little detail is given.

Malik and Mullen. *A First Course in Probability and Statistics*. Don Mills, Ontario: Addison-Wesley, 1973.
This book requires a mature level of pre-calculus mathematics. Examples are profuse and many are worked out in detail.

Mosteller *et al. Probability With Statistical Applications*. Don Mills, Ontario: Addison-Wesley, 1972.
The authors introduce each new concept through examples. Additional examples are given after important definitions or theorems. A thorough knowledge of high school algebra is assumed.

Tanur *et al. Statistics: A Guide to the Unknown*. Toronto, Ontario: Holden-Day, 1972 (available through *NCTM*).
This book consists of a series of essays which illustrate past and current uses of statistics and probability. The text may be used as supplementary reading or as a source of ideas for projects and assignments.

Willoughby, S.S. *Probability & Statistics*. Morristown, N.J.: Silver Burdett, 1968.
This book covers most of the content in Statistics and Probability 305. The concepts involved require some knowledge of algebra and a reasonably high level of maturity.

APPENDIX III *The Board of Education for the Borough of Scarborough*

STATISTICS

COURSE OUTLINE

1. *RATIONALE*

a) The contribution which a study of statistics can make to the more adequate development and understanding of many subjects offered at the post-secondary level has long been recognized. Statistics is literally indispensable in such fields as physiology, medicine, psychiatry, the social sciences, engineering, educational research, marketing research, advertising, agriculture, biology, economics, home economics, the manufacturing industries and in research generally.

b) A very significant number of our year five students do not enjoy studying the type of mathematics offered in the Analysis courses. Some students actually need a mathematics course that is of some immediate practical value to them, and 'relevant' to their future needs and interests. A large number of Grade 13 students hope to major in the humanities; sociology, psychology, economics or business administration. This course might very well be an alternative. The course in statistics would meet the needs and interests of the student who is not oriented to mathematics and/or sciences.

c) Finally, where the student does not continue with his formal education after year 5, but leaves school to join a business firm, his knowledge of statistics would in all probability stand him in good stead.

2. *OBJECTIVES*

a) To introduce students to analysis of data and the drawing of inferences from such analysis.

b) To introduce students to variability and uncertainty.

c) To introduce students to methods such as interval estimation and hypothesis testing for making decisions from observed data.

d) To highlight desired emphases and utilize a variety of approaches to enhance interest and motivation e.g., utilization of computers, experiments, etc.

e) To enable students who are weak in *abstract axiomatic* mathematics to obtain a credit in applied mathematics to satisfy admission requirements of post-secondary educational institutions, if they wished to go on.

f) To give students a sufficient background in statistics to enable them to understand advanced statistics courses in psychology, the social sciences, advertising, marketing research, agriculture, economics and biology.

3. *CONTENT*

TOPIC	*HOURS TO BE DEVOTED TO EACH*	Running Total
Variables and graphs	4	
Frequency distributions	8	12
Averages: Mean, Median Mode and other measures of central tendency	5	17
Measures of Variation, Skewness, Kurtosis	7	24
Elementary probability theory	7	31
Binomial distribution	10	41
Prediction in relation to correlation	6	47
Introduction to statistical decision theory	7	54
Tests of hypothesis and significance	7	61
Chi-squared test	7	68
Rank correlation methods	6	74
Other varieties of correlation	10	84
Multiple and partial correlation	7	91
Transformations: Their nature and purpose	7	98
Structure and planning of experiments	8	106
Analysis of variance, one-way classifications (Optional)	4	110

4. *TEACHING APPROACHES AND STRATEGIES*

a) Normal classroom procedure.

b) Mathematics laboratory approach (experimental)

c) There is a complete film series on 'Statistics' produced by the BBC. This series has been purchased by the Scarborough Board of Education.

d) Many videotapes on 'statistics'.

e) A number of projects to be undertaken by students.

f) Field trips to large manufacturing firms to study quality control.

5. *US TEXTS*

'Basic Concepts of Probability and Statistics' by J.L. Hodges & E. Lehmann, Second Ed. 1970, Holden-Day Publishing Co.

REFERENCE TEXTS

'Statistics' Schaum's Outline Series, McGraw-Hill

'Statistics' Macmillan of Canada has many books

6. *REFERENCE*

Many other appropriate reference books in the Public Libraries in Scarborough, Stephen Leacock Collegiate Institute School Library, Board of Education Professional Library.

(Data: re-exam results), Surveys to be conducted in school.

Ample supply of data in Stephen Leacock Collegiate Institute.

Guest Speakers:

Mr. J.W. Fencott, Co-ordinator of Mathematics, Dr. H. Dilling, Director of Research, Mr. R. Stevens, Research Assistant, would be willing to assist as consultants.

*STATISTICS COURSE 5AO *ST*

To be offered at Stephen Leacock Collegiate Institute.

BASIC IDEAS IN STATISTICS
- statistics as the study of populations
- statistics as the study of variation
- samples and sampling
- parameters and estimates
- variables and their classifications – discrete (discontinuous) and continuous variables; nominal variable, ordinal variable; interval variable; rational variable.
- experimental and correlational investigations, units of measurement.

SUMMATION NOTATION (Σ). Theorems and properties of summation. Frequency distributions and their graphic representations.
- classification of data
- conventions regarding class intervals
- exact limits of the class interval
- distribution of observations within the class interval histograms
- cumulative frequency distributions
- tabular representation
- graphic representation of frequency distributions
- frequency polygons; cumulative frequency polygons
- skewness

AVERAGES
- arithmetic mean; calculating the mean from frequency distributions; change of origin and unit
- alternative method of calculating the mean
- the mean of combined groups
- some properties of arithmetic means and theorems
- median; calculating the median from frequency distributions
- mode; comparison of the mean, median and mode

MEASURES OF VARIATION, SKEWNESS AND KURTOSIS
- the range
- the mean deviation
- the sample variance and standard deviation
- calculating the sample variance and the standard deviation from ungrouped data
- calculating the standard deviation from a frequency distribution
- effects of grouping
- the effect on the standard deviation of adding or multiplying by a constant
- standard deviation of the first n integers
- the variance of combined groups
- standard scores
- advantages of the variance and standard deviation as measures of variation
- moments about the mean
- measures of skewness and kurtosis.

PROBABILITY AND THE BINOMIAL DISTRIBUTION
- appropriate theorems in permutations and combinations
- that lead into probability theory
- joint and conditional probabilities
- addition and multiplication of probabilities
- the Binomial theorem; Pascal's triangle
- the Binomial distribution; properties of the binomial
- a hypothetical experiment

THE NORMAL CURVE
- functions and frequency curves
- the normal curve
- areas under the normal curve, properties of the normal curve

CORRELATION
- definitions of terms; scatter diagrams
- relations between paired observations
- the correlation coefficient
- calculation of the correlation coefficient from ungrouped data
- bivariate frequency distributions
- theorems of variance of sums and differences

PREDICTION IN RELATION TO CORRELATION
- linear regression of Y on X
- linear regression of X on Y
- relation of regression to correlation
- errors of estimate
- the variance interpretation of the correlation coefficient

SAMPLING
- methods of sampling
- sampling errors
- sampling distributions
- sampling distribution of means from a finite population
- sampling distribution of means from an indefinitely large population
- sampling distribution of proportions
- sampling distribution of difference

ESTIMATION
- properties of estimates; terminology defined
- confidence intervals for means of large samples
- the distribution of t
- degrees of freedom
- standard errors and confidence intervals of proportions
- standard errors and confidence intervals of other statistics

TESTS OF SIGNIFICANCE: MEANS:
- null hypothesis
- two types of errors
- levels of significance
- directional and nondirectional tests

- significance of the difference between two means for independent samples
- significance of the difference between two means for correlated samples
- significance of the difference between means where population variances are unequal
- significance of the difference between means when the population distributions are not normal

CHI SQUARED:
- sampling distribution of Chi Squared
- test of independence
- contingency table of R rows and C columns; degrees of freedom
- the application of χ^2 (Chi Squared) in testing the significance of the difference between proportions

RANK CORRELATION METHODS:
- Spearman's coefficient of rank correlation ρ
- Spearman's ρ with tied ranks
- testing the significance of Spearman's ρ

OTHER VARIETIES OF CORRELATION:
- contingency coefficient
- Phi coefficient (ϕ)
- point bi-serial correlation

TRANSFORMATIONS:
- transformations to standard measure
- calculation of percentile points and ranks

APPENDIX IV *The Province of Nova Scotia*

STATISTICS

COURSE OUTLINE

Rationale

People in our society need to make sense of the bewildering array of statistics and data with which they are constantly confronted. The individual has only to open a newspaper or watch television to be confronted by numerical information of all sorts; graphical data, forecasts, advertisers' claims, opinion polls and technical data. An informed person should know how statistics are used in the solution of questions from local affairs to national problems involving unemployment, health and airplane safety. Education should include the opportunity for students to learn some of the ways in which data can be collected, used and misused. People need training in order to look at this information critically and use it effectively. Therefore, the study of statistics should be introduced in the secondary school.

Aims

The aims of this course are for students to be able to:

(1) understand, then be critical of, the statistical information found everywhere around them
(2) recognize situations that call for statistical reasoning
(3) determine appropriate statistical methods for these situations
(4) understand and use some of these methods effectively

This guide outlines a one-year statistics course to be taught at the Senior High level. Although the only actual prerequisite would be basic mathematics, the nature of the course demands a certain amount of mental maturity as well. We would therefore suggest that the course in the form outlined would be most appropriate for mature grade eleven or twelve students.

General Teacher Notes

Statistics 012 should be a practical course based on applications to real problems using real data. It should involve the students in statistical surveys, experiments and projects. This is not intended as a replacement for the first statistics course at a university. Statistics 012 is planned to involve students in doing statistical experiments not in just reading about them. An elementary and intuitive understanding is the goal.

At present there is no textbook, particularly one with Canadian content, available for this course. In fact, the nature of the course might be destroyed if a specific text was chosen and used year after year. The course depends on fresh, realistic in-put. A number of books and publications are listed in the Reference section below. These books offer a well rounded selection of problems and resource material. It is important that a wide variety of pro-

blems be used including some from social sciences, biology, medicine, sports, economics, etc.

Teachers are urged to develop their course using the topic outline which follows; the recent material from the media, government publications, local data that can be collected from the school or community and problems from the books in the reference section.

TOPIC OUTLINE

1. What is Statistics?
   (a) statistics everywhere
   (b) basic definitions
   (c) use and abuses
2. Collecting Data
   (a) sampling
      (1) simple random
      (2) stratified random
      (3) non-random
      (4) capture and recapture
3. Organizing and Displaying Data
   (a) graphical displays
   (b) frequency distributions
   (c) common forms of the frequency curve
4. Measures of Central Tendency and Dispersion
   (a) mean, median, mode for ungrouped and grouped data
   (b) range and standard deviation
   (c) relationship between the standard deviation and the normal curve
   (d) some measures of an individual in a population (z scores and percentiles)
5. Probability
   (a) simple probability models
   (b) equally likely events
   (c) the complement of an event
   (d) addition and multiplication rules
   (e) expected value of an event (lotteries, business decisions, etc.)
6. Normal Distribution
   (a) properties
   (b) raw scores to z scores and vice versa
   (c) area from z scores
   (d) using the normal curve table
7. Binomial Distribution
   (a) binomial experiments
   (b) Pascal's Triangle and the binomial expansion
   (c) normal approximation to the binomial
   (d) quality control
9. Hypothesis Testing
   (a) binomial one-sample

   (b) binomial two-sample
   (c) sample means – large sample
   (d) sample means – small sample
10. Estimation and Confidence Intervals
   (a) point and interval estimates
   (b) confidence intervals
   (c) maximum error of estimate
11. Test of Variance
   (a) one sample Chi Squared
   (b) two sample
   (c) comparing means of several populations
12. Correlation and Prediction
   (a) scattergrams
   (b) linear correlation
   (c) coefficient of correlation
   (d) linear regression
13. Optional Topics
   (a) non-parametric tests
   (b) conditional probability
   (c) index numbers
   (d) time series analysis
   (e) game theory

Statistics Textbooks and References

1. *Beginning Statistics* by Zirkel and Rosenfeld; McGraw-Hill, 1976.

A good beginning book particularly for the average student. Its examples are simple, meaningful and to the point. Many ideas for students' experiments and projects are included in a book that is easy to read. Available in soft cover only.

2. *Understanding Statistics* by Norman, Rosenfeld and Zirkel; McGraw-Hill, second edition, 1977.

A good book with realistic, up to date problems and ideas for field projects. It covers most of the topics in the outline and it is readable by the 'average' student.

3. *Elementary Statistics*, Robert R. Johnson; Duxbury Press, second edition, 1976.

A good source of problems which covers most of the topics in the outline.

4. *How to Lie With Statistics* by Darrell Huff, W.W. Norton and Co., 1954.

5. *Flaws and Fallacies in Statistical Thinking* by Stephen K. Campbell; Prentice-Hall, 1971.

Both these delightful books give a full range of examples of the misuse and abuse of statistics. An excellent source of material for sharpening students' statistical awareness. Highly recommended reading.

6. *Statistics by Example: Exploring Data*
Statistics by Example: Weighing Chances

Edited by Mosteller, Kruskai, Link, Pieters, Rising; Addison-Wesley, 1973.

These are the first two books in a series of four by Mosteller. The other two are more advanced. These books have some excellent cases using real data. The problems are taken from a variety of disciplines. Depending on the author, the articles can be read by the student with limited direction.

Some good articles are 'Introduction to the Chi-Square Procedure' by Carlson, 'The Cost of Eating' by Kruskal, and 'Estimating the Size of Wildlife Populations' by Chatterjee.

7. *Schaum's Statistics* by Murray R. Spiegel; McGraw-Hill, 1961.

The standard Schaum's format of a summary of the theory followed by a wealth of worked examples and exercises. The theoretical summaries can be useful as a means of consolidating material on a particular topic. As a source of worked-out examples it cannot be surpassed. It should be noted that Schaum's is best used in conjunction with other sources primarily as an excellent summary of the material.

8. *Statistics, A Guide to the Unknown* by Mosteller, Kruskai, Link, Pieters, and Rising, (Ed.) Tanur; Holden Day Inc., San Francisco, 1972.

Contains some interesting readings. Some articles, as the 'Plight of the Whales' by Chapmen, are quite interesting. Others are too laconic or difficult to follow. Students could gain credits by reading or reporting on some of the articles.

9. *Introduction to Linear Programming* by R.S. Stockton; Irwin, Dorsey Limited, 1971.

The book provides an elementary introduction to linear programming and can be used by students interested in doing special topics in the third term. The emphasis is on business and management problems.

Other Sources for Reference

1. Nova Scotia Government Book Store, 1597 Hollis Street, Box 637, Halifax, Nova Scotia has statistical reports providing ideas for 'real' problems or student discussions and projects.
2. Other groups, organisations or businesses which might be consulted for ideas, guest speakers, etc., are:
   1. universities
   2. consumer association in your area
   3. consumer reporters in the media
   4. Institute of Public Affairs
   5. Agricultural Station at Kentville

REFERENCES

Berkeley, E. (1972). *Probability and Statistics: an introduction through experiments*. Berkeley Enterprises, Newtonville, Mass. USA.

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Cox, W.L. (1977). *STAT-TREK*. Cherry Creek Schools, Aurora, CO, 80014, USA.

Del Grande, J.J. (1980). *Pre-College Statistics for the 80's*. Precollege statistics committee of Canada and the Beatty Fund.

Hoffer, A. (1978). *Statistics and Information Organisation*. Creative Publications.

Newmark, J. (1975). *Statistics and Probability in Modern Life*. Holt, Rinehart and Winston, Toronto.

Pieters, R.S. (1976). Statistics in the High School Curriculum. *The American Statistician*, **30**, 134–139.

Råde, L. (1975). *Statistics at School Level.* Armqvist and Wiskell International, Stockholm.

Willoughby, S. (1977). *Statistics and Probability*. G.L.C. Publishers.