PART 3

Australasia

CHAPTER 8

Teaching Statistics in Schools in Australia

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8.1 THE AUSTRALIAN SCHOOL SYSTEM

Each of the six States (and the Australian Capital Territory) has its own Education Authority set up under State legislation, and each issues various guides to teachers and schools regarding courses. As well, there are Statutory Authorities more or less distinct from the overall Education Authorities which are responsible for certification that students have completed satisfactory courses of study, sometimes simply for the award of a Certificate and sometimes as an entry qualification for further study (e.g. for Matriculation purposes): these operate in the upper levels of Secondary School programmes.

Primary education (Year 1 to 6, ages 5 to 6 to 11 or 12) is less prescriptive in its details; there is a transition to a secondary school (Years 7 to 12, ages 12 to 18) in most cases for nearly all pupils, with education being compulsory from age 6 until age 16, though the proportion going on beyond this age is still increasing. Running parallel with the later stages of Secondary Schools are Technical and Further Education Colleges, but these are not further discussed here.

Most school children are in State controlled schools (79 per cent), with a substantial minority in Roman Catholic schools, and a smaller proportion in other private schools mostly affiliated with religious persuasions. The further up the age range the more likely are the published syllabuses of the various statutory bodies to be closely followed: especially because of competitive entry to tertiary institutions, there is in practice a great deal of uniformity within a State at the upper secondary level. However, since there is little leakage at tertiary level across State boundaries there is correspondingly little uniformity across States in syllabus details. It might also be noted that various Universities have begun to base entry not wholly or even partly on the Public Examinations conducted by Statutory Authorities, using assessment substantially based on school records.

8.2 STATISTICS IN AUSTRALIAN SCHOOLS

Generally speaking, most of the Statistics, and all of the Probability, taught in schools is within Mathematics syllabuses. The syllabus details which are given below (§8.3) are therefore extracts from the full Mathematics syllabuses, which are themselves aimed at differing groups of students, differing in

abilities, achievements and future intentions. It is thus not easy to estimate the amount of weight, or time, given to these topics even when the syllabuses are read prescriptively – as an example, in the Western Australian courses the Mathematics I component might occupy 30 hours teaching time, Mathematics III about twice this, and Mathematics about that for Mathematics I.

Primary school syllabuses emphasize descriptive statistics especially applied to self-collected data, a very valuable introduction. There are of course the, by no means trivial, difficulties relating to accurate definition of the sampled population that professional statisticians do not always surmount, and it is not realistic to expect primary school teachers to be expert samplers. Nevertheless, experience with 'real' data can, and should, provide a much firmer foundation for the interpretation of long-run frequencies in relation to probability models – in several cases this is part of the primary syllabus, but whether or not such models occur at the primary stage is not necessarily important. There is, universally, a great emphasis on graphical representation of data.

At secondary levels the higher degree of prescription of course detail is accompanied by a great variety of alternative courses, so that a wide range of subject matter is still available. Not further mentioned here is the relatively recent development of some form of computer studies – these tend to concentrate on broad outlines of computer architecture and computer algorithms, often paying little regard to what has happened recently in the development of genuinely high-level languages and the little contact most users will ever have with the internal structure of computers; consequently such courses have little bearing on the effective manipulation of statistical data.

At the junior Secondary level, Years 7 to 10 (ages 12–16), practical descriptive Statistics is (again) dealt with in detail – this is expanded to deal more formally with calculations of statistics such as the mean and median, and also measures of dispersion. A formal introduction to probability, perhaps via sets, is common, perhaps with an introduction to formal counting techniques involving simple permutations and combinations, and the identification of a relative frequency as a realization of a probability from some model (not necessarily representing equally likely outcomes).

Senior Secondary level, Years 11 and 12 (ages 17 and 18), is beyond compulsory attendance requirements, and the degree of specialization many students have had to pursue by then may mean that little or no mathematics or statistics is then studied. Those who continue with mathematics or statistics have available courses which range from valuable, but sometimes mathematically unsophisticated, Social Mathematics to courses which include the probability axioms. While it is obviously difficult to generalize, there is for many students a quite substantial syllabus in statistics – indeed, given the backgrounds of many teachers, with little formal tertiary study of statistics, topics such as *t*- or chi-squared tests run very close to the limits of their training. This point exemplifies a general difficulty which tends to occur when Statistics is incorporated in a Mathematics course: there are many important parts of Statistics which do not fit at all well into the formal deductive structure of Mathematics and are often unattractive to those trained exclusively in Mathematics. Especially at the higher levels of Mathe-

matics courses, this easily produces a distortion of the Statistics taught. Since there are no separate Statistics 'subjects' in any State, and there is little likelihood of any being proposed, the difficulty will remain well into the future.

A view beyond the snapshot of the position in 1979 can be obtained by reading 'On the teaching of probability and statistics at the pre-college level in Australia', J.B. Douglas, in the *Teaching of Probability and Statistics*, 303–317, edited by Lennart Råde, 1969 (Wiley): this gives a review of the probability and statistics components of the same education systems ten years earlier.

8.3 STATISTICS COMPONENTS IN THE SYLLABUSES FOR DIFFERENT STATES

8.3.1 *Australian Capital Territory*
(Population 220 000, School Population 55 000)

Schools in the ACT do not have common prescribed syllabuses, each school being responsible for its programme. It is probably broadly true that these resemble those of NSW, the syllabuses of which used to be followed; though particular schools certainly do more (e.g. normal distribution, *t*-tests, correlation, sample surveys), it is also likely that others do less.
Course descriptions are lodged with the Schools Authority, and must be approved by the ACT Accrediting Agency.

8.3.2 *New South Wales*
(Population 5 100 000; School Population 1 040 000)

Primary School Syllabus (Years 1 to 6)
Constructing and reading picture, column, bar and line graphs. Introduction to sets.

Secondary School Syllabus (Years 7 to 12)
School Certificate (End of Year 10) Core Course, taken by nearly all students.
Practical statistics: collection and tabulation of data, pictorial, bar, sector, column and line graphs. Measures of central tendency (mean, median, mode), range. Optional: simple experimental and theoretical (equally likely) probability.
Practical statistics with grouped data, cumulative frequencies (median, deciles, quartiles), weighted mean, disperson, range and interquartile range. Grouping data.
Probability: counting techniques (optional: use of sets).
Higher School Certificate (End of Year 12)
Mathematics 2 Unit A (Terminal course in Mathematics)
Probability: in a finite equally weighted sample space, with interpretation as relative frequencies. Probability model. The 'addition' theorem, and simple conditional probabilities.

Mathematics 2 Unit (Standard non-scientific course)
Theory of probability: statistical regularity; random experiments; finite sample spaces; simple, composite and mutually exclusive events; probability; theorem of total probability. Two stage random events, independent events, the product rule.

Mathematics 3 Unit (Standard scientific, engineering course)
As for Mathematics 2 Unit, plus: Systematic enumeration in a finite sample space (permutations, combinations), the binomial distribution, random variables, expectation.

Mathematics 4 Unit (Specialist course)
As for Mathematics 3 Unit.

8.3.3 *Queensland*
(Population 2 220 000, School Population 440 000)

Primary School Syllabus (Years 1–7)
Statistics: tally marks, picture graphs for personally collected data.
Bar graphs, circle (pie) graphs, line graphs. Measures of central tendency: mean, median, mode.
Histograms, frequency polygons.
Empirical investigation of random events. Probability as ratio: (number of successful outcomes) ÷ (number of possible outcomes).

Secondary School Syllabuses
Lower Secondary (Years 8, 9, 10)
Advanced
The application of sets and algebra to probability. (Sample space, events, definition of probability and application to single and compound events.)
Ordinary
Tabular and graphical techniques to describe real situations.
General Mathematics
Interpreting tables and graphs.
Upper Secondary (Years 11, 12)
Mathematics
1. Collection of data in experiments and surveys. Discrete and continous variables. Sigma notation. Measures of central tendency (arithmetic mean, median, mode).
2. Measures of disperson (range, mean deviation, variance, standard deviation).
3. Frequency tables for both grouped and ungrouped data. Graphical representation of such tables by bar diagrams, histograms and frequency polygons.
4. Probability. Addition theorem for mutually exclusive and overlapping events. Conditional probability and independent events. Multiplication theorem.
5. Permutations and combinations.
6. Binomial distribution: mean and variance, histograms.
7. Normal distribution. Standard normal distribution and use of tables to find areas. Normal approximation to the binomial distribution. (Equation of normal curve not required.)
8. Simple linear regression and correlation.

Social Mathematics
Graphs – presenting information pictorially.
Collecting information: counting, ordering, tabulating, tallying, grouping.
Summarizing or condensing data: pictorial or graphical representation, pie diagrams, ideograms, valid and misleading frequency polygons.
Analysis of data: averaging, with mean, median, mode.
Sampling.
Range, variance, standard deviation.
Correlation.
Probability: simple probability in terms of long-run frequency, methods of determining sample spaces (tree diagrams, etc.), simple examples of a binomial distribution (Pascal's triangle).

8.3.4 *South Australia*
(Population 1 300 000; School Population 270 000)

Primary School Syllabuses (Years 1 to 6)
The emphasis is on Statistics and Graphs: construction of picture, column and bar graphs, reading such graphs (including pie graphs), construction of tables from data. Probability – simple problems.

Secondary School Syllabuses (Years 7 to 12)
Year 11
Mathematics 2 (With Mathematics 1, leads to Matriculation Mathematics)
Probability and Statistics.
Simple extensions of the sum and product laws of probability.
Binomial probability distribution. Frequency graphs; mean; median, quartiles, percentiles. Measures of dispersion: range, interquartile range.
Mathematics 3 (Selection from Mathematics 1 and 2)
Statistics.
Frequency graphs. Frequency tables and cumulative frequency tables (discrete and continuous): histograms and ogives, with absolute, relative and percentage frequencies. Histograms approximating normal, rectangular, skewed distributions. Mean by coding, median from cumulative distribution, quartiles, percentiles. Measures of dispersion: range, interquartile range.
Mathematics 4 (Less mathematical content than Mathematics 3)
The scope of statistics; picturegrams, histograms, pie diagrams, column graphs, continuous graphs. The concept of statistical distribution; collection and tabulation of data, relative frequencies of classes, expected class frequencies for extensive data. Calculation of mean and mode (grouped data). Cumulative frequency graphs: median, quartiles, interquartile range, percentiles.
Year 12
Mathematics 1 (With Mathematics 2, a preparation for Tertiary Mathematics).
Counting of Probability.

Notation of sets (union, intersection). Numbers of elements in sets, selections, permutations and combinations; combinatorial proof of binomial theorem. Sample space, event, probability in finite sample spaces, disjoint events, conditional probability, independence, binomial probability function. Optional topics: Inverse probability, random variables, binomial and normal distributions. Markov chains, transition matrices, powers of the transition matrix, steady state.

Mathematics 1S (Non-specialization in Mathematics)

As for Mathematics 1.

8.3.5 *Tasmania*

(Population 420 000; School Population 93 000)

Primary School Syllabuses (Years 1 to 6)

Construct and interpret graphs, present data in tabular form, and use simple statistical measures.

Bar graphs, circle graphs, empirical investigation of random events, simple measures of central tendency.

(The above is not prescriptive, but widely followed.)

Secondary School Syllabuses

School Certificate (End of Year 10)

Advanced Mathematics (Pre-requisite for further study in Mathematics)

Sampling and presentation of samples for grouped data.

Measures of central tendency: mean, median, mode.

Cumulative frequency and percentiles.

Measures of variability – range, interquartile range, standard deviation.

Optional topics:

Basic concepts of probability:

a) Sample space, event, mutually exclusive events, complement.
b) Probability of an event where outcomes are equally likely.
c) Counting techniques – simple permutations and combinations.
d) Sum and product rules for probability.

Mathematics

Level I

Practical surveys, tallying, tabular and graphical presentation. Simple average. Interpretation of tables and graphs, samples.

Level II

Practical surveys, counting techniques, frequency distributions, grouping. Sampling. Range, cumulative frequency. Mean, median and mode for grouped distributions: effect of extreme observations.

Level III

Probability; long-run frequency. Sample space, events, mutually exclusive, complementary. Probability models in equiprobable case.

Higher School Certificate (End of Year 12)

Mathematics Level II (Not mathematics at tertiary level)

Optional topics: probability and statistics. Games of chance, equally likely events, tree diagrams, tabular and graphical presentation, measures of central

tendency and variability. Paired observations, correlation and regression (cause and effect).

Mathematics Level III

10. Statistics
    Populations and samples, frequency distributions, parameters, normal distributions (tables), statistics, tabular and graphical presentation, measures of central tendency and variation, association, two-dimensional representation.
11. Probability
    Sample space, mutually exclusive, complement, equally likely points. Elementary permutations and combinations.
    Tabular and graphical presentation, measures of central tendency and variability, association, two-dimensional representation. Union and intersection, probability of compound events, conditional probability, independence, mention of infinite sample spaces.
12. Statistical Inference
    Sampling: random samples from distributions, use of random number tables. Hypothesis testing and estimation: principles of hypothesis testing, estimation of a parameter.

(The subject Data Processing includes the topic Statistical Packages.)

8.3.6 *Victoria*

(Population 3 850 000; School Population 830 000)

Primary School Syllabuses (Years 1 to 6)

There are no prescriptive State-wide syllabuses, but Guides on various topics are issued by the State Education Department for use by teachers. These include general aims and detailed teaching notes. Among the topics treated are:

Collection and sorting of data, tallying, pictorial and graphical representation (pictograms, bar graphs, line graphs, histograms) interpretation of tables and representations. Summaries: average, median, mode. Random events: introduction to probability.

Secondary School Syllabuses (Years 7 to 12)

Each secondary school is responsible for developing curricula which are relevant to that school's view of its role in the community, and so these are widely diversified. At the end of Year 12, the Higher School Certificate examination has prescribed syllabuses, under the control of the Victorian Institute of Secondary Education, and details of these follow.

Years 11 and 12

Applied Mathematics

1. Probability
   (a) Elementary ideas: event space; events and their union, intersection and complements. Probability axioms. Conditional probability, simple applications of the law of total probability and Bayes' theorem.
   (b) Discrete random variables. Probability function, expectation, descriptive measures (mean, variance, standard deviation),

$\Pr(\mu - 2\sigma \leqslant X \leqslant \mu + 2\sigma) \doteq 0.95$ for a variety of distributions.

(c) Probability distributions. Simple examples involving permutations and combinations. Sampling: binomial and hypergeometric distributions. Sequences of independent trials: the binomial and geometric distributions, with the Poisson distribution as a limit. Normal distribution: informal treatment, as approximating distribution, and as the distribution of 'errors'.

General Mathematics

1. Probability and Statistics
   (a), (b), (c) as above.
   (d) Statistics. Descriptive statistics, sample estimates of the probability function, mean, variance, standard deviation. Central limit theorem (no proof) with applications.
2. Arrangements and selections

General Mathematics: Computing option

As for General Mathematics.

8.3.7 *Western Australia*
(Population 1 230 000; School Population 250 000)

Primary School Syllabuses (Years 1 to 6)
Chance Processes, Statistics and Graphs
(Very detailed notes, with examples selected below)
'Lucky dips', simple spinner, sampling with tallying and graphs (picture histograms), guessing games, throwing dice including examples such as sum of scores for two dice, bar graphs, listing sample spaces (e.g. how obtain 10 spots in throwing two dice), pie diagrams, range and average, mode.

Secondary School Syllabuses (Years 7 to 12)
(The Education Department of W.A. produces very detailed information in the form of books to be put in students' hands: e.g.

Introductory Statistics (1st Ed., 1977) 206 p.
Arithmetic 6 Statistics (1973) 111 p.

These include topics such as:
Collecting and sorting, picture graphs, mean, column and bar graphs, histograms, mode, line graphs, pie graphs, median, misleading graphs, categorical data and graphs, frequency tables and polygons, relative frequency, cumulative frequency, the range as a measure of scatter, probability and relative frequency.)
For the upper high school the syllabuses for the Certificate of Secondary Education/Tertiary Admissions Examination are summarized:

Mathematics I

Statistics
11. Measures of central tendency
    Median, mode, mean from grouped data (notation $\bar{x}$ and μ)
12. Measures of dispersion
    Variance (S^2 and σ^2), standard deviation, range, interquartile range
13. Measures of relationship (covariance, correlation)

14. Frequency distributions for grouped data

Counting Techniques
15. Permutations and combinations

Probability
16. Sample space, long-run frequency, conditional probability
17. Probability laws
18. Normal probability distribution (use of tables)

Mathematics III

Probability and Statistics
8. Counting techniques
9. Definition of probability (equally likely, long-run frequency), conditional probability
10. Probability laws
11. Measures of central tendency
12. Measures of dispersion
13. Measures of relationship (covariance, correlation)
14. Binomial probability distribution (expected value)
15. Normal distribution

Mathematics IV

Statistics
1. Methods of representing data
2. Measures of central tendency
3. Measures of spread
4. Measure of relationship (correlation)
5. Frequency distributions for grouped data
6. Sampling a population (random and stratified sampling)
7. Interpretation of statistical reports (difference between proportions)

Counting techniques and probability
1. Counting techniques
2. Probability and sample spaces
3. Calculating probabilities
4. The normal distribution (tables)

Acknowledgment
The material in this Report could not have been assembled without the co-operation of Departments of Education around Australia, to whose officers the statistical community is grateful.