CHILDREN AS EXPLORERS:

EXPLORATORY DATA ANALYSIS

BY JUNIOR HIGH SCHOOL STUDENTS

IN A COMPUTER ASSISTED ENVIRONMENT

By

Dani Ben-Zvi

Department of Science Teaching

PH.D. THESIS

Submitted to

The Feinberg Graduate School

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Supervisors: Dr. Abraham Arcavi

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ABSTRACT

The overall goal of this dissertation is to advance the understanding of learning and teaching Exploratory Data Analysis (EDA) in a carefully designed computer assisted learning environment for the junior high school level. This thesis, presented as a collection of papers, documents and analyzes three dimensions of students' activity, their evolving conceptions, their interactions, and their adoption of points of view. The theoretical framework informing the analysis of students' learning assumes that competence in a new and complex domain, such as statistics, involves more than particular set of skills, strategies, or knowledge. Following Resnick (1988), the framework includes viewing competence of a domain as adopting the habits, language and dispositions of interpretation and sense making (enculturation).

Firstly, I analyze at a very fine level of detail the ways in which a pair of students began to make sense of data and data representations, as well as the process of adopting and exercising the habits and points of view that are common among EDA experts. The focus is on the ways they started to develop global views of data and their representations on the basis of their previous knowledge and different kinds of local observations. I examine how knowledge was gradually constructed through complex cognitive and socio-cognitive processes, which included their interactions with each other, the teacher, the materials and the computerized tool. I analyze the ways in which the same 'pieces' of students' prior knowledge which seemed to hinder progress, ultimately became the support for the construction of new meanings. Of special interest were the teacher's interventions, which though short and not necessarily directive had catalytic effects, can be characterized in general as interesting instances of appropriation.

Secondly, through the analysis of students' 'research projects', I suggest an initial framework of student reasoning in the domain of EDA with an emphasis on handling data representations. I then use written assessments to characterize student sense making of statistics after the end of their experience with the curriculum. Finally, I analyze the interrelationships between curriculum design and research in order to characterize the nature of the instructional activities, including the role and impact of computerized tools, which have the potential to promote meaningful learning of EDA.

The research methodology is mainly qualitative in nature with some quantitative aspects. The subjects for this study were seventh grade students (13-year-old) of mixed ability from the experimental classrooms that used the curriculum. The analysis is based on: 1) focused and detailed data on one pair of students, which were videotaped at almost all stages of their learning statistics; 2) classroom data that was gathered in three experimental classes; and 3) summative assessment data that consist of students' 'research projects', written assessments, students' evaluations, and teacher's comments.

These data were used to characterize important phenomena related to the following questions: 1) how students choose, interpret, design, transform and use data representations?; 2) what are the contributions of student interactions with their peers and their teacher to their understanding of data representations?; and 3) how students adopt the habits and points of view that are common among EDA experts, in particular the experts' point of view on local-global approaches to data interpretation and their representations?

The dissertation shows how meaningful learning of EDA took place through complex socio-cognitive processes of enculturation, the processes of teacher-student (and, possibly, student-student) appropriation, the students' exposure to carefully designed learning arenas and relevant computerized tools, and the students' long-term involvement in constructing a 'research project'.

CONTENTS

LIST OF PUBLISHED AND NON-PUBLISHED RESEARCH PAPERS

This thesis includes synopses and full manuscripts of the following five published (or in press) research papers, and one non-published research paper.

ACKNOWLEDGEMENTS

I would like to first thank my dissertation supervisors, Abraham Arcavi and Alex Friedlander, for their ongoing encouragement and guidance. I would especially like to thank them for many hours of viewing videotapes, discussing conjectures and interpretations, and contributing their complex understanding of learning and teaching processes. I thank them for reading early drafts, struggling through telegraphic text, giving me feedback on half-baked ideas, and having to deal with convoluted prose. They encouraged me to consider statistical content; cognitive, socio-cognitive and later socio-cultural approaches to learning and research; and various aspects of the innovative educational environment. I also thank my dissertation examining committee, Lee Segel and Michal Yerushalmy. My meetings with them were important and stimulated new ideas and directions.

I am deeply indebted to the teachers and students who welcomed me into their classrooms. I especially thank *A* and *D* who agreed to be part of the case study including being videotaped, and repeatedly being 'nudged'. I owe many of the insights in this dissertation to the open and candid discussions of these students and their peers. Their curiosity, perseverance, and creativity were inspiring. I hope that the insights in this thesis will make these and other students' classroom experiences in statistics more meaningful and interesting. I would especially like to thank the three skillful and dedicated teachers, Michal Tabach, Hannah Stein, and Gila Ozruso, who always welcomed me with patience and provided every possible support and advice.

My conversations with colleagues were essential in the writing of the thesis. Conversations with Maxim Bruckheimer, Tommy Dreyfus, Ruhama Even, Nurit Hadas, Rina Hershkowitz, Anna Sfard, Tsippi Resnick, Baruch Schwarz, and others in the Weizmann Science Teaching Department, helped me to put many of the ideas in the thesis into words, and to attempt to explore new directions. During my work on the thesis, I was very fortunate to meet many colleagues and friends from all over the world. My discussions with them were also important and supportive while I was conducting the research and writing the papers. I mention here just a few, who read and reviewed earlier versions, and provided support in many helpful ways: Janet Ainely, Carmen Batanero, Bob delMas, Iddo Gal, Joan Garfield, Brian Greer, Jonathan Moritz, Maxine Pfannkuch, Chris Reading, Jane Watson, and many others. I especially would like to thank Joan Garfield. Her dedication to statistics education, research, and curiosity were inspirational and influenced my work on this thesis.

The Weizmann Science Teaching Department provided a supportive environment where I could present 'pieces' of the thesis as 'work in progress' and watch the videotape in great company providing helpful feedback. I would also like to thank the department administrative staff doing their outmost efforts to help in any service needed. This dissertation would not have been possible without various forms of institutional support provided by the Feinberg Graduate School and the Science Teaching Department of the Weizmann Institute of Science. Kibbutz Be'eri and its members were also supportive along the way.

And lastly, I would like to thank my family, my wife and friend, Hava, my children, Michal, Dagan, Nir, and Noa, and my parents, who understood that this thesis was an important project for me, and suffered my frequent and extended 'disappearances' for so many years. They have been my main source of energy, always thinking I could do this, even when I was most discouraged and anxious.

I am thankful to the opportunity given to me in writing this thesis. The development and growth that came with it were often painful, but ultimately rewarding.

A NOTE TO THE READER

This thesis developed through extended planning, trying ideas, testing and implementing educational materials, gathering data, writing and presenting, collecting feedback, discussing and more discussing, and so forth. Through this process, my conceptions and understandings of the issues related to children learning statistics have evolved and changed. Thus, the introductory overview of the study (Chapter 1) mostly reflects my intentions and views of what is researchable, interesting, and original at the 'start point'. However, the research evolved to show that the issues selected as my research questions were richer and wider, and thus the scope of my initial questions got enlarged and immersed in 'larger issues', without leaving aside the initial questions. I could have arranged that introduction to completely fit the 'end point', but I preferred the readers to get a feeling of how the study evolved. I hope this structure preserves this sense of growth, and provide you, the reader, with a perspective on those changes that this thesis and myself have gone through during this scholarly endeavor.

"*I learnt a lot from my teachers,*
and even more from my colleagues,
but from my students - I learnt the most."
(Talmud)

INTRODUCTION

Statistics is becoming ever more pervasive, and we live in a society which is ever more dependent on information. Major political, social, economic and scientific decisions are made on the basis of data. Statistics is a discipline which provides means for dealing with data. Statistical reports affecting virtually all aspects of our lives appear regularly in all the news media. Therefore, the importance of statistics literacy is becoming a major goal of the school curriculum, regardless of the professional future of the student (Gal, 2000). This thesis joins the increasing efforts and interest in the study of statistics education.

The presentation of this thesis is the final part of a 'direct Ph.D. program' at the Feinberg Graduate School, of The Weizmann Institute of Science. I began with studies leading to the M.Sc. Degree, focusing primarily on developing junior high school statistics curriculum. I reviewed relevant statistics curricula, software (Ben-Zvi, 1997a) and research literature, established a set of curriculum design principles, and wrote and tested instructional materials to be used in an innovative educational environment.

With the change of status from the M.Sc. to the Ph.D. study, I continued the cycles of writing - field-testing - research - improvements (for an account of the curriculum development, see Hershkowitz, Dreyfus, Ben-Zvi, Friedlander, Hadas, Resnick, Tabach, and Schwarz, in press). The first field experiments culminated in the production of an improved version of the student textbook (Ben-Zvi & Friedlander, 1997a, see Appendix III), the development and implementation of teachers' courses, the raising of numerous research issues involving many variables, as well as a complex pedagogical situation, and the narrowing down to the main themes of investigation (see Chapter 1).

This Ph.D. thesis is a collection of papers presenting in a coherent description how junior high school students begin to learn *Exploratory Data Analysis* (EDA, in the sense of Shaughnessy, Garfield and Greer, 1996) in a carefully designed, computer assisted learning environment. The thesis consists of five published (or in press) research papers, in which I was the only author or the major contributor. In addition, I include a non-published study, presented in a published paper format. Each of the six research papers is introduced by a brief *synopsis*[1] and is linked to the research questions and the thesis 'story' to form a logical continuum (Chapters 2, 3 and 4). At the end, I review and discuss the study, and suggest five major conclusions, theoretical implications, and practical implications for education (Chapter 5).

The first two papers analyze at a very fine level of detail the ways in which two students (*A* and *D*) began to make sense of data and data representations, as well as the process of adopting and exercising the habits and points of view that are common among EDA experts (Chapter 2). The third paper analyze typical *statistical thinking modes* of junior high school students, that is, an initial taxonomy of novices' reasoning in the domain of EDA, with an emphasis on handling data representations (Paper III, in Chapter 3). In the fourth paper, I complement the former analysis by assessing students' understanding of statistics after the end of the *Statistics Curriculum* (*SC*) based on their responses to a 'real' data-based assessment task (Paper IV, in Chapter 3). The fifth paper analyzes the mutual relationships between the design of the *SC* and research on learning, in order to characterize the nature of the instructional activities, which promoted meaningful learning of statistics (Paper V, in Chapter 4; see also, Hershkowitz et al., in press). Finally, I study the impact of computerized tools in enhancing learning and understanding statistics (Paper VI, in Chapter 4). Thus, this thesis includes five chapters:

1. Overview of the study: aims and research questions, methods, a review of literature on teaching and learning EDA, and the *SC* learning environment.
2. *A* and *D*'s case, concentrating on the construction of meanings and global views of data and data representations by one pair of students.
3. Students' statistical reasoning and their understanding of data at the end of the *SC*.
4. The impact of curriculum design and technology in enhancing the learning of EDA.
5. Discussion of conclusions, theoretical and educational implications.

[1] The synopsis of each paper is presented below *before* the full manuscript.

Chapter 1

OVERVIEW OF THE STUDY

Part 1 **Aims**

Part 2 **Methods**

Part 3 **Literature review**

Part 4 **The learning environment:**
The *Statistics Curriculum* (*SC*)

CHAPTER 1

OVERVIEW OF THE STUDY

Part 1

Aims, Research Questions

The overall goal of this Ph.D. thesis is to advance the understanding of learning and teaching statistics (EDA) by junior high school students in a computer-assisted, carefully designed learning environment. My purpose is to follow, at a very detailed level, the ways in which students begin to make sense of data and data representations, as well as the process of adopting and exercising the habits and points of view that are common among EDA experts. The observations and analysis are expanded beyond the learning of basic statistical concepts, tools and processes, to include also the EDA 'culture', i.e., perspectives and points of view of statistics experts involved in the practice of data based enquiry (described in Paper II).

More specifically, the purpose is to investigate the following themes in the context of statistical problem solving processes.

1. *Student understanding of data representations*, namely, how junior high school students choose, interpret, design, transform and use data representations in the context of open-ended EDA problem solving situations with computerized tools. I chose to analyze the following.

   Choosing data representations. In a computerized environment, a diverse collection of tabular, numerical, or graphical representations of data is readily available to students. When and how are students' choices made and for what purpose? What are students' considerations and difficulties in making these choices?

Interpreting data representation. In the experimental classes, students were engaged extensively and intensively in making sense of data representations. What 'data sense' (Friel, Bright, Frierson, & Kader, 1997) do students make of data representations? How do they interpret and critique them? What are students' considerations and difficulties in making these interpretations?

Designing/transforming representations. Computerized tools allow students to design, or transform a chosen representation in a relatively simple and straightforward manner. How, why and when do students take advantage of these facilities, and in which ways? What are students' purposes, considerations and difficulties in transforming and designing representations?

Using representations. How do students use representations to see patterns and to support, refine, refute or critique claims and/or conclusions based on the data?

2. *The contribution of student interactions to their understanding of data representations*. The interactions -- within-pair interactions, group interactions, and student-teacher interactions -- can be used to characterize student understanding and learning processes. Therefore, I chose to analyze the following.

   Within-pair interactions and group interactions. Students' collaborative work includes use of the computer, conversation, division of roles, resolution of conflicts, mutual guidance, reflection, critique of others' work, and presentation of reports. What are the main characteristics of these interactions? How do they contribute to develop meanings?

   Student-teacher interactions. The student-teacher dialogue in the experimental classes was radically different from the traditional one in content and format. Since students are engaged for long periods of time in independent exploratory activities, they receive less 'teacher time'. However, these interactions with the teacher seemed to be catalytic. What is the nature of student-teacher interactions, and how/when do they promote student work on EDA problem situations?

In my establishment of the educational environment and the research setting and in conducting the analysis, I rely on both cognitive and socio-cultural perspectives. The *cognitive perspective* leads me to focus on the development and change in students' conceptions and the evolution of their statistical reasoning on the basis of what they knew (cf., Piaget, 1976; Davis, Maher & Noddings, 1990). The *socio-cultural perspective* guides me to focus on learning (of a complex domain, such as EDA) as the adoption of the point of view of a community of experts, in addition to skills and procedures (cf., Resnick, 1988; Yackel and Cobb, 1996). Thus, I look at learning as an '*enculturation*' process, i.e., entering and picking up the points of view of a community or culture (Schoenfeld, 1992) with two central components: students engaged in doing, investigating, discussing and drawing conclusions; and teachers engaged in providing role models by being representatives of the culture their students are entering through timely interventions.

The two perspectives are intertwined: conceptions evolve within a purposeful context in a social setting. On the other hand, developing an expert point of view and interacting with peers or with a teacher, imply undergoing mental actions within specific tasks related to complex ideas. These actions over time are a central part of the meaningful experience within which the 'culture' of the field is learned.

In light of the above, I wanted to investigate what it meant for a student to be deeply involved in open-ended, complex, and long problem-solving processes related to the learning of basic statistics (EDA), interacting with peers and teachers, while using various tools, such as relevant computer software (spreadsheets). I asked what it meant for a student to become a '*data explorer*', who is able to meaningfully investigate interesting questions, make relevant conjectures, and use data to answer them.

Part 2

Methodology

Data

I collected a diverse body of data, to characterize important phenomena related to the research questions. The data types used in this thesis are pre-experiment data, statistics prior-knowledge data, statistical exploration data, post-experiment data, and delayed post data (Appendix I). Within these types, I analyzed three main groups of data:

1. *Focused and remarkably detailed data on one pair of students*. Two students, *A* and *D* (13-year-old boys) were videotaped at almost all stages of their learning statistics (20 hours of tapes). (For a detailed sequence of their statistical explorations, see Appendix II.) The students were considered by their teacher to be both able and verbal[2]. They were asked to talk aloud and explain their actions. Additional data of the pair include statistics notebooks, pre and post mathematics portfolios, their 'research project', final assessment task, teacher's report, and post experiment evaluation forms, and delayed post interviews (see full details in Appendix I).

2. *Classroom data that was gathered during the experimental period in three classes of the same school*. These data include classroom observations of other pairs and whole class discussions (not necessarily captured in video but reported by the observers or teachers[3]), interviews with the teachers, and the researcher's notes. These data are used, when appropriate and relevant, to check for coherence of interpretations emerging from the different sources. Even when a phenomenon seems important and the data interpretation was validated and

[2] There are several good reasons to focus on verbal students. Firstly, their descriptions and explanations provided valuable data on their actions, thoughts and considerations. Furthermore, I did not have at my disposal video equipment to record simultaneously students' faces and their computer's screen. Therefore I used one video camera posted in front of the students, which framed the students, their notebooks, and a teacher on intervening sessions. Thus, students had to provide additional information on their unseen computerized actions and screens.

[3] The observers included two (or sometimes three) researchers, the video photographer, who was a member of the researchers' team, and the three mathematics teachers.

agreed upon, the question of the idiosyncrasy of the identified phenomenon may remain open. Therefore, the data and interpretations from pairs of students in the same class or from other classes assist in checking for generalizability of the phenomena.

3. *Summative assessment*. These data consist of students' 'research projects', final assessment activity (described in paper IV), and students' evaluation forms, which were collected in the same school and other schools, and are analyzed in order to compare, contrast, and validate the key findings from the above.

Students

The students in this study were 80 seventh graders (13-year-old) in three classes, in a progressive experimental school[4], taught by skillful and experienced teachers, who were aware of the spirit and goals of the *SC* (*Statistics Curriculum*) *Project*. The students were of mixed ability, and were used to work collaboratively in small groups in mathematics classes. They participated in this study, within their regular classroom period. A detailed description of the pair of students, *A* and *D*, the subjects of the microgenetic case studies, is given in Chapter 2.

Prior knowledge. When the students started to learn the *SC*, they had limited in-school statistical experience. However, they had some informal ideas and positive dispositions towards statistics, mostly through exposure to statistical jargon in the media. In primary school, they had learned only about the mean and the uses of some diagrams. Prior to, and in parallel with, the learning of the *SC*, they studied beginning algebra based on the use of spreadsheets to generalize numerical linear patterns and represent them in graphs (Resnick & Tabach, 1999).

Data Analysis

The nature of this thesis is mainly qualitative with some quantitative aspects. The *qualitative* analysis of the data was based on *interpretive microanalysis* (see, for example, Schoenfeld, Smith & Arcavi, 1989; Meira, 1991, pp. 62-3; Schoenfeld, 1994):

[4] This selective 'magnet' school was established in 1986 in Tel-Aviv. It emphasizes nature and interdisciplinary studies, the use of innovative curriculum, cooperative learning, students' responsibility of their learning, and independent investigative projects. The team of mathematics teachers is supported by staff member from the Department of Science Teaching of the Weizmann Institute of Science in order to help design the content and the teaching of its mathematics courses (grades 1-9).

a qualitative detailed analysis of the protocols, taking into account verbal, gestural and symbolic actions within the situations in which they occurred. The goal of such an analysis is to infer and trace the development of cognitive structures and the socio-cultural processes of understanding and learning.

The validation of the data analysis underwent two stages, one within the researchers' team and one with fellow researchers, who had no involvement with the data or the *SC* project (*triangulation* in the sense of Schoenfeld, 1994). In both stages, the researchers discussed, presented, advanced and/or rejected hypothesis and interpretations, and inferences about the students' cognitive structures. Advancing or rejecting an interpretation requires: (a) the provision of as many pieces of evidence as possible (including past and/or future episodes, and all sources of data); and (b) attempts to produce alternative interpretations equally strong in terms of the available evidence.

The goal of the *quantitative*-oriented studies (Chapter 3) is to assess at the end of the *SC*: (a) students' typical reasoning about data representations; (b) students' sense making of data and data representations, use of data analysis skills, and understanding of basic statistical procedures and concepts; (c) if and how they adopted the dispositions and points of view of certain aspects of the 'EDA culture'; and (d) to provide a quantitative indication of the extent and scope of the phenomena previously identified in the microgenetic case study.

Part 3

Literature Review

Teaching and learning EDA

Exploratory Data Analysis (EDA, or data handling) is the current name for what has been traditionally called descriptive statistics. EDA is the discipline of organizing, describing, representing, and analyzing data, with a heavy reliance on visual displays and, in many cases, technology. Its goal is to make sense of data, analogous to an *explorer of unknown lands*[5] (Cobb & Moore, 1997).

EDA studies patterns, centers, clusters, gaps, spreads, and variations in data, and its essence can be captured by the slogans - look at the data (preliminary analysis), look between the data (comparisons), look beyond the data (informal inference) and look behind the data (context) (Curcio, 1989; Shaughnessy et al., 1996). Pedagogically, EDA is an opportunity for open-ended data exploration by students, aided by basic concepts of descriptive statistics.

In the past decade EDA has gained recognition as an important component of the school curriculum. Recommendations were made to introduce stochastics (statistics and probability) concepts for *all* students, throughout the school years, beginning at an early stage (National Council of Teachers of Mathematics, 1989 and 2000; Australian Education Council, 1991; American Association for the Advancement of Science, 1993[6]). This led to the production of new instructional materials for elementary and secondary schools in many countries, with two principal approaches: the 'series approach' and the 'stand alone unit approach'. In the 'series approach' curriculum efforts concentrate on developing EDA materials that can be used continuously for an extended period of time; e.g., *Used Numbers* (Russell & Corwin, 1989; Corwin & Friel, 1990; Friel, Mokros &

[5] This is the source of inspiration for the title of this thesis: '*Children as Explorers*'. The EDA expert is portrayed as an explorer of unknown 'lands of data'. Learning EDA is considered as becoming a '*data explorer*'.

[6] In Israel, a statistics curricular reform is currently been suggested in grades 1-6, by a committee of the Israeli Ministry of Education and Culture. The committee recommendations will instruct primary schools to allocate 15% of the mathematics syllabus and time to statistics. If applied, a production of instructional materials and teacher development programs will follow.

Russel, 1992), and *The Quantitative Literacy Series* (Landwehr & Watkins, 1986; Newman, Obremski & Schaeffer, 1986; Landwehr, Watkins & Swift, 1987; Gnanadesikan, Schaeffer & Swift, 1987; Barbella, Kepner & Schaeffer, 1994). In the 'stand alone unit approach' EDA materials are developed as periodic chunks in a larger development effort; e.g., *Mathematics in Context*[7], *Oxford Mathematics* (Greer, Yamin-Ali, Boyd, Boyle & Fitzpatrick, 1995); *Core Plus Mathematics Project (CPMP)*[8], and *The Connected Mathematics Project* (Lappan, Fey, Fitzgerald, Friel Phillips, 1996). In these curricula there is growing emphasis on graphical approaches, on students gathering their own data and carrying out investigations, on the use of probability simulations to generate data, and sometimes on a cross-curricular approach. Another example is *Data Visualization* (de Lange & Verhage, 1992), in which statistics misuses and distortions are used as points of departure for the study of EDA.

At the university level in the USA, dissatisfaction with student outcomes across most introductory statistics courses led to a task force report, commissioned by the Mathematical Association of America (Cobb, 1992). This report made recommendations for the 'reform' of most courses, captured in the basic message: 'more data and concepts, less theory, fewer recipes'.

In Israel, the official junior high school mathematics syllabus (Israeli Ministry of Education and Culture, 1990) assigns 15 hours in grade 7 (age 13) to cover basic statistics topics, and an additional 10 hours in grade 8 (age 14) to introduce basic concepts of probability. High school matriculation exams include only limited topics in stochastics at the three and four credit points levels. Many texts used in Israeli schools (e.g., Geva, 1990) emphasize definitions and computation and neglect the development of a broader integrated view of statistical problem solving. Thus, statistical concepts rarely originate from real problems, and in general, there is just one correct answer to each problem. Many teachers ignore the grade 7 statistics unit altogether, due to lack of knowledge, interest, and enthusiasm, maintaining that there is no time.

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[7] Math in Context (MiC) is the name of the project and of the curriculum. It consists of a series of booklets, each authored by different sets of authors, individually packaged and available independently.

[8] Core Plus Mathematics Project (CPMP) was the project of a set of high school of three books, for grades 9, 10, and 11. Each book consists of separate chapters that have names such as 'Patterns in Data'. Chris Hirsch is the overall editor, but each chapter is written by a different team of authors.

In response to this situation, attempts to improve the teaching of statistics (using hand held calculators) were made by the Weizmann Institute Mathematics Group (Taizi, 1991; Robinson, 1992; Halevi, 1992). The *Machar 98* report (Israeli Ministry of Education and Culture, 1992) stressed the importance of mathematical skills, including numeracy and graphical interpretation ability, and recommended the use of computers to measure, collect, analyze, and present data. In response to the report, a comprehensive junior high school development and research program - *CompuMath* - was established. *The Statistics Curriculum*, which is the development component of this study, is part of the *CompuMath* project.

Research on the teaching and learning of statistics

Several researchers have reviewed research on the teaching and learning of stochastics over the past decade (Garfield & Ahlgren, 1988; Shaughnessy, 1992; Shaughnessy, et al., 1996). Research on EDA teaching and learning has not yet developed as richly as probability research, however, it is beginning to emerge as an important topic in the world community of statistics educators, and is being fostered by international organizations.

Since 1982, The International Statistical Institute (ISI) and The International Association for Statistical Education (IASE) has sponsored and organized five International Conferences on Teaching Statistics (ICOTS), international research forums on Statistical Reasoning, Thinking and Literacy (SRTL-1, 1999; SRTL-2, 2001), and a number of roundtable meetings on statistical education[9]. A review of the proceedings of these conferences reveals the following themes, which emerged in a fairly chronological order:

1) clarification of goals for teaching statistics;
2) preparation of teachers to teach statistics;
3) use of technology to teach statistics; and
4) empirical research on teaching and learning statistics.

[9] The topic of the most recent IASE roundtable meeting was ‘training researchers in the use of statistics’ (Batanero, 2001).

ICOTS-5 (Pereira-Mendoza, 1998) saw the broadening of these research issues in statistics education with many influences coming from current trends in mathematics education and new technologies (including Internet). In the following, I shall use these categories to describe briefly existing research on EDA.

1) Clarification of goals for teaching statistics

The ICOTS-1 (Grey, Holmes, Barnett & Constable, 1983) focused on the importance of teaching statistics, its goals, and the difficulty of integrating statistics into the already crowded mathematics curriculum. Kapadia (1983) suggested that students should become aware of and appreciate (a) the role and use of statistics in society and in other academic subjects, and (b) the scope of statistics; i.e., the sort of questions that an intelligent user of statistics can answer, and the power and limitations of statistical thought.

Gal and Garfield (1997a) also offer two common overall goals of statistics education: students should become informed citizens who are able to (a) contribute to the production, interpretation, and communication of data pertaining to problems they encounter in their professional life, and (b) comprehend and deal with uncertainty, variability, and statistical information in the world around them, and participate effectively in an information-laden society.

To achieve this broad vision over several years of schooling, they suggest eight interrelated basic subgoals for statistics instruction:

1) understand the purpose and logic of statistical investigations;
2) understand the process of statistical investigations;
3) master procedural skills;
4) understand the relevant mathematical relationships;
5) understand probability and chance;
6) develop interpretive skills and statistics literacy;
7) develop ability to communicate statistically; and
8) develop useful statistical dispositions.

2) Preparation of teachers to teach statistics

The following aspects of training teachers to teach statistics were discussed at ICOTS-2 (Davidson & Swift, 1986), and at the ISI roundtable conference (Hawkins, 1990):

a) difficulties of teachers with inductive and exploratory data analysis instruction; and

b) the need to develop professional programs to prepare teachers willing and able to be
a 'guide on the side' instead of a 'sage on the stage'.

Biehler (1990) suggests that teachers should view statistics as interactive and experimental 'detective' work with data, which involves openness, flexibility, multiplicity, and uncertainty of results. The professional development needs of statistics teachers at school level, as well as those in other subject areas with statistics literacy requirements, have slowly been recognized and are only now beginning to be met comprehensively (c.f., Hawkins, Jolliffe, & Glickman, 1992; Watson, 1998; Gal 2000).

3) Use of technology to teach statistics

The implementation of computerized tools in statistics education was the main theme of the 1996 International Association for Statistical Education (IASE) roundtable conference (Garfield & Burrill, 1997). In this forum, I reviewed and proposed a categorization of the different types of software that have been typically used in statistics instruction (Ben-Zvi, 1997a). These uses have the potential to renew statistics instruction on the basis of strong synergies between content, pedagogy and technology (Moore, 1997) but raise both theoretical and practical issues.

Biehler (1993) describes the use of technology in empowering students to do data analysis that is interactive and exploratory, using visualization and simulations to understand statistical concepts and methods. The potential of software lies in relieving the user from technical aspects of computation and graphing and providing dynamic and interactive tools such as linked representations, and simulations (Kaput 1992). Thus, a statistics curriculum which takes advantages of the technology can stress conceptual understanding, mathematical modeling and problem solving, real-world applications, and new methods of analyzing data. Moreover, it can support a learning environment in which students take an active role, and their social interactions may not merely amplify cognitive capacities, but brings about a reorganization of cognitive capacity (e.g., National Council of Teachers of Mathematics, 1989 and 2000; Hawkins et al. 1992; Heid, 1995, Biehler, 1997).

4) Empirical research on teaching and learning statistics

Empirical research studies include students' naive conceptions of statistical concepts, the development of graphical understanding in students, and case studies documenting the teaching and learning of statistics.

a) Naive conceptions of statistical concepts

The misunderstanding of the concept of *mean* has been investigated by Pollatsek, Lima & Well (1981), Mevarech (1983), Strauss & Bichler (1988), Mokros & Russell (1992), and recently in a more global approach by Konold & Pollatsek (in review). Pollatsek and Mevarech found that students use strategies, such as active balancing or representativeness when dealing with the mean, inappropriately. Strauss & Bichler identified different developmental paths of children's reasoning on some tasks measuring seven predetermined properties of the mean. Mokros & Russell concluded that the mean is a mathematical object of unappreciated complexity (belied by the 'simple' algorithm for finding it) and that it should only be introduced relatively late in the middle grades, well after students have developed a strong foundation of representativeness.

Batanero, Estepa, Green & Godino (1996) report a variety of student interpretations of the cells in 2x2 contingency tables. Landwehr (1989) summarizes a number of misunderstandings about elementary statistical concepts, including the tendency to believe that any difference in means between two groups is significant, and the belief that there is no variability in the 'real world'. Recent works focused on statistical ideas and reasoning beyond the most basic ones, such as, variability, inference and sampling (e.g., Watson & Moritz, 1999 and 2000; Reading and Shaughnessy, 2000).

b) The development of graphical understanding

Few studies on the understanding of statistical graphs exist (cf., Ainely, 1995). Curcio (1987) distinguished three difficulty levels in the comprehension of graphical data:

a) reading *the* data without any interpretation, attending only to facts explicitly represented in graphs;

b) reading *within* the data, which requires comparisons, mathematical concepts and skills; and

c) reading *beyond* the data, which requires extension, prediction, or critique.

The understanding of particular types of graph has been investigated by a number of researchers, e.g., bar-graphs (Pereira-Mendoza & Mellor, 1991); bar-charts and line graphs (Aberg-Bengtsson and Ottosson, 1995); stem-and-leaf plots (Dunkels, 1994; Pereira-Mendoza & Dunkels, 1989); box-plots (Carr & Begg, 1994); and scatter plots (Estepa and Batanero, 1994).

Pereira-Mendoza (1995) discussing graphing at the elementary level, suggested that children should explore the assumptions underlying the classification of data and interpretation of the meaning of data, discuss and explore the possibility of alternative representations, and predict from the data. Recently, Bakker (2001) has analyzed how concepts and symbolizations co-develop in the case of statistical data analysis. The study shows how 11 to 12 year-old students developed their concept of *distribution* in relation to the graphs they used. In particular symbolization of data into a so-called ‘bump’ is discussed.

c) Case studies documenting the teaching and learning of statistics

In a teaching experiment conducted with lower secondary school students by Biehler & Steinbring (1991), data analysis was introduced as ‘detective’ work. Teachers gradually provided students with a data ‘tool kit’ consisting of tasks, concepts, and graphical representations. The researchers concluded that all students succeeded in acquiring the beginning tools of EDA, and that both the teaching and the learning became more difficult as the process became more open. There appears to be a tension between directive and non-directive teaching methods in this study. In addition, this case study and others (e.g., de Lange, Burrill & Romberg, 1993) point to the crucial need for professional development of teachers in the teaching of EDA in the light of the difficulties teachers may find in changing their teaching strategy from expository authority to guide. It is also a challenge for curriculum developers to consider these pedagogical issues when creating innovative EDA materials. Recent experimental studies in teaching basic EDA concepts in middle school classes have been conducted by Cobb (cf., 1999) with an emphasis on socio-cultural perspectives of teaching and learning.

Another evolving area of research is related to methods of alternative assessment in statistics (c.f., Garfield, 1993; Gal and Garfield, 1997b). New ideas about learning and the way that assessment is viewed in education in general have started to impinge on statistics education and its research (Begg, 1997).

Future research on EDA

Research related to teaching and learning statistics still needs to achieve academic recognition in the different disciplines or educational programmes (Jolliffe, 1998). Batanero, Garfield, Ottaviani, and Truran (2000) attempt to define some important questions to be researched in statistical education and the theoretical foundations of such research. Scientific efforts and extended dialogs among researchers and educators are required to define “what is research in statistics education”, and to convince others of its validity as a research discipline.

Since EDA is a relatively new topic in statistics education and in the mathematics curriculum in schools, the existing research does not yet provide full documentation and analysis of the domain. Many research questions remain open, especially those pertaining to students learning and understanding (with the assistance of technological tools), the student-teacher and student-student interactions with open-ended data investigation tasks, and the role of enculturation in learning. This thesis is intended to be a contribution to the study of such questions.

Part 4

The Learning Environment: *The Statistics Curriculum* (*SC*)

Response to the research questions of this thesis cannot be accomplished in a 'vacuum'. A carefully designed and systematically researched learning environment - imbued with a genuine spirit of EDA - is required. The challenge is to develop a coherent sequence of learning situations, together with the necessary materials and computerized tools, whose implementation has the potential to impact the daily practices of teaching and learning statistics in classrooms. The ultimate intention is to transform classrooms from academic work factories where students perform assigned tasks under the management of teachers into communities of learning and interpretation, where students are given significant opportunity to take charge of their own learning, and reflective practice among students, teachers, and researchers is encouraged (in the spirit of Brown, 1992).

The *Statistics Curriculum* (*SC*) is one component of a large and comprehensive mathematics curriculum for the junior high school level called *CompuMath*, of the Science Teaching Department, Weizmann Institute of Science. These curricula are characterized by the teaching and learning of mathematics using open-ended problem situations to be investigated by peer collaboration and classroom discussions using computerized environments. In the following, I first describe the three-stage development of the *SC*, and then present and characterize the curriculum.

The development of the Statistics Curriculum

The main goal of the curriculum development process was to design and create a learning environment in which students are engaged in meaningful statistics, i.e., statistical processes that arise for the students in familiar problem situations as natural means for investigating and solving the problem, rather than as ritual procedures or ready-made algorithms that are imposed by the teacher or the textbook. The curriculum development process consisted of the following three stages. (More details on this process are provided in Hershkowitz, et al., in press.)

The first stage involved design considerations, before starting the actual development and research work, taking into account statistical content and syllabi, national standards and international trends, the participants in the process and their background, the state of the art of teaching and learning EDA, theoretical approaches to learning, etc. To illustrate the type of activities and considerations in this stage, I focus on the choice of the computerized tool to be incorporated in the *SC*.

Our criteria for choosing technological tools in the *CompuMath* project were: (a) *generality*, that is, applicability in different content areas, availability, and cultural status; (b) *mathematization*, i.e., amplification and reorganization of students' work in the sense of Pea (1987) and Dörfler (1993); and experience new 'mathematical realism', in the sense of Balacheff and Kaput, 1996); and (c) *communicative power,* meaning the power of the tool to support the development of statistical language. Against these criteria, I considered the types of software generally used in statistics instruction: statistical packages, microworlds, tutorials, resources (including Internet), and teacher's meta-tools (Biehler, 1993, 1997; and Ben-Zvi, 2000, Paper VI). At this preliminary stage, I chose *Stats!* (LOGAL® Software, Inc.), an educationally modified *statistical package*, which is a basic and simple data analysis program intended for introductory statistics courses in school level. I hypothesized that the dynamic, manipulative power of *Stats!* would foster mathematization and that its simultaneous displays would give it communicative power for all students. Further discussion on the use of technological tools in statistics education is provided in Paper VI below.

The second stage consisted of a first design of the activities and their trial in a few classrooms, accompanied by classroom research on learning and teaching practices (observations, data collection and analysis). Some of the findings of this extensive pilot are described in papers, which are not part of this thesis (cf., Ben-Zvi, 1997a; 1997b; 1999b; Ben-Zvi & Arcavi, 2001). The study itself, and the comments and critiques that arose from local and international experts, helped me to redesign the learning materials (to make them more attuned to student interests, motivation, capacities, and interactions); add an extended statistical 'research project' to the curriculum; reexamine the software, and, as a result, replaced it with spreadsheet (see details below); select, focus on and refine the research questions of this thesis; and choose and improve the methods used informally, in order to apply them more systematically to answer the research questions.

During the experimental implementation of the curriculum I was forced to reexamine the type of software used, which although seemed to help students in using data to solve real problems, had limited statistical power and was uncommon in schools. Therefore, I replaced *Stats!* by a spreadsheet package (Excel®), for the following reasons:

1. Spreadsheets are *common* and *familiar*. Spreadsheets, especially Excel®, are now recognized as a fundamental part of computer literacy (Hunt, 1996). They are used in many areas of everyday life, as well as in other domains of mathematics curricula, and are available in many school computer labs. Hence, learning statistics with a spreadsheet helps to reinforce the idea that this is something connected to the real world. Moreover, prior and in parallel to the learning of the EDA course, the students used spreadsheets in their studies of algebra.

2. Spreadsheets provide *direct access*, that allows students to view and explore data in different forms (data representations or inscriptions), investigate different models that may fit the data, manipulate a line to fit a scatter plot, etc.

3. Spreadsheets are *flexible* and *dynamic* allowing students to experiment with and alter displays of data. For instance, change, delete or add data entries in a table and consider the graphical effect of the change, or manipulate directly data points on the graph and observe the effects on a line of fit. Further, they are *adaptable*; namely, they provide students and teachers with control over the content and style of the output.

One major concern at this stage was the degree of *openness* of the activities. Our initial inclination, when planning a 'virtual activity', was to engage students in the investigation of data in a given context, and then to give them the freedom to choose research questions, tools and strategies to analyze the data. Thus, the 'virtual activity' was very open, instructions being minimal. The actual design of activities was however realized as an ongoing compromise, by trying to find the appropriate blend between open and closed.

Further, I experimentally added at this stage to the EDA course an extended activity - a '*research project*' (described below), in which each pair of students explore a real data-based problem of their choice. At this stage, I had scarce knowledge about the implementation of projects in class. For instance, how and when to introduce the project to students, how to guide their work and assess it.

The third stage comprises the creation of coherent sequences of redesigned activities forming a complete curriculum and its implementation, including the dissemination of the curricular aims and 'spirit' on a national scale, and focused research initiatives. I focus my research on the compound relations between the design of curricular activities and learning (Paper V: 'learning arenas').

In summary, the *Statistics Curriculum* was developed in an extended and complex process, in which practices were fed by theory and research, and vice versa (in the spirit of 'developmental research', cf., Cobb, 1988). It was implemented in schools and in teacher courses, and subsequently revised in several development cycles. A description of the *SC* follows.

The Statistics Curriculum

The *SC* was developed in order to introduce junior high school students (grade 7, age 13) to statistical thinking and the 'art and culture' of EDA. The *SC* stresses: (a) student active participation in organization, description, interpretation, representation and analysis of data situations on topics close to the students' world, with a considerable use of visual displays as analytical tools (in the spirit of Garfield, 1995; and Shaughnessy et al., 1996); and (b) incorporation of technological tools for simple use of various data representations and transformations of them (cf., Biehler, 1993, and in Paper VI below).

The design of the learning activities (Ben-Zvi & Friedlander, 1997b, Appendix III) was based on the creation of 'small scenarios' in which students can experience some of the processes involved in the experts' practice of data based enquiry. The learning activities are mostly open-ended situations using spreadsheets, in which statistical *content* and *concepts* are emphasized. For example, types of data, posing questions and collecting data, basic statistical measures, handling and interpretation of data representations, and intuitive notions of inference and correlation. Emphasis is also put on 'big ideas', such as, need for data, proof and certainty in statistics, the existence of variability, and the underlying points of view, processes and methods.

The form of *instruction* for this curriculum emphasized a community of learners communicating with each other and a new role for the teacher as an 'enculturator', who guides the students in adopting and picking up EDA experts' points of view (Paper II). The curriculum content and pedagogy is based on the PCAIC investigative cycle (*pose*, *collect, analyze, interpret, and communicate*) proposed by Graham (1987), and Kader & Perry (1994) (see Figure 1).

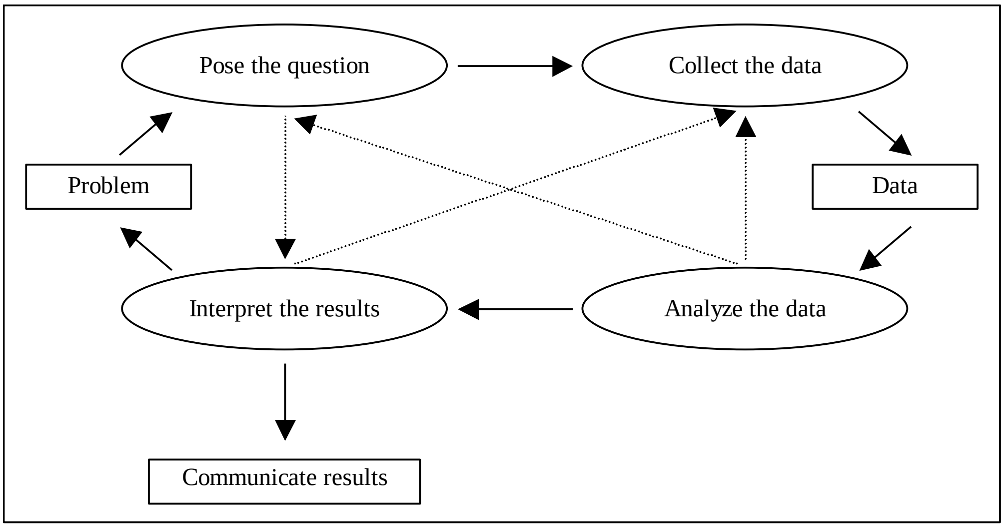

Figure 1. The PCAIC cycle for statistical investigation. Dotted arrows illustrate possible research paths.

The student is expected to:

- become familiar with the problem at hand, identify research questions and hypothesize possible outcomes;
- collect, organize, describe, and interpret data;
- construct, read, and interpret data representations;
- develop a critical attitude towards data;
- make inferences and arguments based on data analysis;
- understand and apply measures of central tendency, variability (and correlation[10]);
- use curve fitting to predict from data;
- communicate the results of his investigation.

[10] Correlation and use of curve fitting to predict from data were presented only in some of the seventh grade classes, or postponed to the higher-grade class.

Students work on two parallel strands - (1) classroom activities, and (2) a 'research project' (see Figure 2). Each of these will be discussed in turn.

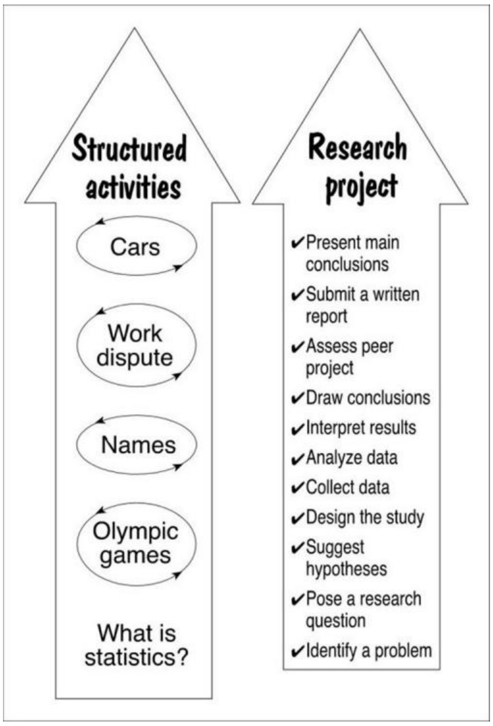

Figure 2. The two strands of the *Statistics Curriculum.*

1. The classroom activities

The *classroom activities* are semi-structured open-ended statistical investigations, in which considerations and processes involved are also pursued. The situations focus on topics close to the students' world (sport, people's names, salaries, cars, etc.), and provide the background for statistical concepts and methods. The students are encouraged to hypothesize about possible outcomes, choose tools and methods of inquiry, representations, conclusions, and interpretation of results. Most activities are designed to be done in pairs.

The role of the teacher is to introduce the investigation, to foster communication among groups, to raise questions, to guide the students through technical and conceptual difficulties, and to conduct the introductory and summary discussions. The activities are interspersed with more traditional class work, designed to reinforce statistical concepts, procedures and routines. A lesson starts usually with a short introduction by the teacher on the investigation topic, and a discussion of background, prior knowledge, data, research question and hypotheses. Later, students, in pairs, spend most of the class time in analyzing the data using a computer. Once the investigation is finished (it may take more than one class period), the class gathers for a synthesis session with the teacher, in which main results and concepts are argued and summarized. The '*Work Dispute'* activity illustrates the above.

The '*Work Dispute'* activity concerns workers in a printing company: The workers are in dispute with the management, who has agreed to a total increase of 10 percent in the salary bill. The dispute is about how this increase is to be divided among the employees. The students are given the present salary list of the one hundred employees, and an instruction booklet to guide them in their work. They are also provided with information about the national average and minimum salaries, Internet sites to look for data on salaries, newspaper articles about work disputes and strikes, and a reading list of background material. In the first part of the activity, groups of students are required to take a position in the dispute, and to clarify their arguments. Then, using the computer, they have to describe the distribution of salaries and use appropriate measures (median, mean, mode and range) to support their position. They learn about the effects of grouping data and the different uses of statistical measures in arguing their case. In the third part, their task is to suggest changes to the salary structure which satisfy the 10 percent constraint. They produce their proposal to solve the dispute, and design representations to support their position and refute opposing arguments. Finally the class meets for a general debate and votes for the winning proposal. The time spent on this activity is about seven class periods.

The computerized tool helps students to develop expertise in using data to solve real problems. It frees them from computations and graph drawing. It allows students to view and explore data in different forms, e.g., highlight a specific point (or sets of points) in a data set, alter dynamic displays, and explore different models that may fit the data. The impact of technology is studied in Paper VI.

2. The 'research project'

The *'research project'* is an extended activity, in which the students act as independent and responsible learners. Students identify a problem and the questions they wish to investigate, suggest hypotheses, plan and design the investigation, collect and analyze data, interpret the results and draw conclusions. At the end they submit a written report and present their main results to fellow students and parents in a 'statistical happening'.

The teacher schedules dates for each stage, guides the students individually, and actively supports and assesses student progress using alternative assessment methods (Gal and Garfield, 1997b). Some of the topics students have chosen to investigate were: superstitions among students, attendance at football games, student ability and the use of Internet, students' birth month, formal education of students' parents and grandparents, and road accidents in Israel.

The time spent on the full *SC* unit in the pilot experimental classes was 25-30 hours (roughly two months), of which two thirds were spent in the computer lab. Students spent a similar amount of time out of class working on home assignment and their *'research projects*'. The implementation of this curriculum also included community involvement (parents, school authorities, etc.), and use of school surveys, media, and data sets archives in the Internet.

Chapter 2

A and *D*'s Case

Paper I **Constructing an understanding of data graphs**

Paper II **Construction of global views of data and data representations**

CHAPTER 2

A AND *D*'S CASE

The following chapters (Chapters 2, 3, and 4) introduce the scope of the Ph.D. research work, presented in a collection of six papers. The goal is to form a coherent description of how junior high school students begin to learn *Exploratory Data Analysis* (EDA) in a carefully designed, computer assisted learning environment (the *SC*).

In this chapter, I analyze at a very fine level of detail the ways in which two students (*A* and *D*) began to make sense of data and data representations, as well as the process of adopting and exercising the habits and points of view that are common among experts. In this process, I consider the role of peer and student-teacher interactions to enhance learning. The focus in the second paper is on the ways they started to develop global views (and tools to support them) of data and their representations on the basis of their previous knowledge and different kinds of local observations. I realize how knowledge was gradually constructed through complex cognitive and socio-cognitive processes, which included their interactions with each other, the teacher, the materials and the computerized tool. I analyze the ways in which the same 'pieces' of students' prior knowledge not only hindered, but ultimately also supported the construction of meaning for data and data representations.

The students: A and D

A and *D*, the subjects of this chapter, were students in a progressive experimental school, who come from an affluent background. They were above-average ability students, verbal and willing to share their thoughts, attitudes, doubts and difficulties. They had worked collaboratively in various school subjects, and had acquired good cooperative working praxis. They agreed to participate in this study, including being videotaped and interviewed (after class), and furnishing their notebooks for analysis. They were familiar with computerized environments and spreadsheets, and resourceful in their work with unfamiliar features of the software.

A and *D* engaged seriously with the curriculum, trying to understand and reach agreement on each task. They were quite independent in their work, and called the teacher when technical or conceptual issues impeded their progress. The fact that they were videotaped did not intimidate them; on the contrary, they were pleased to speak out loud, address the camera explaining their actions, intentions, and misunderstandings, and share what they believed were their successes. Michal, their mathematics and science teacher, characterizes them:

> "*I is a grown-up, easy-going and polite student. He always likes to explore puzzling issues, until they are fully exhausted. He is bright and diligent, but not extremely quick. His friend, D, is a good student and has excellent verbal and writing abilities. He tends to perform in class and in daily life of school on a 'large scale'. He frequently invents quizzes, puzzles, and poems and includes them in his homework assignments. Quiet often, D seems to be trying hard to impress the adults around him*."

My classroom observations of the pair show that they are both hard-workers, cooperating very well together. They have open and good communication with Michal (their mathematics teacher), consider her to be "*the best mathematics teacher*", and ask often for her help or corroboration. They fully complete their home assignments, try to solve the complementary challenging tasks, and happily report to the teachers on mistakes, missing data, or contradictions in the assignments.

The differences between them are well captured in their mathematics portfolios, which were submitted at the end of the first semester of the seventh grade (right before the beginning of the *Statistics Curriculum*). In his portfolio, *I* managed to 'get to the bottom' of the algebraic conceptions taught in class, and arranged and analyzed the issues brilliantly. *D*, using his writing talents, described chronologically his learning, in a very pleasant, lengthy and clear style.

In sum, *A* and *D* approach the *Statistics Curriculum*, with extensive and good collaborative learning habits, social oriented classroom praxis, understanding of basic concepts of algebra, including multiple representation approach to problem solving, basic knowledge base in computers and spreadsheets, and positive attitude towards the use of computers in mathematics.

Paper I

Constructing an Understanding of Data Graphs

Ben-Zvi, D. (1999). Constructing an understanding of data graphs. In O. Zaslavsky (ed.), *Proceedings of the Twenty-Third Annual Conference of the International Group for the Psychology of Mathematics Education, II*, 97-104. Haifa, Israel: PME.

Paper I

Constructing an Understanding of Data Graphs

In this first case study I focus on the microevolution of students' incipient understandings of graphs as data representations. To understand students' discussions, considerations, difficulties, solutions, and interactions, I used *interpretive microanalysis* (described in the Methodology section above), which will be systematically used and improved in Paper II.

The purpose of this paper is to study how students construct their understanding of graphs as displays of real life data, and learn to re-design them to support certain claims. I describe episodes of two 13-year-old students (*A* and *D*) working on the activity - '*The Same Song, with a Different Tune*' (about four lesson periods). The context is the Olympic 100 meters race. The students were given, in a spreadsheet, the men's 100 meters winning times, and the years in which they occurred (from 1896, the first modern Olympiad, to 1996, the 23^{rd} Olympiad).

In the first part of the activity, the students were introduced to the context of the investigation and were asked to describe the data graphically and verbally. In the second part, the students were asked to manipulate data graphs, i.e., change scales, delete an outlier, and connect points by lines. In the third part, they were asked to design a graph to support claims, such as: *"Over the years, the times recorded in the Olympic 100 meters improved considerably"*.

I describe and analyze the students' development from a stage in which they did not understand the requirements of the task and the notion of *trend*, to the successful completion of the design task, in which they were able to make sense of statistical concepts and ideas. At the beginning, they interpreted the effect of changing scales as a movement of the graph downwards rather than an effect on its shape. However, the purpose of the re-scaling was to enable the students to visualize the graph as a whole in a different sense. In order to support a claim in a debate, the transformation was aimed at visually supporting the position that there are significant improvements in the records.

A teacher's intervention, which seemed to be a miscommunication, helped the students to make sense of their tasks and they started to consider how scaling of both axes affects the shape of the graph. Moreover, they were able to develop manipulations for these changes to occur in order to achieve the ‘most convincing’ design. Briefly stated, they transferred and elaborated, in iterative steps, ideas of changing scales, from one axis to the other until they finally arrived at a satisfying graph, without any further intervention from the teacher.

In response to the second research question, the description also includes the teacher’s interventions, the students' discussions and collaborative attempts to solve the tasks (descriptions, self-explanations, questions to a colleague and the teacher, transfer of ideas, etc.), which have contributed to the construction of students' understanding of certain characteristics of data graphs.

CONSTRUCTING AN UNDERSTANDING OF DATA GRAPHS

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I describe episodes of two 13-year-old students working on Exploratory Data Analysis (statistics) developed within an innovative curriculum. I analyze the microevolution of their incipient understandings of some features of graphs as data representations. The description includes the role of the instructional materials, the students' discussions and collaborative attempts to solve the tasks, and the teacher's intervention. Although her intervention seemed to be a miscommunication, it appears to have helped the students to make sense of their tasks.

BACKGROUND

The teaching of *Exploratory Data Analysis* (statistics) is mostly based on: (a) organization, description, representation and analysis of data, with a considerable use of visual displays (Shaughnessy et al., 1996); (b) a constructivist view of learning (Garfield, 1995); and (c) incorporation of technological tools for making sense of data and facilitating the use of various data representations (Biehler, 1993).

With these perspectives in mind, we developed a middle school statistics curriculum[11] (Ben-Zvi & Friedlander, 1997a), implemented it in schools and in teacher courses, and undertook research on learning (Ben-Zvi & Arcavi, 1997; Ben-Zvi & Friedlander, 1997b). The curriculum is characterized by: (a) a use of extended real (or realistic) problem situations; (b) collaboration and communication in the classroom; and (c) a view of the teacher as "a guide on the side" (Hawkins et al., 1992). The students pose, collect, analyze, interpret data, and communicate (Graham, 1987) using a spreadsheet. The classroom activities are semi-structured investigations, in which students, working in pairs, are encouraged to hypothesize about possible outcomes, choose tools and methods of inquiry, design or change representations, interpret results, and draw conclusions.

THE STUDY

A pair of 13-year-old students (*A* and *D*) was videotaped at different stages of their learning statistics (20 hours of tapes). I focus here on 15 minutes of their work with brief teacher interventions. The students were considered by their teacher to be both very able and very verbal. They were asked to talk aloud and explain their actions.

The purpose of the following analysis is to study how students construct their understanding of graphs as displays of real life data, and learn to design them to support certain claims. I used interpretive microanalysis (see, for example, Meira, 1991, pp. 62-3) to try to understand students' discussions,

[11] The project is part of *CompuMath*, an innovative and comprehensive curriculum (Hershkowitz & Schwarz, 1997).

considerations, difficulties and solutions. In this analysis I consider socio-cognitive aspects, taking into account verbal, gestural, and symbolic actions, in the context in which they emerged -- comparing and contrasting the data with other pieces of data, written records, and conversations with the teacher.

The Problem Situation

The extended (four lesson) activity - *The same song, with a different tune* - occurs early in the curriculum. The context is the Olympic 100 meters race. The students were given, in a spreadsheet, the men's 100 meters record times, and the years in which they occurred (from 1896, the first modern Olympiad, to 1996). In the first part of the activity, the students were introduced to the context of the investigation and were asked to describe the data graphically and verbally. In the second part, the students were asked to manipulate data graphs, i.e., change scales, delete an outlier, and connect points by lines. In the third part, they were asked to design graph to support the following claim: *"Over the years, the times recorded in the Olympic 100 meters improved considerably"*.

In the following, I present and analyze the students' work through the activity.

DEVELOPING UNDERSTANDING OF DATA GRAPHS: THE 'STORY' OF *A* AND *D*

In this section, I present three parts of the activity chronologically: (a) getting acquainted with the context, (b) acquiring tools, and (c) designing graphs.

(A) Getting Acquainted with the Context

In the first part of the activity, *A* and *D* analyzed the table of results, compared the records of consecutive Olympiads, considered the issue of extreme data, sorted the data, and created a graph with a spreadsheet (Figure 1). In their written summaries, they wrote that (a) the best record is 9.48 sec. and the worst is 12 sec., (b) the greatest improvement is from 10.25 to 9.48 sec., and (c) the differences between records are not constant. The first two conclusions are wrong: the best record is 9.84 sec., and the greatest improvement is from 12 to 10.8 sec. When requested to describe the data *patterns*, they did not seem to understand the meaning of the question. With the teacher's help, they concluded correctly that "the record times seem to improve, yet there was occasionally a lower (slower) result, than the one achieved in previous Olympiads".

Although *A* and *D* seemed to notice the general trend of improvement in the records, their view was mostly local and focused on discrete data points, or, at most, on two consecutive records. I claim (based on data not detailed here) that their difficulty to discern general data *patterns* was caused by: (a) the students' lack of experience with the notion of *pattern*; (b) the discrete nature of the graph; (c) the non-deterministic and disorganized nature of statistical data, which is very different from the deterministic formulae, they had met in algebra.

(B) Acquiring Tools

In the second part of the activity, the students became acquainted with three strategies for manipulating graphs (changing scales, deleting an outlier, and connecting points), and considered the effect of these changes on the shape of the graphs. The objective was to prepare for the *design* task (Part C below).

Changing Scales

The following transcript describes the students' comments on the effect of changing the vertical scales of the original graph from 0-12 to 0-40 (Figures 1& 2):

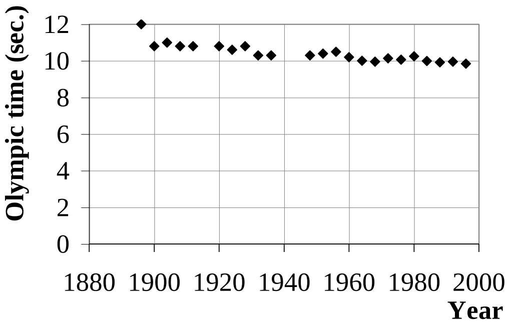

Figure 1: Given graph

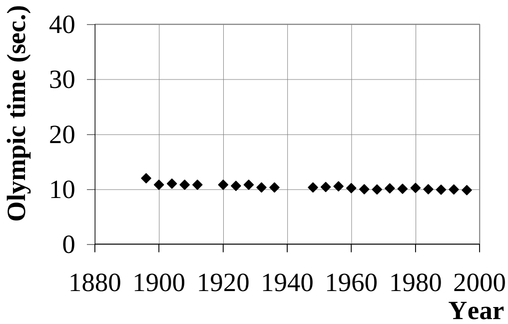

Figure 2: Manipulated graph

A. Now, the change is that… that the whole graph stayed the same in shape, but it went down…

D. The same in shape, but much, much lower, because the column *[the y-axis]* went up higher. Did you understand that? *[*D *uses both hands to signal the down and up movements of the graph and the y-axis respectively.]*

A. Because now the 12, which is the worst record, is lower. It used to be once the highest. Therefore the graph started from very high. But now, it *[the graph]* is already very low.

The students' perception of the change is restricted to the overall relative position of the graph; they considered the shape itself as remaining "the same". Their description includes: global features of the graph ("The whole graph … went down"), an interchange of background and foreground (the graph went down and/or the y-axis went up), and local features (12 as a "starting point" of the graph). These descriptions are linked and complement each other. *A* wrote the following synthesis in his notebook: "The graph remained the same in its shape, but moved downward, because before, 12 - the worst record - was the highest number on the y-axis, but now it is lower".

Deleting an Outlier

In this task, the students were asked to delete an outlying point (the record of 12 sec. in the first Olympiad, 1896) from the graph (Figure 2), and describe the effect on its shape. First, *D* justified why 12 can be considered an outlier:

D. It [the record of 12 sec.] is pretty exceptional, because we have here [in the rest of the data] a set of differences of a few hundredths, and here [the difference is] a whole full second.

Then, they struggled to interpret the effect of the deletion on the graph (Figure 3):

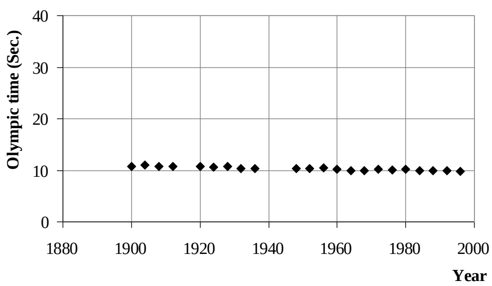

Figure 3: Outlier deletion

D. The change is not really drastic … Now, however, the graph looks much more tidy and organized.

A. One point simply disappeared. The graph… its general shape didn't change.

They wrote in their notebooks different descriptions of the change: "The graph became straighter" (*D*); "One point in the graph disappeared" (*A*). Thus, the students struggled between different views of the effect: global and significant change (the graph is tidy and organized), no change at all (the general shape didn't change); or just a mere description (one point disappeared).

Although the dispute about the outlier was not resolved, it served another purpose: it drew *A*'s attention to a mistake in their conclusions in the first part of the activity, and corrected it: "the greatest improvement is from 12 to 10.8 seconds".

<u>Connecting Points</u>

In the third task, they were asked to connect the points to obtain a continuous graph. The new graph (Figure 4) elicited many comments from the students, who tried to make sense of what they saw. They were particularly intrigued by the fact that the connected graph included both the original points, and the connecting line.

D. OK. You see that the points are connected by lines. Now, what's the idea? The graph did not transform to one line. It transformed to a line, in which the points are still there. It means that the line itself is not regarded as important.

A. This line is OK. We previously thought that if we connect the points with a line, they might disappear. But now, there is a graph, and there are also the points, which are the important part.

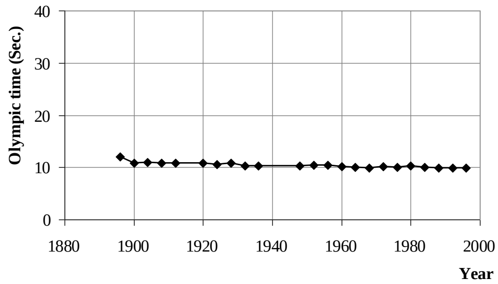

Figure 4: Connecting line

In their view, the connecting line (as provided by the spreadsheet) not only did not add any new meaning, but also contradicted the context, as *D* observed: "Olympiads occur only once in every four years" (namely, there is no data between the points). The students did not see the line as an aid to detect or highlight patterns in the data, and this is consistent with their previous difficulties in recognizing data patterns.

So far *A* and *D* were practicing manipulations (changing scales, deleting an outlier, and connecting points), and discussing their effect on the graph's shape. The intention was to provide students with the means to design a graph, in order to support a particular claim. In the following section, I discuss in what sense this preparation helped them achieve this purpose.

(C) Designing Graphs

I present here a fragment of the students' work on the third part of the activity. The students were asked to design a graph to support the statement: "*Over the years, the times recorded in the Olympic 100 meters improved considerably*". I bring first a teacher intervention, which eventually helped the students understand the task. Then, I focus on five attempts (*Stages 1-5* below) to obtain a satisfactory form of the graph.

The Teacher Intervention

A and D did not understand the task and requested the teacher's (T) help:

T. [Referring to the 0-40 graph displayed on the computer screen -- see Figure 4.] How did you flatten the graph?

A. [Surprised] How did we flatten it?

T. Yes, you certainly notice that you have flattened it, don't you?

D. No. The graph was like that before. It was only higher up [on the screen].

The teacher and the students are at "loggerheads". The teacher assumes that the students (a) had made sense of the task, but just did not know how to perform it, (b) had acquired the necessary tools, and understood their global effect on the graph's shape to be used to support the claim. Thus, her hint consisted of reminding them of what they had already done (scale change). However, the students did not regard what they had done, as changing the graph's shape. Although this intervention seemed to be a case of miscommunication, it apparently had a catalytic effect, as reflected in the dialogue, which took place immediately afterwards:

T. How would you show that there were very very big improvements?

A. [Referring to the 0-40 graph -- see Figure 4.] We need to decrease it [the maximum value of the y-axis]. The opposite…[of what we have previously done].

D. No. To increase it [to raise the highest graph point, i.e. 12 sec.].

A. The graph will go further down.

D. No. It will go further up.

A. No. It will go further down.

D. What you mean by increasing it, I mean - decreasing.

A. Ahhh… Well, to decrease it… OK, That's what I meant. Good, I understand.

Even though their use of language is not completely clear, their previous perception that the graph shape remains the same was not mentioned at this stage. Moreover, *D* expressed what appears to be a new understanding:

D. As a matter of fact, we make the graph shape look different, although it is actually the same graph. It will look as if it supports a specific claim.

At this point, *D* seems to discern that a change of scales may change the perceptual impressions one may get from the graph. Thus, they seemed to understand the purpose of the activity, and started to focus on its goal. In the following, the students' five attempts to design corresponding graphs are presented.

Stage 1 (The scales are changed to x: 1880-2000; y: 0-5)

D suggested (Figure 4) changing the scale on the y-axis to 0-11. It seems that he chose 11, since he had previously deleted the outlier, making 11 the maximum data point. They didn't implement this change, because he immediately proposed another scale change: 0-5. This suggestion seems to be based on his assumption that the smaller the range the larger the decline in the record time would look *(Idea I).* However, when they implemented this change, the graph disappeared.

A. *We don't see the graph at all, since there is no graph in 5.*

Stage 2 (x: 1880-2000, y: 0-12)

Having failed to present a new graph, they returned to the 0-12 range (see Figure 1):

A. The graph looks more curved, because the difference between records is much bigger, since we increased the… now the "Olympic time in seconds" [y-axis] is from 0 to 12, and every record– as much as it descends – it is bigger than the record… the line is more…[D. interrupts] Wait a second, the line is bigger than it used to be from 0 to 40.

The effect of changing scales on the graph's global features (straight, curved), which were not noticed initially, and started to be considered after the teacher's intervention, were now being fully considered. Still, the students struggle to verbalize and explain what they do, or want to do.

Stage 3 (x: 1896-1996, y: 0-12)

At this stage, it seemed that *A* and *D* had exhausted the changes on the y-axis. So they turned to the x-axis. *D* suggested changing the upper limit of x from 2000 to 1000 (*Idea I* above). They realized, however, that this would cause the graph to disappear again (the year's range is 1896-1996). Thus, *D* proposed using 1996 (instead of 2000) as the upper limit of x. Although the effect[12] was marginal, *D* commented:

D. One can really see, as if there are bigger differences in the graph… Very interesting!

Although they had presumably understood how changing scales effects the graph's shape, *D*'s wrong impression of this horizontal change, seems to originate from his ambiguous distinction between vertical or horizontal "differences" and/or distances. However, having focused their attention on the x-axis, they realized that it does not start at zero, which triggered the following idea *(Idea II).*

Stage 4 (x: 1896-1996, y: 8-12)

A transferred attention from the x-axis to the y-axis, and suggested changing the lower limit for y from 0 to 8 (to get a scale of 8-12). Observing the resulting sharp visual effect, he reacted immediately:

A. It looks much bigger.

[12] The lower limit for y changed automatically to 1896, resulting in a final range of 1896-1996, instead of 1880-2000, which were the default values provided by the software.

Stage 5 (x: 1896-1996, y: 9.48-12)

D suggested applying Idea II to the x-axis, but withdrew, when *A* indicated that it already started at a non-zero value. Instead, *A* suggested using the minimum record time (9.48 sec.) as the lower limit of y *(Idea III)*. The resulting graph (Figure 5) satisfied them, and they made the following final comments:

D. This way we actually achieved a result [graph] that appears as if there are enormous differences.

A. To tell you the truth, this booklet is lovely.

D. Right, it is nice!

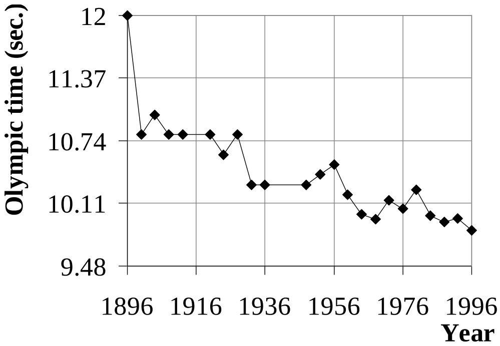

Figure 5: Final design

DISCUSSION

This 'story' of *A* and *D* traces the microevolution of incipient understanding of some features of graphs as displays of real life data (see also Bright & Friel, 1997). It describes the students' perceptual development from a stage in which they did not understand the requirements of the task and the notion of data *pattern*, to the final successful completion of the *design* task. The following elements seem to have contributed to the construction of students' understanding of certain characteristics of data graphs.

Careful instructional engineering. The students worked with semi-structured guidance to solve open-ended questions. First, they acquired tools to modify graphs and then, they employed these tools in the design of graphs, to support a certain claim.

Close collaboration between the pair of students. The students:

a) verbalized almost every idea that crossed their minds. At times this spontaneous verbalization produced mere descriptions, but later served as stepping stones towards a new understanding, and at times, it served as self-explanation (Chi et al., 1989) to reinforce ideas;

b) complemented and extended each other's comments and ideas, which seems to have "replaced" some of the teacher's role in guiding their evolution;

c) decided to request the teacher's help when faced with a difficulty, which could not be resolved among themselves; and

d) transferred and elaborated, in iterative steps, ideas of changing scales, from one axis to the other.

The teacher's main intervention. At a first glance, the teacher's intervention to help the students make sense of the task, can be considered unfortunate. She did not grasp the nature of their question, misjudged their position, and tried to help by reminding them of their previous actions. The students, however, did remember the acquired tools, but perceived them differently.

Nevertheless, this miscommunication itself contributed to their progress. At first, *A* and *D* were surprised by her use of the notion of *flattening the graph* as a description of what they had done. Then, they started to direct their attention to the shape of the graph, rather than to its relative position on the screen. Although puzzled by the teacher's language, the students appropriated (Moschkovich, 1989) her point of view on what to look at. Their previous work and their "struggle" with language seems to have prepared them for the reinterpretation of what they had done, triggered by their teacher's comments.

In sum, the microevolution of the students' understanding of data graphs was influenced by the instructional engineering, the students' ways of making sense (descriptions, self-explanations, questions to a colleague and the teacher, transfer of ideas, etc.), and the teacher's intervention and the use students made of it.

Acknowledgment. I thank Abraham Arcavi and Alex Friedlander for their helpful comments and suggestions.

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Paper II

Construction of Global Views

of Data and Data Representations

Ben-Zvi, D., & Arcavi, A. (2001). Junior high school students' construction of global views of data and data representations. *Educational Studies in Mathematics, 45,* 35–65.

Paper II

Construction of Global Views of Data and Data Representations

The purpose of this second case study is to describe and analyze the first steps of a pair of 7th grade students (*A* and *D*) working through the *SC*. The verbal abilities of these students allowed us to follow, at a very fine level of detail, the ways in which they begin to make sense of data and data representations, as well as the process of adopting and exercising the habits and points of view that are common among experts. I focus on the ways they started to develop global views (and tools to support them) of data and their representations on the basis of their previous knowledge and different kinds of local observations.

As presented above (in Chapter 1), in establishing the setting and in conducting the analysis, I relied on both socio-cultural and cognitive perspectives. The socio-cultural perspective, which is emphasized in this paper, guided me to focus on learning of a complex domain, such as EDA, as the adoption of the points of view of a community of experts, in addition to skills and procedures (Resnick, 1988). Thus, I looked at learning as an '*enculturation*' process, and the design of the *SC* is seen as an attempt to bring *"the practice of knowing mathematics* (statistics) *in school closer to what it means to know mathematics* (statistics) *within the discipline*" (Lampert, 1990, p. 29). To shed light on the common practice of the discipline, I briefly present a framework of the general processes, perspectives and points of view of statistics experts involved in the practice of data based enquiry (Wild and Pfannkuch, 1999). In this framework, I focus on one important perspective of experts' view of data: an inclination to see generalities, or *propensities* (in the sense of Konold, Pollatsek & Well, 1997, p. 151[13]) in data, and a flexible and dynamic shift between local observations and global observations. Briefly stated, *local understanding* of data representation involves focusing on an individual value (or a few of them) within a group of data. *Global understanding* refers to the ability to search for, recognize, describe and explain general patterns in a set of data (e.g., change over time, trends).

I describe how the views of *A* and *D* slowly changed and evolved from a novice to an expert perspective. I focus on how their use of local information promote the

[13] Propensity refers to group tendency, namely, the intensity or rate of occurrence of some characteristic within a group composed of elements that vary on that characteristic.

development of global points of view of data sets in different representations (table, graph). The ‘story’ includes descriptions of how *A* and *D* developed an understanding of:

- On what to focus: from irrelevant towards relevant information,
- How to read and make sense of local (pointwise) information in tables and in graphs,
- How to look at differences between adjacent local entries,
- How to look globally by struggling to learn the ideas underlying global perspectives, the language to describe those ideas, and the difficulties in applying both,
- How to construct global views through handling very special local entries, and
- How to flexibly transfer between local and global views through re-scaling of graphs.

On the basis of the analysis, I claim that learning to look globally at data can be a complex and devious process from a local-pointwise view towards a flexible combination of local and global views of data and data representations. The local pointwise view sometimes restrained the students from ‘seeing globally’, but on other occasions it served as a basis upon which the students started to see globally. In a certain context, even looking globally may indicate different meanings for a novice than for an expert.

The data show that most of the learning took place through dialogs between the students themselves and in conversations with the teacher. Of special interest to me were the teacher's interventions, at the students' request. These interventions, which though short and not necessarily directive had catalytic effects, can be characterized in general as ‘negotiations of meanings’ (in the sense of Yackel and Cobb, 1996). More specifically, I regard them as interesting instances of *appropriation*, as a non-symmetrical two-way process (in the sense of Moschkovich, 1989). This process takes place, in the *zone of proximal development* (Vygotsky, 1978, p. 86), when individuals (expert and novices, or teacher and students) engage in a joint activity, each with their own understanding of the task. Students take actions that are shaped by their understanding; the teacher ‘appropriates’ those actions - into her own framework - and provides feedback in terms of her understandings, views of relevance, and pedagogical agenda. Through the teacher's feedback, the students start to review their actions and create new understandings for what they do.

The data show the teacher appropriating students' utterances with several objectives: to legitimize their directions, to redirect their attention, to encourage certain initiatives, and also implicitly to discourage others (by not referring to certain remarks). The students appropriate from the teacher a reinterpretation of the meaning of what they do. For example, they appropriate from her answers to their inquiries, from her unexpected reactions to their request for explanation, and from inferring purpose from the teacher's answers to their questions.

Appropriation by the teacher (in order to support learning) or by the students (in order to change the sense they make of what they do) seems to be a central mechanism of 'enculturation'. As shown in this study, this mechanism is especially salient when students learn the dispositions of the subject matter rather than its skills and procedures.

Finally, I propose that in learning environments such as the *SC*, the students' learning involves the following: (a) prior knowledge is engaged in multifaceted and sometimes unexpected ways - possibly hindering progress in some instances, but making the basis for construction of new knowledge in others; (b) during the learning process, many questions either make little sense, or, alternatively, are interpreted and answered differently from the original intention; (c) students' work is inevitably based on partial understandings, which grow and evolve towards more complete meanings; and (d) most of the learning takes place through dialogs between the students themselves and in conversations with the teacher.

Our study confirmed that even if students initially do not make more than partial sense of their tasks, through the support of appropriate teacher guidance, class discussions, peer work and interactions, and ongoing cycles of experiences with realistic problem situations, students slowly build meanings and develop experts' points of view on local-global approaches to data and data representations.

JUNIOR HIGH SCHOOL STUDENTS' CONSTRUCTION OF GLOBAL VIEWS OF DATA AND DATA REPRESENTATIONS

DANI BEN-ZVI AND ABRAHAM ARCAVI

ABSTRACT. The purpose of this paper is to describe and analyze the first steps of a pair of 7th grade students working through an especially designed curriculum on Exploratory Data Analysis (EDA) in a technological environment. The verbal abilities of these students allowed us to follow, at a very fine level of detail, the ways in which they begin to make sense of data, data representations, and the 'culture' of data handling and analysis. We describe in detail the process of learning skills, procedures and concepts, as well as the process of adopting and exercising the habits and points of view that are common among experts. We concentrate on the issue of the development of a global view of data and their representations on the basis of students' previous knowledge and different kinds of local observations. In the light of the analysis, we propose a description of what it may mean to learn EDA, and draw educational and curricular implications.

KEY WORDS: Exploratory Data Analysis, learning and instruction, enculturation, local and global views of data.

The real voyage of discovery consists not in seeking new landscapes but in seeing with new eyes.

Marcel Proust (French Novelist, 1871-1922)

INTRODUCTION

Research on mathematical cognition in the last decades seems to converge on some important findings about learning, understanding and becoming competent in mathematics. Stated in general terms, research indicates that becoming competent in a complex subject matter domain, such as mathematics or statistics, "*may be as much a* *matter of acquiring the habits and dispositions of interpretation and sense making as of acquiring any particular set of skills, strategies, or knowledge*" (Resnick, 1988, p. 58). This involves both cognitive development and 'socialization processes' into the culture and values of 'doing mathematics' ('*enculturation*'). Many researchers have been working on the design of teaching in order to "*bring the practice of knowing mathematics in school closer to what it means to know mathematics within the discipline*" (Lampert, 1990, p. 29).

This study is intended as a contribution to the understanding of these processes in the area of Exploratory Data Analysis - EDA (Tukey, 1977; Shaughnessy, Garfield & Greer, 1996). We focus on two 13-year-old students working on the first activity of a statistics curriculum, and concentrate on the growth and change of the students' conceptions as they entered and learned the 'culture' of EDA and started to develop a global view of data and data representations.

We first outline the theoretical perspectives of this study, by presenting aspects of learning as a process of *enculturation.* We then describe a framework of the general processes, perspectives and points of view of statistics experts, and in particular the distinctions between local and global understandings of data.

Secondly, we describe the *Statistics Curriculum* (SC) Project that was developed to introduce junior high school students to statistical thinking and the 'culture' of EDA (Ben-Zvi & Friedlander, 1997b). We then present the first classroom activity. In the methodology section we present details of the two students who were the subjects of the analysis, the experiment, the data collection and the methods we used to analyze their interactions. The analysis, which is the main part of this paper, focuses on excerpts from the interactions. On that basis of the analysis, we propose a characterization of what starting to become competent in EDA may entail. Finally, we propose some instructional implications.

THEORETICAL PERSPECTIVES

Enculturation Processes in Statistics Education

A core idea we use in this study is that of *enculturation*. Recent learning theories in mathematics education (cf., Schoenfeld, 1992; Resnick, 1988) include the process of enculturation. Briefly stated, this process refers to entering a community or a practice and picking up their points of view. The beginning student learns to participate in a certain cognitive and cultural practice, where the teacher has the important role of a mentor and mediator, or the '*enculturator*'. This is especially the case with regard to statistical thinking, with its own values and belief systems, and habits of questioning, representing, concluding and communicating. Thus, for *statistical enculturation* to occur, specific thinking tools are to be developed alongside collaborative and communicative processes taking place in the classroom.

Statistics Experts' Points of View

Bringing the practice of knowing statistics at school closer to what it means to know statistics within the discipline requires a description of the latter. On the basis of in-depth interviews with practicing statisticians and statistics students, Wild and Pfannkuch (1999) provide a comprehensive description of the processes involved in the practice of data based enquiry from problem formulation to conclusions. They suggest that a statistician operates (sometimes simultaneously) along four dimensions, which can be summarized as follows.

- *Investigative cycles:* actions and thinking processes during (a) the selection and definition of a 'real' problem, (b) the making of a plan to tackle it, (c) the data collection, (d) the analysis, and (e) the formulation of conclusions.
- *Types of thinking:* general problem solving strategies (e.g., seeking explanations, modeling) and specific statistical thinking modes (e.g., learning from data via continuously forming and changing data representations, searching for evidence on which to base a judgement, integrating the purely statistical with the contextual constraints and meanings, considering variation);

- *Interrogative cycles*: actions and thinking processes aimed at questioning, judging, criticizing, continuing to entertain or discarding, and the like.
- *Dispositions*: such as imagination, skepticism, curiosity, a tendency to see generalities, and openness to challenge preconceptions.

It is clear that these characteristics of an expert statistician develop over time during the ongoing intensive engagement with the professional practice. In the following, we focus on one important perspective of the expert's view of data: a flexible and dynamic shift between local observations and global observations.

Local and Global Views of Data and Data Representations

The distinction between local and global perceptions/understandings in mathematics education in general has been documented and discussed. For example, Monk (1988), in his study of tertiary students' understanding of functions in calculus courses, distinguishes between *Pointwise* understanding and *Across-Time* understanding. *Pointwise* understanding of functions involves observing particular numerical values of the dependent variable as corresponding to the particular numerical values of the independent variable in a table, graph or formula. *Across-Time* understanding of functions refers to the ability to answer global questions such as: How does change in one variable lead to change in others? Monk suggests that expertise involves both the competence within each view (Pointwise and Across-Time) and the competence to flexibly shift from one view to another, according to what is needed. Similar descriptions in the subject of linear relationships are also presented in Moschkovich, Schoenfeld & Arcavi (1993).

In EDA, *local understanding* of data involves focusing on an individual value (or a few of them) within a group of data (a particular entry in a table of data, a single point in a graph). *Global understanding* refers to the ability to search for, recognize, describe and explain general patterns in a set of data (change over time, trends) by 'naked eye' observation of distributions and/or by means of statistical parameters or techniques. Looking globally at a graph as a way to discern patterns and generalities is fundamental both to statistics and mathematics. In statistics it also includes the production of explanations, comparisons and predictions on the basis of the *variability* in the data. By attending to where a collection of values is centered, how those values are distributed or how they change over time, statistics deals with features not inherent to individual elements, but to the aggregate that they comprise.

The interplay between local and global views of data is reflected in the tools experts use. Among such tools, which support data based arguments, explanations and (possibly) forecasts, are *time plots*. Usually, time plots are Cartesian-like graphical representations, which display change of a chosen variable over time. Time scales of choice (years, months) are represented at regular intervals on the x-axis, and the variable of interest on the y-axis. Time plots highlight features such as center, rates of change, fluctuations, cycles and gaps.

When looking at time plots, one of the main observations of experts consists of looking at *trends* (upward, downward or cyclic 'movements' over time) or *propensities*. Propensity refers to group tendency, namely, the intensity or rate of occurrence of some characteristic within a group composed of elements that vary on that characteristic (Konold, Pollatsek & Well, 1997, p. 151). Experts may connect single data points by line segments in order to aid the search or to highlight the existence of trends, being aware that the rate of change between two data points is not necessarily constant. *Outliers* – individual data points that fall outside the overall pattern of the graph, represent deviations from a trend. Experts may flexibly handle outliers in different ways, using their experienced (and honest) judgment according to context and goals. For example, in some cases, outliers can be of special interest as evidence of an extraordinary event, whereas in others, outliers may be deleted when mismeasurement is suspected (Moore & McCabe, 1993, p.15).

For the purpose of reflection (or even dishonest manipulation), trends can be highlighted or obscured by changing the scales. For example, in Cartesian-like graphs the vertical axis can be 'stretched' (e.g., the distance between two consecutive scale marks may be changed from representing a unit to represent the tenth of that unit) and the time axis can be 'compressed'. In such a case, the graph conveys the visual impression of a steep slope for segments connecting consecutive points, giving a visual impression that the rate of change is very large. Experts propose standards in order to avoid such visual distortions (see, for example, Cleveland, 1994, pp. 66-7).

THE STATISTICS CURRICULUM

The *Statistics Curriculum* (SC) Project, as described in more detail in Ben-Zvi & Friedlander (1997b), was developed in order to introduce junior high school students (grade 7, age 13) to statistical thinking and the ‘art and culture’ of EDA. The design of the Project was based on the creation of ‘small scenarios’ in which students can experience some of the processes involved in the experts’ practice of data based enquiry. The SC was implemented in schools and in teacher courses, and subsequently revised in several curriculum development cycles. It is one component of a large and comprehensive mathematics curriculum for grades 7-9 called *CompuMath* (Hershkowitz, Dreyfus, Ben-Zvi, Friedlander, Hadas, Resnick, Tabach and Schwarz, in press), which is characterized by the teaching and learning of mathematics using open-ended problem situations to be investigated by peer collaboration and classroom discussions using computerized environments.

The SC was designed on the basis of the theoretical perspectives on learning and the expert view of statistical thinking described above. It stresses: (a) student active participation in organization, description, interpretation, representation and analysis of data situations (on topics close to the students' world such as sport records, lengths of people's names in different countries, labor conflicts, car brands), with a considerable use of visual displays as analytical tools (in the spirit of Garfield, 1995; and Shaughnessy et al., 1996); and (b) incorporation of technological tools for simple use of various data representations and transformations of them (as described in Biehler, 1993 and Ben-Zvi, 2000). The scope of the curriculum is 20-30 classroom hours, and it includes a ‘research project’ to be carried out by the students (mostly out of the classroom) with the teacher’s guidance.

The following is the first classroom activity of the SC, in a worksheet format, freely translated from the Hebrew student’s textbook (Ben-Zvi & Friedlander, 1997a).

‘The Men’s 100 Meters Olympic Race’

Part I: ‘Looking at the 100 Meters Data’

Figure 1 shows the introduction, usually presented by the teacher in dialog with the students (about half an hour).

Introduction: Olympics in Short ***(A whole class discussion)***

In ancient times. The Olympic Games were athletic festivals that originated in ancient Greece. Only men took part in the event, which was held every four years in Olympia from 776 BC to AD 393.

In modern times. Since the Olympic Games were revived in 1896 in Athens, 23 Games were held every four years, in various places over the globe. It was Pierre de Coubertin, the founder of the modern Olympic Games, who conceived the idea of the Olympic Flag - five colored interlocking rings on a white background, which represent the union of the five continents and the meeting of the athletes of the world at the Olympic Games. Women began to take part in 1912. The 23rd Olympic Games were held in Atlanta, USA and the next in Sydney, Australia (2000). One of the most interesting and exciting events in the Olympics is the 100 meters race.

In this activity you are going to examine real data about the winning times in the men’s 100 meters during the modern Olympic Games. With the help of the spreadsheet you will analyze the data in order to find trends and interesting phenomena.

Discuss your hypotheses regarding the following questions:

- Are men improving their times in the 100 meters?
- If they improve, what is the rate of improvement?
- Will women ever outrun men in the 100 meters?
- Can you predict the gold-medal running time in the future?

Figure 1. Excerpts from the introduction to ‘*The Men’s 100 meters Olympic Race*’.

Figure 2 shows the first worksheet for students, to work in pairs, using a spreadsheet file, with the men's 100 meters winning times, and the years in which they occurred from 1896, the first modern Olympiad, to 1996 (about two lesson periods).

Part I: Looking at the Data

Open the '*100-meters*' file, which holds the data of the winning times in the men's 100 meters race during the modern Olympics. The data was obtained from *The World Almanac and Book of Facts* (1993) and from the official Website of the Atlanta 1996 Olympic Games.

Table

- What do you learn from the 100 meters table?
- What does a row in this table stand for?
- What do the columns in this table stand for?
- Based on the data in this table, write down your hypothesis regarding trends or interesting phenomena in the winning times.

Time Plot

- Use the computer to display the 100 meters data in a time plot.
- What do the axes stand for? What does a single graph point represent?
- What do you learn from this graph?
- Does it make sense to connect the graph points by lines?

Describe briefly what you have learnt about the men's 100 meters data. Use the table or the time plot to explain your observations. You may use some of the words from the '*Word Basket*' below. [This 'basket' includes, in random order, phrases such as: the best/worst time, interesting phenomena, changes between two consecutive Olympic years, the average change, the biggest improvement, number of improvements/retrogressions, trends, and outlying values.]

Figure 2. Excerpts from the student's textbook in '*The Men's 100 meters Olympic Race*'.

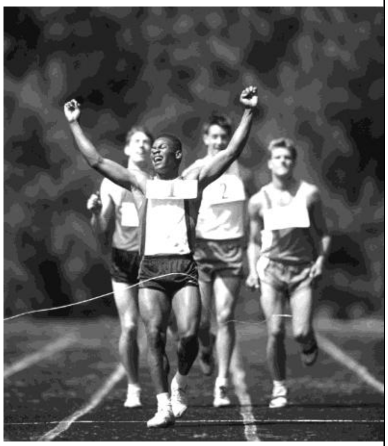
Part II: 'The Same Song, with a Different Tune*'*

Figure 3 presents the second part of the activity (about two lesson periods).

Part II: 'The Same Song, with a Different Tune'

The Journalists' Debate

Two sports journalists argue about the record times in the 100 meters. One of them claims that there seems to be no limit to human ability to improve the record. The other argues that sometime there will be a record, which will never be broken. To support their positions, both journalists use graphs.



Change the Graph (using a computer)

Change the original 100 meters graph, and describe the effect of the change in the following cases:

- Delete an outlier: the record of 12 sec. from the graph. (Can you justify the deletion of this point in the graph?)
- Change the vertical scales from 0-12 to 0-40, and then to 9-12.
- Change the horizontal scales from 1880-2000 to 0-2000.
- Connect the graph's points by lines.

Support Statements

What changes in the graph would you make in order to support the following statements? Draw a graph's sketch under each statement.

- Over the years, the times recorded in the 100 meters improved considerably.
- Throughout the years, the changes in the Olympic times for the 100 meters were insignificant.
- Between 1948 and 1956, the times in the 100 meters worsened considerably.

Presenting Arguments *(First in pairs, and then a whole class discussion)*

Whom do you support in the *journalists' debate*? Support your arguments by using what you have learnt in the analysis and by presenting appropriate graphs.

Figure 3. Excerpts from '*The Same Song, with a Different Tune'.*

METHODOLOGY

Students

At the time of this study, *A* and *D* were seventh graders (13-year-old boys) from a class of 35 mixed ability students (in a progressive experimental school), taught by a skillful and experienced teacher, who was aware of the spirit and goals of the SC Project. They were above average ability students, very verbal and willing to share their thoughts, attitudes, doubts and difficulties. They had worked collaboratively since first grade (age 6), in various school subjects, and had acquired good cooperative working praxis. They agreed to participate in this study, which took place within their regular classroom periods and included being videotaped and interviewed (after class), and furnishing their notebooks for analysis. They were familiar with computerized environments and spreadsheets, and resourceful in their work with unfamiliar features of the software.

When they started to learn this curriculum, *A* and *D* had limited in-school statistical experience. However, they had some informal ideas and positive dispositions towards statistics, mostly through exposure to statistics jargon in the media. In primary school, they had learned only about the mean and the uses of some diagrams. Prior to, and in parallel with, the learning of the SC, they studied beginning algebra based on the use of spreadsheets to generalize numerical linear patterns (Resnick & Tabach, 1999).

They engaged seriously with the curriculum, trying to understand and reach agreement on each task. They were quite independent in their work, and called the teacher only when technical or conceptual issues impeded their progress. The fact that they were videotaped did not intimidate them; on the contrary, they were pleased to speak out loud, address the camera explaining their actions, intentions, and misunderstandings, and share what they believed were their successes.

The Experiment and the Data Collection

The total class time for the whole SC was about 30 periods, spread over two and a half months. *A* and *D* were videotaped at almost all stages (20 hours of tapes) and their notebooks were collected. We interviewed the teacher as an additional source of data. In this paper, we concentrate on the work of *A* and *D* during the first four class periods.

Methods

The analysis of the videotapes was based on interpretive microanalysis (see, for example, Meira, 1991, pp. 62-3): a qualitative detailed analysis of the protocols, taking into account verbal[14], gestural and symbolic actions within the situations in which they occurred. The goal of such an analysis is to infer and trace the development of cognitive structures and the socio-cultural processes of understanding and learning.

In order to validate the analysis, it underwent two stages, one within the researchers' team and one with fellow researchers, who had no involvement with the data or the SC Project (triangulation in the sense of Schoenfeld, 1994). In both stages, the researchers discussed, presented, advanced and/or rejected hypotheses and interpretations, and inferences about the students' cognitive structures. Advancing or rejecting an interpretation requires: (a) the provision of as many pieces of evidence as possible (including past and/or future episodes, and all sources of data as described above); and (b) attempts to produce equally strong alternative interpretations in terms of the available evidence.

[14] The dialogues are translated from Hebrew, therefore they may not sound as authentic as in the original.

TOWARDS THE CONSTRUCTION OF GLOBAL VIEWS OF DATA AND DATA REPRESENTATIONS

The 'Story' in Brief

In this paper, we describe how *A* and *D*'s novice views slowly changed and evolved towards an expert perspective. We focus on how they started to make use of local information in different ways as stepping-stones towards the development of global points of view of data sets in different representations.

The 'story' includes descriptions of how *A* and *D* developed an understanding of:

- On what to focus: from irrelevant towards relevant information,
- How to read and make sense of local (pointwise) information in tables and in graphs,
- How to look at differences between adjacent local entries,
- How to look globally by struggling to learn the ideas underlying global perspectives, the language to describe those ideas, and the difficulties in applying both,
- How to construct global views through handling very special local entries, and
- How to flexibly transfer between local and global views through re-scaling of graphs.

The 'Story' in Detail

When the teacher introduced the whole class to the 100 meters problem situation, she asked the students to hypothesize about trends in the records in general (following the questions presented in Figure 1), without first providing them with any data. The teacher considered some sample quick responses (e.g., "*the record times improve*") as an indication that the students had enough familiarity with the context of the task in order to engage meaningfully with the data. When the introduction was over, *A* and *D* moved to working in pairs in the computer lab to work through the questions of the first worksheet (Figure 2).

On What to Focus?

First *A* and *D* faced the question "*What do you learn from this table?*" They had in front of them the computer screen displaying the whole table, a part of which is presented in Table 1. After a short discussion they agree on "*Which country has longer names?*"

TABLE 1

Part of the table of the men's 100 meters winning times in the 23 Olympiads from 1896 to 1996.

Year	City	Athlete's name	Country	Time (sec.)
1896	Athens	Thomas Burke	USA	12.0
1900	Paris	Francis Jarvis	USA	10.8
1904	St. Louis	Archie Hahn	USA	11.0
1908	London	Reginald Walker	South Africa	10.8
1912	Stockholm	Ralph Craig	USA	10.8
1920	Antwerp	Charles Paddock	USA	10.8
1924	Paris	Harold Abrahams	UK	10.6

D [Reading the question out loud] *"What do you learn from this table?" We learn that there are unusual names here.*

This first observation did not match either the intention of the curriculum, or what might have been the first relevant remark made by an expert in data handling. It is also completely unrelated to the remarks made in the opening whole class discussion regarding the improvement in record times. A further indication of their inexperience (and possibly awareness that "*unusual names*" is not a relevant observation) was *D*'s call to the teacher for help, complaining:

D *We don't learn anything special* [from this table]; *there is nothing special here!*

The initial focus on irrelevant features of the data or the inability to focus on anything at all, re-appeared (although not for long) at the beginning of the second problem situation in the SC – *Surnames* (not analyzed here). In that activity, students were asked to compare the length of surnames collected in their own class (Hebrew names) with surnames provided to them from a U.S. class (English names). The data were also provided in a table, provoking the following exchange.

A *We now have to phrase a hypothesis regarding interesting phenomena in the data*.

D *Interesting phenomena, interesting phenomena. O.K., we should find interesting phenomena. We'll find interesting phenomena.* [Reads the task again:] *Formulate a hypothesis about interesting phenomena in the length of surnames. I didn't understand what it means exactly.*

A *O.K., lets skip this* [task], *since we don't have anything interesting. We may shortly find something.*

D *I don't think we should skip this, we'll simply ask what the precise intention is. I didn't really understand: Shall we hypothesize about 'Mc's'?* [There are three surnames in the American class, beginning with the letters Mc.] *No! I don't understand*. [Laughing] *This isn't funny. I will call M* [their teacher].

Their remarks at the beginning of both the first and the second joint activity (Olympic records and surname lengths respectively) indicate that questions like "*identify interesting phenomena in the data*" may encounter an initial inability to focus attention on relevant (even informal) views of the data. *A* and *D* seemed to be unable to make full sense of the intention of the question and its formulation. In both activities, they were aware that their observations (e.g., Mc's) might not be relevant. Namely, they somehow recognized what not to focus on, but were uncertain about what may qualify as 'interesting phenomena' in this context, or how to reply to questions such as "*what do you learn from this table?"*

We propose that this finding is consistent with many other research findings (see, for example, Moschkovich et al., 1993; Magidson, 1992): novices may be either at a loss (when asked these kinds of questions), or their perceptions of what is relevant are very different from the experts' view.

In both episodes described above, *A* and *D* were able to make progress following the teacher's intervention, which they requested. In the first situation (looking at the 100 meters table), the following interaction with the teacher (T) took place.

D [Addresses the teacher for help] *What do we learn from this table? We don't learn anything special. There is nothing special here! For example, the record time here is smaller; here it's bigger*.

A *There isn't anything constant here.*

T *That means, from the table, you learn that there isn't anything constant here. Where exactly?*

A *In the Olympic record times.*

T *So, why do you say that you didn't learn anything? You did learn indeed…*

We observe that, in the presence of the teacher, they started to focus on the numerical value of the record times. They did so on the basis of discrete data points, or on the basis of individual records ("*The record time here is smaller, here it's bigger*").

The only general observation they produced at this initial stage was what seems an allusion to the irregularity in the records: "*There isn't anything constant here*". Although this was a surprising and not very clear remark, it reflects progress with respect to their first irrelevant observation (on the athletes' names). The precise meaning of this remark and its source became clearer to us when we considered it in the context of other pieces of data, as we describe later on in the analysis.

The teacher's role in this interaction has some important features of guidance provided by an expert to novices. Firstly, we can speculate that the teacher's presence may have helped them to abandon their initial observation, which they themselves sensed as irrelevant, in favor of another trial. Secondly, the legitimization of their observation by the teacher helped them to keep going. From conversations with the teacher, we learned that that was precisely what she felt at the time: she wanted them to be in the 'right ballpark', rather than leading them to 'right answers'.

The teacher knew that their preliminary observation ("*There isn't anything constant here*") was not very clear and it seemed far from the intended global perception of a generality or propensity. However, she encouraged them to see their observation as relevant and legitimate, and even strengthened them by saying "*why do you say that you didn't learn anything? You did learn indeed*."

At the beginning of *Surnames*, when *A* and *D* requested the teacher's help, the following dialog took place.

A *What is 'interesting phenomena'?*

T *Are there no interesting phenomena in the data?*

A [Cynically] *It's very interesting that this man's name is Michael.*

T *You are asked about the length of the surnames!*

The teacher's help this time consisted of calling their attention to the question asked: lengths of surnames. In fact, it helped them to re-focus and propose a hypothesis: "*In the USA, the surnames will be usually longer*".

In sum, *A* and *D* started to learn to make sense of general questions normally asked in EDA. Their learning included trying irrelevant answers, feeling an implicit sense of discomfort with them, asking for help, getting feedback, trying other answers, working on a task even with partial understanding of the overall goal, and confronting the same issues with different sets of data and in different investigational contexts. The role of the teacher included reinforcing the legitimacy of an observation, as being of the right 'kind' in spite of not being fully correct, or simply refocusing attention on the question.

We regard these initial steps in an unknown field as an aspect of the *enculturation* process (e.g., Schoenfeld, 1992; Resnick, 1988), to which we previously alluded: entering and picking up the points of view of a community or culture. In this process, we consider the teacher as an 'enculturator'.

How to Read and Make Sense of Local (Pointwise) Information in Tables and in Graphs

When asked to explain what a row in the table stands for (second question in the worksheet, Figure 2), *A* and *D* produced a straightforward orderly reading: "*A row describes when the Olympiad took place, in what place, the winning athlete's name…*". The teacher nudged them to see each row with all its details as one whole case (one Olympiad) out of the many shown in the table. She also focused their attention on the entries which were important for the curricular goal of this activity: the record time, and the year it occurred. This view of each single row, with its two most relevant pieces of information, will be reinforced immediately afterwards in the graph (next question in the worksheet, Figure 2), since the graph (as opposed to the table) displays just these two variables. Also, this understanding of pointwise information may serve later on as the basis for developing a global view, as an answer to "*how do records change over time*?"

Following the questions on time plots, *A* and *D* used the spreadsheet to create a time plot (Figure 4) of the 100 meters data, and were asked to explain what they learned from it.

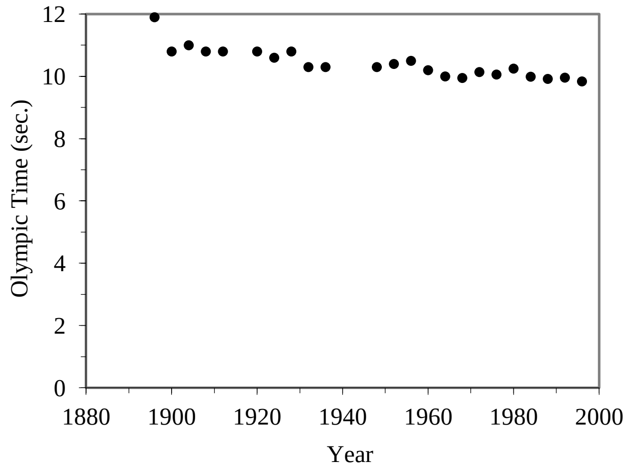


Figure 4. Time plot showing winning times for men's 100 meters.

D *But we have to remember again that the change is not at a constant rate. It doesn't increase constantly* [*D* uses his hand to signal an increasing linear line]; *it can decrease in between, and then increase again, as can be seen here in the graph.*

A [Reading the question] *What do you learn from this graph? We learn from the graph, in which year there was which running time.*

D *What running time was achieved in what year.*

A *Yes, that's what I meant.*

What *D* spontaneously explained here is related to the local - global development, and it reiterates the same comment made before about 'constancy'. We analyze it later. Here we focus on their agreed observation. Instead of looking at the graph as a way to discern patterns in the data, their response focused on how the graph shows what the table displays ("*What running time was achieved in what year*"). This comment indeed can be considered as a reply to the question they were asked: "*What do you learn from the graph?*" It focuses on the nature and language of the graph as a representation, how it displays discrete data, rather than as a tool to display a generality (propensity, trend). In fact, they did with the graph what they had done with the table (with the teacher's help): understand how each entry is represented, especially because the graph - as opposed to the table - displays only two pieces of information. Thus, from their point of view, this is a very satisfactory answer.

Reading a single entry also required learning, which in this case consisted of identifying relevant information according to the context (out of the many pieces in a single case), its tabular and graphical representations, and the ways in which information is encoded in each.

When working on connecting points (last question of the time plot section of the worksheet, Figure 2) they obtained a new graph (Figure 5).

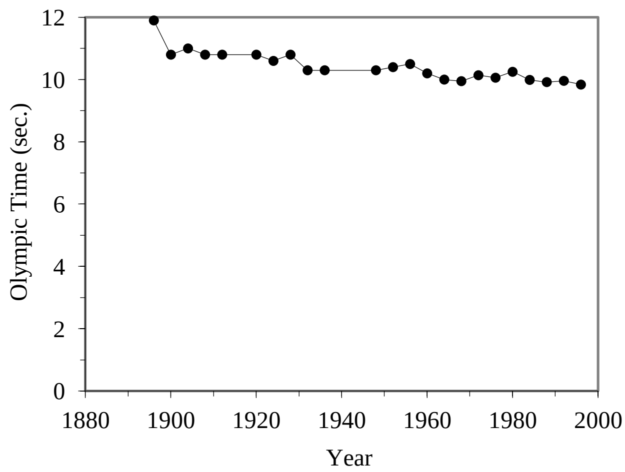


Figure 5. The 100 meters graph with connecting line.

This graph elicited many comments from *A* and *D*, who tried to make sense of what they saw. They first objected and claimed that "*Olympiads occur only once in every four years*", namely, there are no data between the points. Thus, in their view, the connecting line (as provided by the spreadsheet upon request) not only did not add any new meaning, but also contradicted their understanding of the representation in relation to the context of the data. This further supports our claim that they were much more occupied with understanding how the graph represents pointwise data, than about *patterns*. Nevertheless, we regard this comment as a quite sophisticated observation of the graph and its nature in relation to discrete data.

They were also intrigued by the fact that the connected graph included both the original points, and the connecting line.

D *O.K., You see that the points are connected by lines. Now, what's the idea? The graph did not transform to one line. It transformed to a line, in which the points are still there. It means that the line itself is not regarded as important.*

A *This line is O.K. We previously thought that if we connect the points with a line, they might disappear. But now, there is a graph, and there are also the points, which are the important part.*

What the students observed was that the line (a) did not 'swallow' the points (the points remained and did not disappear), and (b) added meaningless segments. Thus, they did not conceive of the line as an artifact that could help them to observe patterns, but as something foreign and disruptive. Paradoxically, their correct understanding of the graphical representation of discrete data was the main factor that impeded their making sense of the line as an artifact, which lacks 'representational meaning', but may support the visualization of trends.

In sum, when looking at relevant features of the graph, *A* and *D* started to make sense of pointwise observations. First, they refined their view of a local entry in the table, and then they identified single entries in the graph and reminded themselves how these entries displayed the tabular information. When invited to use the line as an artifact to support a global view, they rejected it because it lacked any meaning in terms of the pointwise view they had just learned, and with which they felt comfortable.

How to Look at Differences between Adjacent Local Entries

We now return to *A* and *D*'s observation of 'constancy' ("*There isn't anything constant here*") made at the very beginning of their joint work, when asked to describe what they learned from the 100 meters table. When they returned to the question, the following dialog took place:

D *Let's go back now and answer the first question: 'What do you learn from this table?'*

A *There are no constant differences between…*

D *We learn from this table that there are no constant differences between the record times of*…[looking for words]

A *The results of…*

D *The record times of the runners in…*

A *There are no constant differences between the runners in the different Olympiads…*

At this point, they seemed satisfied and wrote in their notebooks: "*There are no constant differences between the record times* [*A* wrote instead: *results*] *of 100 meters runners in the different Olympiads* [*A* added: *through the years*]".

This time after the teacher's reinforcement about the legitimacy of their observation, they explained more clearly what they meant by constancy. Their attention focused on differences between adjacent pairs of data entries, and they noticed that these differences are not constant. What led them to focus on the differences?

In their short-term learning history, we found a possible explanation for their observation. *A* and *D* used spreadsheets in their algebra studies (immediately before they started to learn the EDA unit), to explore patterns, to generalize, to model mathematical problems, to create and use formulae, and to draw tables and graphs. Most of the tables handled were linear correspondences between two sets of values. They were used to generating tables with the spreadsheet by 'extending' constant differences between adjacent cells, and 'dragging' a pair of cells to duplicate this constant difference to the rest of the cells in the column, which resulted in long tables with clearly defined patterns. Thus, from their point of view, and using the same exploratory learning environment in both algebra and statistics, they invoked the deterministic nature of the relationship between variables in algebra, in order to make sense of data. Thus, their first focused observation referred to what was very salient to them and a familiar part of their practices: the 'differences' between adjacent data entries as being not constant.

We claim that *A* and *D* began by focusing on a very particular local view of the data. They studied the presentation of single entries, and compared between two adjacent entries and their differences. These comparisons stemmed from their previous knowledge and experiences with a spreadsheet in algebra towards finding a formula. In other words, we may say that one of the factors, which moved them forward towards observation of patterns, was their application of previous knowledge. Thus, the general pattern they observed and were able to express was that the differences were not constant. Maybe they implicitly began to sense that the nature of these data in this new area of EDA, as opposed to algebra, is disorganized, and it is not possible to capture it in a single deterministic formula.

How to Look Globally by Struggling to Learn the Ideas Underlying Global Perspectives, the Language to Describe those Ideas, and the Difficulties in Applying Both

A and *D* seemed satisfied with their first conclusion on the 100 meters data, but when they were about to start working on the next question, the following exchange took place.

D *Let's do the next question.*

A *Hold on for a second! And we also learn that the results* [100 meters running times] *don't always decrease. Let's see for a second.*

D *They don't always increase - that's what you mean. Let's see. You have here 9.95, 10 seconds* [Points at two adjacent cells in Table 1].

A *Not always decrease.*

D *No. But the records do not always increase.*

A *Oh, Yes. Records do increase.*

D *Yes, right. The records do not always increase.*

A&D *No! Here it increases, here it decreases, here it increases, the same, the same, increases, decreases, ...* [They continue thus for a while].

A felt the need to say more about his understanding of the 100 meters data than just to mention that the differences between consecutive record times are not constant. Apparently, when the work was refocused towards describing the data, he remembered what has been said in the class during the introduction about improvements in records.

The observation that "*the results don't always decrease*" seemed to challenge the view of the general decreasing trend, by focusing on those single recorded times which increase. At this point, the 'not always increase/decrease' observation did not fully advance new global insights about the data and the existence of variability in it. However, it is a step forward towards a more global perception of the data and its disorganized nature, and it will serve as a basis for the students' next interaction with the teacher.

After they had analyzed the 100 meters data for a while, they worked on the next question: to formulate a preliminary hypothesis regarding the trends in the data. They seemed to be embarrassed by their ignorance - not knowing what trends mean, and asked for the teacher's help.

A	*What are trends? What does it mean?*
T	*What is a trend? A trend is… What's the meaning of the word trend?*
A	*Ah... Yes, among other things, and what is the meaning in the question.*
T	*O.K. Let's see: We are supposed to look at what?*
D	*At the table.*
T	*At the table. More specifically - at what?*
A	*At the records.*
T	*At the records. O.K. And now, we are asked about what we see: Does it decrease all the time?*
A&D	*No.*
T	*No. Does it increase all the time?*
A&D	*No.*
T	*No. So, what does it do after all?*
D	*It changes.*
T	*It changes. Correct.*
A	*It generally changes from Olympiad to Olympiad. Generally, not always.*
T	*Sometimes it doesn't change at all. Very nice! Still, it usually changes. And, is there an overall direction?*
D	*No!*
T	*No overall direction?*

A	*There is no overall declining direction, namely, improvement of records. But, sometimes there is deterioration…*
T	*Hold on. The overall direction is? Trend and direction are the same.*
A&D	*Increase, Increase!*
T	*The general trend is…*
D	*Improvement in records.*
T	*What is 'improvement in records'?*
A	*Decline in running times.*
T	*Yes. Decline in running times. O.K. … But…*
A	*Sometimes there are bumps, sort of steps…*
T	*… But, this means that although we have deviations from the overall direction here and there, still the overall direction is this… Fine, write it down.*

They were unfamiliar with the terminology – *trends*, and vague about the question's purpose and formulation. In response, the teacher gradually tried to nudge the students' reasoning towards global views of the data. Once the intention of the question was understood, the students, who viewed the irregularity as the most salient phenomenon in the data, were somehow bound by the saliency of local values: they remained attached to local retrogressions, which they could not overlook in favor of a general sense of direction/trend.

The teacher, who did not provide a direct answer, tried to help them in many ways. First, she devolved the question (in the sense of Brousseau, 1997, pp. 33-35 and 229-235), and when this did not work, she rephrased the question in order to refocus it: "*We are supposed to look at what?*" and "*more specifically at what?*" She then hints via direct questions: "*Does it increase all the time?*" and "*So, what does it do after all?*" In addition, she appropriated (in the sense of Moschkovich, 1989) their answers to push the conversation forward, by using their words and answers: "*It changes. Correct.*"; "*increase*"; "*decrease*"; or by subtly transforming their language, for example, from "*bumps*" to "*deviations*"; or by providing alternative language to rephrase the original question to be: "*Is there an overall direction*?"

After the above interaction, *A* and *D* wrote in their notebooks the following hypothesis: "*The overall direction is increase in the records, yet there were occasionally lower (slower) results, than the ones achieved in previous Olympiads*". At this stage, it seems that they understood (at least partially) the meaning of trend, but still stressed (less prominently than before) those local features that did not fit the pattern.

How to Construct Global Views through Handling Very Special Local Entries

In Part II - *The Same Song, with a Different Tune,* the students were asked to delete an 'outlying' point (the record of 12 sec. in the first Olympiad, 1896) from the graph (Figure 4), and describe the effect on its shape. The purpose of the curriculum was to lead students to learn how to transform the graph in order to highlight trends. First, *D* spontaneously referred to the exceptionality of 12:

D *It* [the record of 12 sec.] *is pretty exceptional, because we have here* [in the rest of the data] *a set of differences of a few hundredths, and here* [the difference is] *a whole full second.*

The 'differences' between values, first expressed when looking at the table, still played an important role in their view of data when they looked at anything in the graph. Then, they struggled to interpret the effect of the deletion on the graph.

D *The change is not really drastic ... Now, however, the graph looks much more tidy and organized.*

A *One point simply disappeared. The graph… its general shape didn't change.*

Again, the students struggle between reporting the exact description of what happened (as *A* wrote in his notebook: "*One point in the graph disappeared*") and making sense of it (as *D* wrote: "*The graph became straighter*", after saying "*the graph looks much more tidy and organized*", but also "*the change is not really drastic*"). When they were asked to justify the deletion, they struggled and said: "*There is no justification, unless one wants to make the graph look straighter*". This and other comments ("*tidy and organized*") are references to the graph as a whole with its own global features.

We claim that *D*'s focus on the unusual data point, his explanation of its exceptionality (via 'differences', which was a strong criterion for them), put him in a new position to start viewing the graph globally. In other words, focusing on an exceptional point and the effect of its deletion directed their attention to a general view of the graph. This finding seems consistent with Ainley (1995), who also describes how an outlier supported students' construction of global meanings for graphs.

How to Flexibly Transfer between Local and Global Views through re-Scaling of Graphs

The following transcript describes the students' comments on the effect of changing the vertical scales of the original 100 meters graph from 0-12 (Figure 6) to 0-40 (Figure 7) as requested in Part II - *The Same Song, with a Different Tune*.

A *Now, the change is that the whole graph stayed the same in shape, but it went down.*

D *The same in shape, but much, much lower, because the column* [the y-axis] *went up higher. Did you understand that?* [*D* uses both hands to signal the down and up movements of the graph and the y-axis respectively.]

A *Because now the 12, which is the worst record, is lower. It used to be once the highest. Therefore, the graph started from very high. But now, it* [the graph] *is already very low.*

As with the previous graphical transformation, the change of scales also focused the students' attention on the graph as a whole. They talked about the change in the overall relative position of the graph, whereas they perceived the shape itself as "*the same*". Their description included: global features of the graph ("*The whole graph ... went down*"), attempts to make sense of the change via the y-axis ("*Because the column went up higher*"), and referral to an individual salient point ("*Because now the 12, which is the worst record, is lower*"). *A* wrote the following synthesis in his notebook: "*The graph remained the same in its shape, but moved downward, because before, 12 - the worst record - was the highest number on the y-axis, but now it is lower*".

However, the purpose of the re-scaling was to enable the students to visualize the graph as a whole in a different sense. In order to take sides in the journalists' debate, the transformation was aimed at visually supporting the position that there are no significant changes in the records. Although their focus was global, for them the perceptually salient effect of the re-scaling was on relative 'location' of the whole graph rather than on its trend.

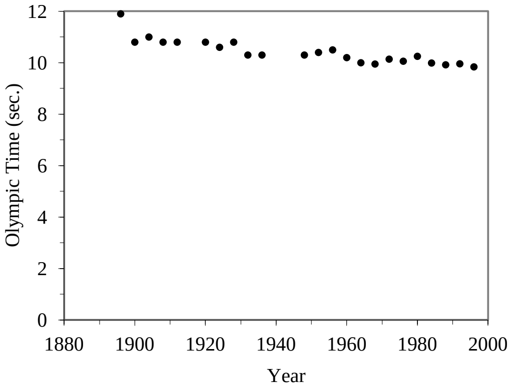


Figure 6. The original 100 meters graph.

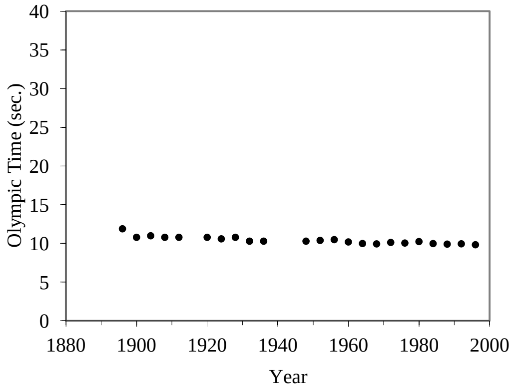


Figure 7. The 100 meters graph after the change of the y-scales.

When *A* and *D* were asked to design a graph to support the (opposite) statement: "*Over the years, the times recorded in the Olympic 100 meters improved considerably*", they did not understand the task and requested the teacher's help:

T [Referring to the 0-40 graph displayed on the computer screen - see Figure 7.] *How did you flatten the graph?*

A [Visibly surprised] *How did we flatten it?*

T *Yes, you certainly notice that you have flattened it, don't you?*

D *No. The graph was like that before. It was only higher up* [on the screen].

The teacher and the students were at cross-purposes. The teacher assumed that the students had made sense of the task in the way she expected, and that they understood the global visual effect of the scaling on the graph's shape. When she asked, "*how did you flatten the graph?*" she was reacting to what she thought was their difficulty: how to perform a scale change in order to support the claim. Thus, her hint consisted of reminding them of what they had already done (scale change). However, the students neither understood her jargon ("*flatten the graph*") nor did they regard what they had done as changing the graph's shape ("*The graph was like that before*"*)*. Although this intervention is an interesting case of miscommunication, it apparently had a catalytic effect, as reflected in the dialog, which took place immediately afterwards, after the teacher realized what might have been their problem.

T *How would you show that there were very very big improvements?*

A [Referring to the 0-40 graph - see Figure 7.] *We need to decrease it* [the maximum value of the y-axis]. *The opposite of…* [what we have previously done].

D *No. To increase it* [to raise the highest graph point, i.e., 12 sec.].

A *The graph will go further down.*

D *No. It will go further up.*

A *No. It will go further down.*

D *What you mean by increasing it, I mean - decreasing.*

A *Ahhh… Well, to decrease it… O.K., That's what I meant. Good, I understand.*

D *As a matter of fact, we make the graph shape look different, although it is actually the same graph. It will look as if it supports a specific claim.*

When the teacher rephrased her comment ("*How would you show that there were very very big improvements?*") they started to make sense of her remarks, although they were still attached to the up-down movement of the whole graph. *D* began to discern that a change of scale might change the perceptual impressions one may get from the graph. The teacher's first intervention ("*How did you flatten the graph?*"), although intended to help the students make sense of the task, can be considered unfortunate. She did not grasp the nature of their question, misjudged their position, and tried to help by reminding them of their previous actions on scale changing. The students seemed comfortable with scale changing, but their problem was that they viewed this tool as doing something different from what the curriculum intended.

The miscommunication itself, and the teacher's attempt to extricate herself from it, contributed to their progress. At first, *A* and *D* were surprised by her description of what they had done as *flattening* the graph. Then, they 'appropriated' the teacher's point of view (in the sense of Moschkovich, 1989) and started to direct their attention to the shape of the graph, rather than to its relative position on the screen. They started to focus on scaling and re-scaling in order to achieve the 'most convincing' design. Briefly stated, they transferred and elaborated, in iterative steps, ideas of changing scales, from one axis to the other until they finally arrived at a satisfying graph (Figure 8), without any further intervention from the teacher. (See, Ben-Zvi 1999 for a detailed description of this rescaling process.) *A* and *D* flexibly and interchangeably relied on pointwise observations and global considerations (both in the table and in the graph), in order to fix the optimal intervals on the axes so that the figure would look as they wished. When done, they made the following comments:

D *This way we actually achieved a result* [graph] *that appears as if there are enormous differences.*

A *To tell you the truth, this book* [the student's textbook] *is lovely.*

D *Right, it is nice!*

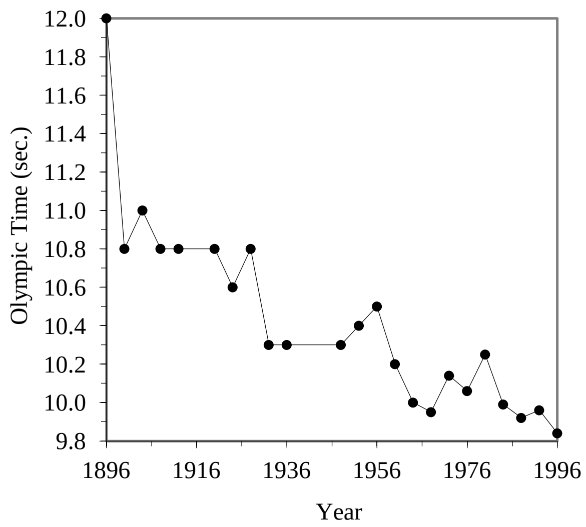


Figure 8. Graph designed to support the statement that the 100 meters times improved considerably.

In sum, at the beginning of this episode they interpreted the effect of changing scales as a movement of the graph downwards rather than an effect on its shape. Following the teacher's intervention, they started to consider how scaling of both axes affects the shape of the graph. Moreover, they were able to develop manipulations for these changes to occur in order to achieve the desired shape. In the process, they began to move between local and global views of the data in two representations.

It is interesting to notice the students' persistent invocation of 'differences' between values ("*This way we actually achieved a result that appears as if there are enormous differences*"). However, their focus here is on the way these differences are 'blown up' by the scaling effect, rather than on them not being constant, as was the case earlier when differences were invoked. We suggest that this important background of their prior knowledge has been adapted to a new use and for a new purpose. The ‘differences’, which were used to drive the way they made sense of patterns in the data, are being successfully used here as a powerful tool to evaluate their success in designing a graph to visually support a certain claim about a trend in the data.

DISCUSSION

The above ‘story’ focuses on the first steps of two students learning an EDA curriculum. It concentrates on the way they started to develop views (and tools to support them) that are consistent with those of EDA experts. Our account is offered as a contribution to understanding the process of EDA learning and the mechanisms involved therein. In our establishment of the setting and in conducting the analysis, we relied on both socio-cultural and cognitive perspectives.

The socio-cultural perspective guided us to focus on learning (of a complex domain, such as EDA) as the adoption of the point of view of a community of experts, in addition to skills and procedures. Thus, we looked at learning as an ‘enculturation’ process with two central components: students engaged in doing, investigating, discussing and making conclusions; and teachers engaged in providing role models by being representatives of the culture their students are entering through timely interventions. The cognitive perspective led us to focus on the development and change in students' conceptions and the evolution of their reasoning on the basis of what they knew.

It is not easy to tease out the two perspectives. Conceptions evolve within a purposeful context in a social setting. On the other hand, developing an expert point of view and interacting with peers or with a teacher, imply undergoing mental actions within specific tasks related to complex ideas. These actions over time are a central part of the meaningful experience within which the ‘culture’ of the field is learned. In light of the above, we propose that the analysis of our data illustrates the following ‘mechanisms’ in the process of learning EDA.

The Role of Previous Knowledge

One of the strongest visible ‘pieces’ of knowledge *A* and *D* applied and referred to repeatedly was the ‘difference’ between single pairs of data, which came from their practices in the algebra curriculum. This background knowledge played several roles. On the one hand, it provided them with the ‘differences’ lens, which conditioned most of what they were able to conclude for quite a while. On the other hand, looking at differences helped them to refocus their attention from ‘pure’ pointwise observations towards more global conclusions (the differences are not constant). Also, we suggest that looking at differences helped them in implicit and subtle ways, to start getting accustomed to a new domain, in which data do not behave in the deterministic way in which they were used to in algebra, in which regularities are captured in a single exact formula.

We stress that *A* and *D*'s focus on the ‘differences’ served more than one function in their learning. It was invoked and applied not only when they were asked to look for patterns in the data, but also in a very fruitful way when they spontaneously evaluated the results of re-scaling the graph. There, they used the ‘differences’ in order to judge the extent to which the re-scaled graph matched their goal of designing a graph to support a certain claim about trends.

Thus *A* and *D*’s previous knowledge not only conditioned what they saw, sometimes limiting them, but also on other occasions, empowered them. Moreover, their previous knowledge served new emerging purposes, as it evolved in the light of new contextual experiences.

In the data presented above, we find other instances of how previous knowledge played out. For example, *A* and *D* were adamant in their rejection of the line segments connecting the discrete data in the graph because of their knowledge of how a single data entry is represented in the ‘Cartesian-like’ representation. The points ‘in between’ have no representational meaning, since “*there were no Olympiads in between*”. In this case, their sophisticated prior knowledge did not allow them to focus on the connecting segments as a mere artifact of the new domain, used by experts to help them notice or highlight a trend, rather than to claim interpolated values which do not exist.

In conclusion, the above illustrates the multifaceted and sometimes unexpected roles prior knowledge may play, some of which may hinder progress, but others may advance it in interesting ways.

From a Local-Pointwise View towards a Flexible Combination of Local and Global Views

Learning to look globally at data can be a complex process. Studies in mathematics education show that students with a sound local understanding of certain mathematical concepts, struggle to develop global views (cf., Monk, 1988). Konold, Pollatsek & Well (1997) observed that high school students – after a yearlong statistics course – still had a tendency to focus on properties of individual cases, rather than about propensities of data sets.

In this study, *A* and *D*'s also very persistently emphasized local points and adjacent differences. Their views were related to their 'history', i.e., previous background knowledge about regularities with linear relationships in algebra. The absence of a precise regularity in a set of statistical data was their first difficulty. When they started to adopt the notion of trend (instead of the regular algebraic pattern expected) they were still attentive to the prominence of 'local deviations'. These deviations kept them from dealing more freely with global views of data. Later on, it was precisely the focus on certain pointwise observations (for example the place and deletion of one outlying point) that helped them to direct their attention to the shape of the (remaining) graph as a whole. During the scaling process, *A* and *D* looked at the graph as a whole, but rather than focusing on the trends they discussed its relative locations under different scales. Finally, when they used the scaling, and had to relate to the purpose of the question (support of claims in the journalists' debate), they seemed to begin to make better sense of trends.

It is interesting to note: (a) the local pointwise view of data sometimes restrained the students from 'seeing globally', but in other occasions it served as a basis upon which the students started to see globally; (b) in a certain context, even looking globally may indicate different meanings for the students than for an expert (position of the graph vs. trend).

Appropriation

The data show that most of the learning took place through dialogs between the students themselves and in conversations with the teacher. Of special interest to us were the teacher's interventions, at the students' request. These interventions, which though short and not necessarily directive had catalytic effects, can be characterized in general as 'negotiations of meanings' (in the sense of Yackel and Cobb, 1996). More specifically, we regard them as interesting instances of *appropriation*, as a non-symmetrical two-way process (in the sense of Moschkovich, 1989). This process takes place, in the *zone of proximal development* (Vygotsky, 1978, p. 86), when individuals (expert and novices, or teacher and students) engage in a joint activity, each with their own understanding of the task. Students take actions that are shaped by their understanding; the teacher 'appropriates' those actions – into her own framework – and provides feedback in terms of her understandings, views of relevance, and pedagogical agenda. Through the teacher's feedback, the students start to review their actions and create new understandings for what they do.

In our data, we see the teacher appropriating students' utterances with several objectives: to legitimize their directions, to redirect their attention, to encourage certain initiatives ("*why do you say that you did not learn anything*") and also implicitly to discourage others (by not referring to certain remarks). The students appropriate from the teacher a reinterpretation of the meaning of what they do. For example, they appropriate from her answers to their inquiries (e.g., what 'trend' or 'interesting phenomena' may mean), from her unexpected reactions to their request for explanation (e.g., "*How did you flatten the graph*"), and from inferring purpose from the teacher's answers to their questions (e.g., "*we are supposed to look at what?*").

Appropriation by the teacher (in order to support learning) or by the students (in order to change the sense they make of what they do) seems to be a central mechanism of 'enculturation'. As shown in this study, this mechanism is especially salient when students learn the dispositions of the subject matter rather than its skills and procedures.

INSTRUCTIONAL IMPLICATIONS

The learning processes described in this paper took place in a carefully designed environment that included:

- the curriculum built on the basis of expert views of EDA as a sequence of semi-structured (yet open) leading questions within the context of extended meaningful problem situations (Ben-Zvi & Arcavi, 1998),
- timely and non-directive interventions by the teacher as representative of the discipline in the classroom (cf., Voigt, 1995),
- computerized tools that enable students to handle complex actions (change of representations, scaling, deletions, restructuring of tables, etc.) without having to engage in too much technical work, leaving time and energy for conceptual discussions.

In learning environments of this kind, students meet and work with, from the very beginning, ideas and dispositions related to the culture of EDA (making hypotheses, summarizing data, recognizing trends, identifying interesting phenomena, and handling data representations). Skills, procedures and strategies (e.g., reading graphs and tables, rescaling) are learned as integrated in the context and at the service of the main ideas of EDA.

It can be expected that beginning students will have difficulties (of the type described) when confronting the problem situations of the curriculum. However, we propose that what *A* and *D* experienced is an integral and inevitable component of their meaningful learning process (with long lasting effects which are now being analyzed on the basis of further data). If students were to work in environments such as the above, the learning would involve the following:

1. their prior knowledge will be (and should be) engaged in interesting and surprising ways – possibly hindering progress in some instances but making the basis for construction of new knowledge in others;
2. many questions will either make little sense to them, or, alternatively, will be re-interpreted and answered in different ways than intended; and

3. their work will inevitably be based on partial understandings, which will grow and evolve.

Our study confirmed that even if students do not make more than partial sense of that with which they engage, appropriate teacher guidance, in-class discussions, peer work and interactions, and more importantly, ongoing cycles of experiences with realistic problem situations, will slowly support the building of meanings.

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Chapter 3

Statistical Reasoning and Understanding

Paper III **Statistical reasoning modes**

Paper IV **Sense making of data at the end of the *SC***

CHAPTER 3

STATISTICAL REASONING AND UNDERSTANDING

In the previous chapter I used a 'magnifying glass' approach to analyze the microevolution of one pair of students entering a new domain and 'picking up' its language, 'culture', concepts and skills. I now enlarge the scope of my view, to examine the learning that took place in the experimental classes that participated in the *SC Project*.

I first analyze students' products - 'research projects' - to study typical junior high school students' statistical reasoning modes, that is, an attempt to suggest an initial taxonomy of novices' reasoning in the domain of EDA, with an emphasis on handling data representations (Paper III).

Then, I assess students' sense making of statistics after the end of their experience with the curriculum, in a 'real' data-based investigation settings (Paper IV). The summative evaluation focus on the ways students made sense of data: handling data representations, interpreting of data and findings, and if and how they adopted the point of view of EDA experts on local-global conceptions of data.

Paper III

Statistical Reasoning Modes

Ben-Zvi, D., & Friedlander, A. (1997b). Statistical thinking in a technological environment. In J. B. Garfield & G. Burrill (eds.), *Research on the Role of Technology in Teaching and Learning Statistics*, (pp. 45-55). Voorburg, The Netherlands: International Statistical Institute.

Paper III

Statistical Reasoning Modes

This study responds to the first research question: how junior high school students choose, interpret, design, transform and use data representations in the context of open-ended EDA problem solving situations, supported by computerized tools. I describe some of the characteristic thinking processes observed in students' handling of data representations in the *SC* 'research projects'[15]. Based on the analysis of more than one hundred projects, I identified the following patterns of statistical thinking in four modes, which were evidenced in all the experimental classes:

Uncritical thinking, in which the technological power and statistical methods are used randomly or uncritically rather than 'targeted'. In this mode, students' choice of data representation is based on its extrinsic features, such as shape or color, rather than its statistical meaning. Thus, graphs are valued as aesthetic illustrations, rather than providing means to analyze data. Students also ignore the patterns suggested by their graphical representations, relate to some obvious or extreme features only, or fail to analyze their graphs altogether. Statistical methods are perceived mostly as meaningless routines, rather than useful tools, which allow one to analyze and interpret data.

Meaningful use of a representation, in which students use an appropriate graphical representation or measure, in order to answer their research questions and interpret their findings. Their choice of data representation is based on its intrinsic features, such as the type of data or the research question. In this mode, students do not tend to go back and reorganize their raw data or group it, in order to achieve a better representation and to draw further conclusions. They typically ignore numerical analysis methods, and mainly justify their inferences graphically. Although students may use many representations, these are merely a transformation in shape, rather than additional information or new interpretation.

[15] The 'research project' (described in Chapter 1) is an extended activity, in which the students act as independent and responsible learners, identify a problem and the question they wish to investigate, suggest hypotheses, design the investigation, collect and analyze data, interpret the results, draw conclusions, and submit a written and oral report.

Meaningful handling of multiple representations, in which students are involved in an ongoing search for meaning and interpretation to achieve sensible results, and in monitoring their processes. They make sensible decisions in selecting data representations and measures, can explain and justify their choice, consider their contribution to the research questions, and make corresponding changes in the data analysis. Students in this mode, reasoned and operated in a flexible and reflective manner, considered ways to verify and prove their initial hypotheses, and to convince others of their results. They manipulated their data and favored one representation to another, according to their goal. They were using multiple representations in order to add extra meaning to their research, and were moving independently back and forth in different directions in the statistical investigative cycle (Figure 1 in Chapter 1).

Creative thinking, in which students decide that an uncommon representation or method would best express their thoughts, and they manage to produce an innovative graphical representation, or self-invented measure, or method of analysis. Naturally, this type of behavior occurs less frequently.

I also consider the following aspects, which relate to the interaction between the learning environment and the students' style of work and modes of thinking.

1. The students' initial lack of experience in conducting a statistical investigation caused difficulties in gathering and tabulating data, and inefficiencies in analysis methods. As they gained experience, they overcame some of these difficulties, and were able to connect their goals to a suitable method of investigation and representation. (This issue is further discussed in Hershkowitz, et al., in press, pp. 45-46.)
2. The context in which their original research question is embedded affected the nature of students work. Some topics, data sets and research questions enable the investigators to 'take-off' to a higher mode of thought (e.g., questions about trends, association), whereas others leave their performance at a descriptive level (questions such as, 'how many…' or 'what's the best…'). The teacher has an important role in guiding students to questions and topics that may push students' statistical thinking to a higher level.

3. In the *SC* learning environment, teachers cease to be the dispensers of a daily dose of prescribed curriculum and must respond to a wide range of unpredictable events. This is particularly evident in guiding students' 'research projects'. They can play a significant role in their interactions with students by encouraging them to employ critical thinking strategies and use data representations to search for patterns and convey ideas; directing them to a potentially stimulating context; expanding and enriching the scope of their proposed work; and providing reflective feedback on their performance (see more about the teacher's role in Paper II.)

STATISTICAL THINKING IN A TECHNOLOGICAL ENVIRONMENT

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> "I learnt a lot from my teachers,
> and even more from my colleagues,
> but from my students - I learnt the
> most." (Talmud)

Abstract

We designed a statistics curriculum for junior high school using spreadsheets, based on the Pose, Collect, Analyze, and Interpret (PCAI) cycle. The curriculum contains two parallel strands: structured activities and a research project. From the analysis of student behavior, four thinking modes were identified and analyzed: uncritical thinking, meaningful use of a representation, meaningful handling of multiple representations, and creative thinking. We also considered several aspects which relate to the interaction between the learning environment described and the students' style of work and modes of thinking.

Key words: Statistical education; Cognitive research; Technology; Curriculum.

BACKGROUND

Traditional Israeli junior high school statistics usually emphasizes computation and neglects the development of a broader integrated view of statistical problem solving. Students are required to memorize isolated facts and procedures. Statistical concepts rarely originate from real problems, the learning environment is rigid, and in general, there is just one correct answer to each problem. Even when the problems are real, the activities tend to be "unreal" and relatively superficial. The only view of statistics students can get from such a curriculum is of a collection of isolated meaningless techniques, which is relatively irrelevant, dull, and routine. Many teachers ignore the compulsory statistics unit, maintaining that there is no time, or that there is pressure to include "more important" mathematical topics, and lack of interest and knowledge. We have developed a statistics curriculum (Ben-Zvi & Friedlander, 1997) in an attempt to respond to the need for more meaningful learning of statistics and have incorporated the use of available technology to assist in this endeavor.

THE POTENTIAL OF TECHNOLOGY

Technology provides the opportunity to create a whole new learning environment, in which computers can be used both as tools in problem solving, and to foster conceptual development. Thus students can take a more active role in their own learning, by asking their own questions and exploring various avenues to solve them (Lajoie, 1993; Heid, 1995). The use of computers allows students to pursue investigations, can make them less reliant on their teachers, fosters cooperation with fellow students, and can provide them with feedback on progress. We would suggest that the openness of a computerized environment can push students to become intelligent consumers as they are "forced" to choose from the available tools and options. Thus, computers have made the creation of graphs and tables equally acceptable problem solving tools, in addition to the more traditional numerical and algebraic methods.

Computers can provide a wider experience with data manipulation and representation as compared to the traditional class work (Biehler, 1993). Real situations supply large data bases, which are hard to handle without a computer and which offer many different opportunities to explore using a variety of methods. Also, students have to learn to perform in conditions of temporary or extended uncertainty, since they often cannot predict the tool's limitations. When students begin an investigation by posing a question and collecting data, they are unlikely to predict any obstacles ahead of them, for example, wrong data type, missing variables or tabulating difficulties (Hancock et al., 1992), so that one of the things they have to learn is to evaluate progress and persevere.

Whereas students in the traditional statistics curriculum were able to "plant a tree," students in the computerized curriculum are able to "plant a forest and plan for reforestration". Using software that allow students to visualize and interact with data, appears to improve students' learning of data analysis concepts (Rubin et al., 1988).

Thus, the creation of a technological learning environment should have considerable impact on the contents of the statistics curriculum, and should be accompanied by a broadening of its focus, towards an emphasis on conceptual understanding, multiple representations and their linkages, mathematical modeling, problem solving, and increased attention to real-world applications (NCTM, 1989). In the following sections we introduce the actual statistics curriculum and preliminary results of our study.

THE STATISTICS PROJECT

The statistics project, is a curriculum development and research program, which began in 1993. A statistics curriculum for junior high school (grades 7-9), was developed and implemented using an interactive computerized environment. The project has three components: (i) the development of sets of activities in statistics; (ii) implementation in classes and in teacher courses; and (iii) research on the learning processes and on the role of teacher and student within this dynamic environment.

In the first year we developed and tested materials in a few experimental classes and began in-service teacher courses. In the following two years we extended the experiment to more classes and improved the learning materials in the light of feedback. All the work has been accompanied by cognitive research.

The instructional activities aim to promote the meaningful learning of statistics through the investigation of open-ended situations, using spreadsheets. Students are encouraged to develop intuitive statistical thinking through activities in which they collect and interpret their own data. A similar approach was reported by two projects in the United Kingdom: *Statistical Investigations in the Secondary School* (Graham, 1987), and *Data Handling* for the Primary School (Green & Graham, 1994). In all three projects the core concept of the curriculum is based on the process of statistical investigation, introduced as Graham's PCAI cycle (1987) - *pose the question and produce an hypothesis*, *collect the data*, *analyze the results*, *interpret the results* (Figure 1).

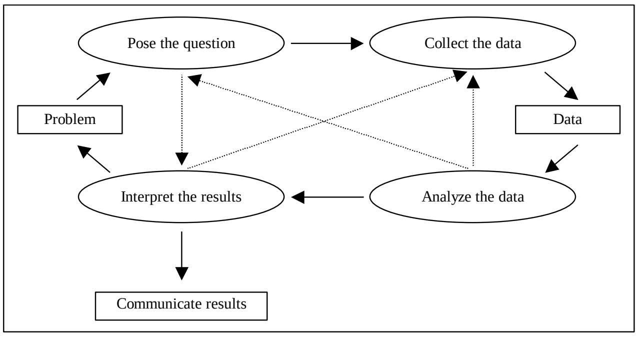

Figure 1. The PCAIC cycle for statistical investigation.
(Dotted arrows illustrate possible research paths.)

Data handling is introduced as "mathematical detective work", in which the student is expected to:

- become familiar with the problem, identify research questions and hypothesize possible outcomes;
- collect, organize, describe, and interpret data;
- construct, read, and interpret data displays;
- develop a critical attitude towards data;
- make inferences and arguments based on data handling;
- use curve fitting to predict from data;
- understand and apply measures of central tendency, variability and correlation.

Our statistics curriculum combines two chronologically parallel strands (Figure 2):

(a) concept learning through a sequence of *structured activities* (basic concepts and skills);
(b) *research project* carried out in small groups (free enterprise).

Each of these will be discussed in turn.

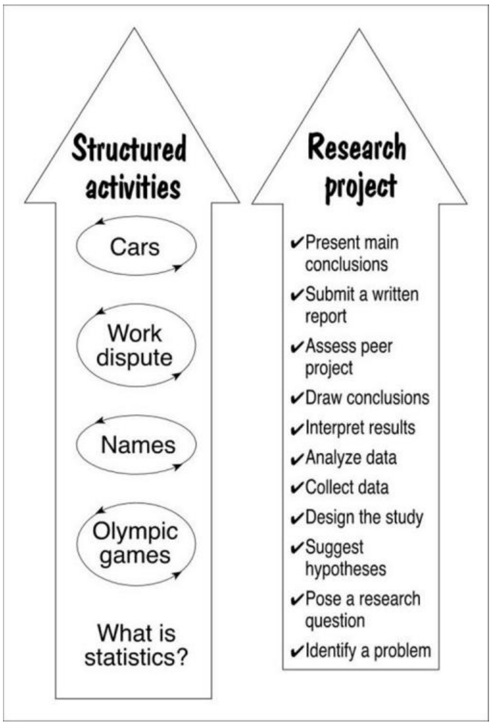

Figure 2. The two strands of the statistics curriculum.

(a) Structured activities

Each structured activity is based on an open-ended problem situation, investigated by students in a complete PCAI cycle. The problem situations focus on topics which are close to the students' world (sport, weather, people's names, salaries, cars, etc.), and present new statistical concepts and methods. The students do not always collect the data on their own; sometimes it is given to them in spreadsheet format. Statistical concepts covered include types of data, critical issues in posing questions and collecting data, statistical measures, graphical representations and their manipulation, and intuitive notions of inference and correlation.

In their first meeting students brainstorm freely on the word statistics, and are asked to complete their first small statistical investigation, based on their intuitive notions. This helps to motivate students and gives the teacher some idea of their prior knowledge. During the following investigations the students are encouraged to make their own decisions about questions to research, tools and methods of inquiry, representations, conclusions, and interpretation of results. Most learning is collaborative in small heterogeneous groups in a computer-based classroom environment. Students get assistance from fellow students as well as from the teacher. The teacher is required to create a holistic, constructivist environment (von Glasersfeld, 1984): the classroom becomes an open "statistical microworld" (Papert, 1980), in which the student is expected to become a responsible learner. When the class is involved in a structured activity, the teacher introduces the investigation theme, fosters communication among groups, raises questions to encourage thinking, guides the students through technical and conceptual difficulties and, at the end, conducts a discussion of the results. The structured activities are interspersed with more traditional class work, designed to reinforce statistical concepts.

To illustrate the structure of an activity, we describe briefly one example - the *Work Dispute* - in a printing company. The workers are in dispute with the management, who has agreed so far to a total increase in the salary bill of 10 percent. How this is to be divided among the employees is a complicated problem - and thereby hangs the dispute. The students are given the salary list of the company's one hundred employees, and an instruction booklet to guide them in their work. They are also provided with information about average and minimum salaries in Israel, Internet sites to look for data on salaries, newspaper articles about work disputes and strikes, and a reading list of background material. In the first part of the activity, students are required to take sides in the debate, and to clarify their arguments. Then, using the computer, they describe the distribution of salaries and use central tendency measures, guided by the position they have adopted in the dispute. The students learn the effects of grouping data and the different uses of averages in arguing their case. In the third part, the students suggest alterations to the salary structure without exceeding the 10 percent limit. They produce their proposal to solve the dispute, and design representations to support their position. Finally the class meets for a general debate and votes for the winning proposal. Thus, throughout this extended activity, students are immersed in complex cognitive processes: problem solving with a 'purpose' in a realistic conflict, decision making and communication.

(b) Research project

The research project is an extended activity, also performed in small groups. Students identify a problem and the question they wish to investigate, suggest hypotheses, design the study, collect and analyze data, interpret the results and draw conclusions. At the end they submit a written report and present their main conclusions and results to fellow students and parents in a "statistical happening". The teacher schedules dates for each stage, guides the students individually to scaffold their knowledge, and actively supports and assesses student progress. Some of the topics students have chosen to investigate are: *superstitions among students*, *attendance at football games*, *student ability and the use of Internet*, *students' birth month*, *formal education of students' parents and grandparents*, *road accidents in Israel*.

The structured activities supply the basic statistical concepts and skills which are applied in the research project, and allow students to get acquainted with the PCAI cycle, the computerized tools, and methods of investigation. On the other hand, the research project motivates students to become responsible for the construction of their knowledge of statistical concepts and methods of inquiry, and provides them with a sense of relevancy, enthusiasm, and ownership.

THINKING PROCESSES

During the three years of experimental implementation, we analyzed student behavior using video recording and classroom observations, student and teacher interviews, and the assessment of research projects. The main objective of this paper is to describe some of the characteristic thinking processes observed. Although our data describes all the phases of the PCAI cycle, we will concentrate on the last two stages, data analysis and interpretation of results. We present the patterns of statistical thinking in four modes, which were evidenced in all the experimental classes. We are still investigating whether or not these developmental stages are hierarchical and whether or not students go through these stages linearly.

Mode 0: Uncritical thinking

Spreadsheets are powerful user-friendly tools, which allow students to generate a wide variety of numbers, statistical measures and, more importantly, colorful and "impressive" graphs in large numbers, quickly and easily. As a result, at the initial stage, students get excited by the technological power and exercise it uncritically. Many students explore the software's capabilities, ignoring instructions or any particular order of steps. Their choice of data presentation is based on its extrinsic features, such as shape, color or symmetry, rather than its statistical meaning. Thus, in this mode, graphs are valued as aesthetic illustrations, rather than providing means to analyze data.

Students ignore the patterns suggested by their graphical representations, relate to some obvious or extreme features only, or fail to analyze their graphs altogether. Statistical methods are perceived as meaningless routines that must be performed to please the teacher, rather than useful tools which allow one to analyze and interpret data. The default options of the software are frequently accepted and employed uncritically, leading to wrong or meaningless information.

> In order to compare the population of different countries, Na. and Ne. created a bar chart. Since China was included, the default option chose a scale on the y-axis which led to a useless chart.

The ease of producing graphical representations led some students to prefer quantity over quality.

I. and R. presented 18 bar charts in a project about models and prices of cars in their neighborhood. Each chart presented information about six models. Thus, they were not able to make an overall analysis of any of their research questions.

Mode 1: Meaningful use of a representation

Choosing a representation from a variety of available options, is a critical process in statistical analysis. An appropriate representation may reveal valuable patterns and trends in the data, supply answers to questions and help justify claims.

Two of the typical features of this mode include the following.

(a) Students use an appropriate graphical representation or measure, and can explain their choice. The reasons for favoring a representation type are based on intrinsic features, such as the type of data, the method of its collection or the research questions.

(b) Students are able to perform modifications and transformations of the representation, in order to answer and justify their research questions and interpret their results. They reflect on their statistical analysis, identify and perform changes of scale, order of variables and titles, according to their needs.

In this mode, students use the statistical techniques with a sense of control, reason and direction. They are able to make changes in their graphs, but do not tend to go back and reorganize their raw data in order to achieve a better representation and to draw further conclusions. In other words, students perform well *within* the stage of data analysis (stage A in the PCAI cycle), but do not make the necessary connections with the C or I stages. We also found that students who operated in this mode, ignored numerical methods of data analysis, and mainly justified their inferences graphically. Although students may use many representations (for example, different types of graphs), these are merely a transformation in shape, rather than adding any additional information or new interpretations.

Typically, the student uses representations meaningfully with an interpretation of results, but there is minimal and poor connection between the PCAI stages.

Mode 2: Meaningful handling of multiple representations: developing metacognitive abilities

In this mode, students are involved in an ongoing search for meaning and interpretation to achieve sensible results. They make decisions in selecting graphs, consider their contribution to the research questions, and make corresponding changes in the data analysis with a variety of numerical and graphical methods. The process of making inferences and reflecting on the results obtained may lead to the formulation of new research questions. Students are able to organize and reorganize data (e.g., changing the number of categories, designing frequency tables, grouping data and analyzing sub-groups of data) in the light of results already obtained. Since students are relieved of most of the computations and graph-drawing load, their learning expands to include task management and monitoring. While working on their investigation, students reflect on the entire process, make decisions on representations and methods, and judge their contribution to the expected results (Hershkowitz & Schwarz, 1996).

G. & A. looked for patterns in the birth month of the students in the school. They hypothesized that spring is the "favorite" birth season. Initially, they produced a pie chart for each of the eight classes in the school. They tried to detect general patterns by adding up the relative frequencies for the spring months. This did not satisfy them and they decided that in order to "prove" their hypothesis, they must aggregate the data for the whole school. They first plotted a bar chart that summarized the data by months for the whole school. Since this graph did not yet yield the expected result, they reorganized their data into the four seasons and drew a second bar chart. This finally satisfied their search for a proof that the "favorite" season is spring.

A. & L. collected data on the number of years of formal education of the parents and grandparents of the students. They expected to find a "relationship" between the level of education of the two generations. They calculated the corresponding statistics (mean, mode, and median) and presented it properly in a tabular format. They noticed that the numerical analysis showed some patterns. However, they preferred to continue their investigation using graphs. They first plotted bar charts, but the picture was too detailed. Nevertheless, they gained the impression that parents studied more than the grandparents. Since the relationship between the two generations was still not clear, they plotted a three-dimensional graph (Figure 3), showing each generation on a separate plane. They deduced that "*in this graph one can clearly see that the number of years of formal education for the parents is higher than that of the grandparents*". They also observed that the two distributions show the same pattern -- i.e., that the two graphs have "*the same 'rises' and 'falls'. It seems as if parents and their children are 'glued' together -- each of them relative to their own generation*".

They then turned to a more conventional method, which had been taught in class. They produced a scatter plot, with a least squares line, which showed a very weak correlation between the two variables. This presented them with a conflict, since visually they "saw" what seemed to be a high correlation. They preferred the visual argument and so changed (distorted) their concept of correlation, claiming that "*high correlation means equality between the variables and low correlation means inequality*". They concluded that "*this graph* [the scatter plot] *reinforces our hypothesis that parents encourage their children to study more...*".

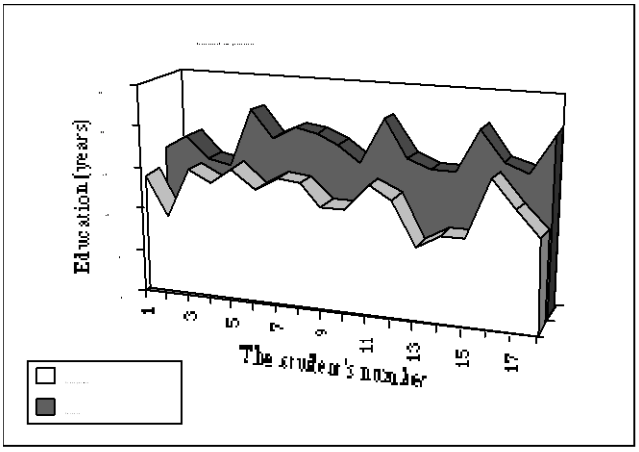

Figure 3. Graph from Number of Years of Formal Education for Parents and Grandparents -- A. & L.'s project.

In this last case, the students' statistical reasoning was incorrect. However, in both cases we observed that the students reasoned and operated in a flexible and reflective manner.

The teams considered ways to verify and prove their initial hypotheses, and to convince others of their results. They manipulated their data and favored one representation to another, according to their goal. They were using multiple representations in order to add extra meaning to their research, and were moving independently back and forth in different directions in the PCAI cycle.

Mode 3: Creative thinking

Sometimes, in their search for ways to present and justify ideas, students decide that an uncommon method would best express their thoughts, and they manage with, or without computers, to produce an innovative graphical representation or method of analysis. Naturally, this type of behavior occurs less frequently.

E., in grade seven, was investigating the frequency of road accidents in Israel over a period of several weeks, using official figures (Israel Central Bureau of Statistics, 1993). After posing a variety of research questions, he found an interesting relationship between the proportion of accidents involving different types of vehicle and their proportion of the Israeli vehicle population. He plotted, a scatter graph of these proportions (Figure 4).

E. added a diagonal line from the origin to the opposite corner, looked at the result, and claimed: "*if a type of vehicle is below the diagonal, it can be considered safer then a type that is above... One can see that the public bus company's campaign to use buses, because of their safety, is not necessarily true, since their share in the total number of accidents* (about 4%) *is almost twice their share of the country's vehicle population* (about 2%)". However, E. concluded his argument by noting that "*I need to check the mean annual distance travelled by each type of vehicle, to be* ***<u>sure</u>*** *that my conclusion is true*".

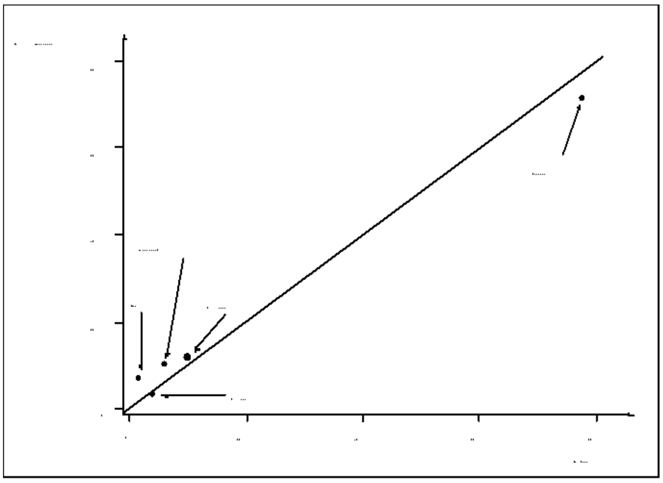

Figure 4. Car Accidents in Israel -- E.'s project. Proportion of accidents involving different types of vehicle versus their proportion of the vehicle population.

E.'s project, evidences two novel cognitive modes. the first is E.'s idea to partition the plane and his understanding of the meaning of what he had done. Second E. identified the need for more data to support a sound conclusion. The first mode reveals flexible unconventional interaction with the graphical representation, and the second reflects his statistical concept understanding.

DISCUSSION

We shall consider several aspects which relate to the interaction between the learning environment and the students' styles of work and modes of thinking described in the previous section.

Cognitive load

The use of computers shifts the students' cognitive load from drawing graphs and calculations to activities which require higher level cognitive skills. Thus, students can apply their cognitive resources to operating the software, applying statistical ideas and methods, and monitoring statistical processes. As the cognitive load increased, some students preferred to choose a familiar method rather than looking for alternative methods of analysis and interpretation. In some cases, the computer's graphical capabilities, and the ease of obtaining a wide variety of representations, diverted student attention from the goals of the investigation to some extrinsic features of the tool.

Experience in statistical investigations

The students' lack of experience in conducting a statistical investigation causes among other things, difficulties in gathering data and tabulating it in spreadsheet format (Kaput & Hancock, 1991), and inefficiencies in analysis methods. Students often failed to foresee the consequences of their strategies on the following stages of statistical problem solving. As they gained experience, they overcame some of these difficulties, and were able to connect their goals and ideas to a suitable method of investigation.

Context of investigation

The context in which the original research question is embedded also affects the nature and statistical methods of student work. For example, as reported in the TERC project (Hancock et al., 1992), deep affective involvement and preconceptions related to the context of the planned investigation, may lead some students to ignore statistical ideas and draw irrelevant conclusions. Similarly in our experience, some topics enable the investigators to "take-off" to a higher mode of thought, whereas others leave their performance at a descriptive level. If, for example, the students' question is in a descriptive format (i.e. How many...?, Who is the most...?), it may not push them to use higher cognitive modes, whereas a question about the relationship between two variables is likely to be more fertile. The teacher can play a significant role in directing students to a potentially stimulating context, and in expanding and enriching the scope of their proposed work.

Combining structured investigations and individual projects

Work on structured statistical investigations, with a given set of data and research questions, helps students to gain experience in data analysis, in the application of statistical concepts and in the process of drawing inferences. Work in parallel on individual projects allows students to experiment, restructure and apply, in a creative and open manner, the ideas and concepts learnt. This combination of the two strands stimulates students to progress in their use of statistical methods, modes of thinking and reflection. They become aware of the wide variety of investigation strategies and possible interpretations of results, and finally, they learn to communicate their ideas in written and oral reports.

Teacher actions

An immediate consequence of working in a technological learning environment is that the teacher has more time to meet students on an individual basis, thereby understanding their needs better. On the other hand, the teacher loses some control -- i.e., is unable to monitor every detail of students' actions. Teachers cease to be the dispensers of a daily dose of prescribed curriculum and must respond to a wide range of unpredictable events. In the initial stages, the teacher has an important role: students need to be encouraged to employ critical thinking strategies, to use graphs to search for patterns and convey ideas, and to become aware of the critical role of choosing an appropriate representation and analysis design. Students also need teacher guidance towards a potentially rich context and reflective feedback on their performance. As students become accustomed to the new setting, gain experience in posing more sophisticated research questions and refine their methods of work and thought, the teacher's role changes from active instructor to fellow investigator.

QUESTIONS FOR FURTHER RESEARCH

We believe that the learning environment described above and the proposed framework for thinking modes in learning statistics, may be useful to statistics educators, cognitive researchers, and curriculum developers.

We would like to suggest several further questions that we have not yet considered.

- Does the student who learns statistics in a technological environment undergo a developmental process, or are the methods of work described above, a set of discrete and not necessarily hierarchical modes?
- What are the contexts of investigation that foster higher level statistical thinking?
- How do students of various learning abilities respond to the proposed learning environment?
- What are the teacher actions that stimulate students to use meaningful multiple representations, and develop metacognitive abilities and creative thinking?

It would be interesting to hear from others whether their observations with similar materials replicate the suggested categorization of student thinking modes. In so doing perhaps some of the questions regarding development may be answered. We ourselves intend to investigate the data gathered over the last three years to answer, if only partially, some of these questions.

Acknowledgment

We would like to thank Abraham Arcavi for his helpful comments and suggestions.

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Paper IV

Sense Making of Data at the End of the *SC*

Ben-Zvi, D. (unpublished paper, thesis version). *Seventh Grade Students' Sense Making of Data and Data Representations at the End of the SC*.

Paper IV

Sense Making of Data at the End of the *SC*

This unpublished 'post experiment' study is included to diversify the body of data, to employ various research methods, and to advance the identified characterization of important phenomena related to the research questions, presented in the previous papers.

The study presents a data investigation assessment task administered to 80 seventh graders two weeks after the end of the *Statistics Curriculum* course. In the assessment task, students were asked to examine in pairs real data on the number of immigrants to Israel since the foundation of the state. The analysis concentrates on students' emerging 'expertise' in making sense of data and data representations, i.e., skills in statistical analysis, understanding and using basic statistical procedures and concepts, as well as, adopting and exercising aspects of the EDA 'culture'. In particular, I focus on the ways they exercised global views of data and their representations. The goal was also to provide a quantitative indication of the extent and scope of the phenomena identified in the case studies (Chapter 2). In the following I present the conclusions.

Students' global views of data and data representations

The results of this study show that most of the students at the end of the *SC* held several important aspects of experts' point of view on local-global approaches to data and data representations. In particular, in the context of a real and meaningful data investigation, the majority was able to formulate global research questions and hypotheses (67.5%), interpret graphs globally (87.5%), and handle cycles in data, while independently learning about 'immigration waves' (45%). Students were fluent in choosing a variety of graphs to display global features of data and their frequency distribution. Further, 46% of the students used a line in a graph as an artifact that could help them to observe and present patterns in the data.

Another important aspect of students' adoption of the EDA point of view, are their frequent manipulations of data representations (e.g., change type of graph, scaling), in order to better present their conceptions and understanding of the data. Global expressions were more frequent in the presence of graphical representations of data, and when prompted by 'guiding' (yet open) questions. Even the minority that still remained attached to local views of data (e.g., 12.5% gave local interpretations of graphs) did better in the presence of supporting prompts and data graphs.

I suggest that this relative success in adopting *global view*, that is, the tendency to notice and describe generalities in data was supported by: (a) the emphasis of the *SC* on *enculturation* processes, i.e., entering and picking up the points of view of a community or culture, through interactions with a teacher, who plays an important role as an '*enculturator*'; (b) the extensive and meaningful learning experiences in handling data within a purposeful context related to complex ideas in a social setting during the classroom activities and the 'research project'; (c) the structure of the assessment worksheet which was similar in contents and style to the textbook's worksheets (but with fewer instructions); and (d) the support provided by the computerized tool, which removed most of the computational/technical load, to allow students focus on becoming *interpreters* of data and findings (cf., Ben-Zvi, 2000; Paper VI in the thesis).

Intertwining the local and global views of data

The data show that students functioned in various levels of understanding within their local or global views, and also variably combining both views. This was evident in the multiple ways they used their views to make sense of data and their representations, in formulating research questions and hypotheses, handling and interpreting graphs, dealing with new concepts, etc.

I suggest that sophistication in students' understanding of data developed both within each point of view (local and global) and within the dynamic and flexible integration of those views. Thus, I see two trajectories of development, which may occur simultaneously: (a) '*vertical*' - growth in sophistication within a view (local and global); and '*horizontal*' - growth in the webbing of the local and global views. These ideas require further research.

Students' emergent 'expertise' in data based enquiry at the end of the SC

Students' emergent statistical knowledge and dispositions at the 'end point' made the basis for their meaningful engagement in the following:

1. making sense of questions normally asked in EDA, being able to interpret and answer them, even if in an incomplete way;
2. choosing, transforming, designing and interpreting data representations to make sense of 'real' situations;
3. interacting with statistical concepts in complex ways combining statistical and contextual perspectives;
4. intertwining flexibly local and global views on data and their representations to see multiple perspectives in real-world situations; and
5. constructing new knowledge (e.g., about cycles) on the basis of their previous knowledge.

Assessment

Many challenges exist in the assessment of outcomes of students' work in such a complex learning environment: the existence of multiple goals for students, the mishmash between the contextual (real-world) and the statistical, the role of the computer-assisted environment, the group vs. the individual work, etc. The assessment approach employed in this study consists in the use of similar settings during the 'test' to those exercised during the learning, i.e., similar design of worksheet with open, semi-structured questions, work in pairs, and use of computers. Although shown beneficial in many respects, this method still needs further investigation, in particular to find efficient ways to evaluate the knowledge of the individual within a group (in the sense of Hershkowitz, 1999), and to assess students' dispositions and points of views.

SEVENTH GRADE STUDENTS' SENSE MAKING

OF DATA AND DATA REPRESENTATIONS

AT THE END OF THE *SC*

Unpublished study (thesis version)

BY

Dani Ben-Zvi

ABSTRACT. The paper describes 7th graders' cooperative work on a real data based assessment task in a computer-assisted environment. The task was administered at the end of a carefully designed Exploratory Data Analysis (EDA) course. The purpose of the study was to assess students' sense making of data and data representations: (a) use of data analysis skills, and understanding of basic statistical procedures and concepts; (b) if and how they adopted the dispositions and points of view of certain aspects of the 'EDA culture'. Additional purpose was to provide a quantitative indication of the extent and scope of the phenomena previously identified in a microgenetic study of two 13-year-old students (Ben-Zvi and Arcavi, in press). In this study, I use the 'local-global lens' to assess students' (a) formulation of research questions and hypotheses; (b) choices, manipulation, design and use of data representations; and (c) interpretations of artifacts and observations.

KEY WORDS: Exploratory Data Analysis, learning and instruction, enculturation, local and global views of data.

INTRODUCTION

This study presents a data investigation assessment task administered to 80 seventh graders two weeks after the end of the Statistics Curriculum (*SC*), a carefully designed 10-week course on Exploratory Data Analysis - EDA (Shaughnessy, Garfield & Greer, 1996) using spreadsheets. In the assessment task, students were asked to examine in pairs real data on the number of immigrants to Israel since the foundation of the state. The analysis of students' responses is proposed to describe their learning, understandings and dispositions at the end of the *SC*, and to provide quantitative data of the extent and scope of the phenomena identified in a previous microgenetic study of two students (Ben-Zvi and Arcavi, in press).

I focus on what students can do independently (i.e., without teacher's help and the support of classroom discussions) while formulating research questions and hypotheses; choosing, manipulating, designing and using data representations; and interpreting the findings of their work. One of the key criteria I use to assess students' handling of data is their local / global observations of data, and the implications thereof.

I first list the theoretical perspectives of this study: learning as a process of *enculturation*, statistics experts' points of view, local and global views of data and data representations, the development of a global view (findings from the previous microgenetic study), and current assessment trends in statistics education. These theoretical perspectives and the *Statistics Curriculum* (*SC*) Project (Ben-Zvi & Friedlander, 1997) are presented only in brief, since they were detailed in other parts of this thesis. I then describe the *Immigration to Israel* assessment task. In the methodology section, I present the students who were the subjects of the analysis, the experiment, the data collection and the methods I used to analyze their responses. The analysis, which is the main part of this paper, focuses on quantitative data and interpretations of students' responses. On the basis of the analysis, I propose a description of students' learning of EDA from both cognitive and cultural perspectives.

THEORETICAL PERSPECTIVES

The following core ideas are the theoretical basis of this study[16].

Enculturation processes in statistics education.

Briefly stated, *enculturation* refers to the process of entering a community or a practice and picking up their points of view. The beginning student learns to participate in a certain cognitive and cultural practice, where the teacher has the important role of a mentor and mediator, or the '*enculturator*' (cf., Schoenfeld, 1992; Resnick, 1988). This is especially the case with regard to statistical thinking, with its own values and belief systems, and habits of questioning, representing, concluding and communicating (Ben-Zvi and Arcavi, in press).

Statistics experts' points of view.

Wild and Pfannkuch (1999) provide a comprehensive four-dimensional description of the processes involved in the practice of data based enquiry from problem formulation to conclusions: investigative cycles, types of thinking, interrogative cycles, and dispositions. In this study (and in the previous microgenetic study, Ben-Zvi and Arcavi, in press), I focus on one important perspective of the statistics expert's view: *a flexible and dynamic shift between local observations and global observations of data*.

Local and global views of data.

In EDA, *local* understanding of data involves focusing on an individual value (or a few of them) within a group of data (a particular entry in a table of data, a single point in a graph). *Global* understanding refers to the ability to search for, recognize, describe and explain general patterns in a set of data (e.g., change over time, trends) by 'naked eye' observation of distributions and/or by means of statistical parameters or techniques. Ben-Zvi and Arcavi (in press) suggest that EDA expertise involves both the competence *within* each view (local and global) and the competence to *flexibly shift* from one view to another.

[16] When I submit this paper to a refereed publication, I intend to complete the description of the thoretical perspectives, which are presnted here only in brief.

The development of a global view of data and their representations.

Ben-Zvi and Arcavi (in press) suggest that learning to look globally at data can be a complex process for students. That study concentrated on the development of a global view of data and their representations by *A* and *D* (seventh graders, 13-year-old boys) on the basis of their previous knowledge and different kinds of local observations.

Alternative assessment in statistics education

In light of the complex goals for statistics education, an adequate assessment of student outcomes is not possible using only traditional methods, such as, multiple choice or short answer questions. Instead, a variety of imaginative and sophisticated assessment methods is suggested, such as, open-ended questions, performance tasks, observations, conversations, journals, portfolios, projects, and media-based or 'real-life' assessment tasks (Gal and Garfield, 1997).

In light of the complex goals for students in the *SC*, I used in this study an extended *performance task* to better illuminate students' capacity to apply statistics in complex and new situation. The focus of the task was on students' cognition (skills, knowledge) and 'culture' (views, dispositions). I assessed their ability to manage meaningful, realistic situation, as *interpreters* of data and findings. To maximize the value of assessment, the task was administered in similar settings to those of the *SC*, namely, computer-assisted work in pairs on extended, open-ended, real data based investigation.

THE STATISTICS CURRICULUM

The *Statistics Curriculum* (*SC*) Project was developed in order to introduce junior high school students (grade 7, age 13) to statistical thinking and the 'art and culture' of EDA. It is described in detail in Ben-Zvi & Friedlander (1997) and in the thesis (Chapter 1).

The 'Immigration to Israel' assessment task

In the final assessment task, students were asked to interpret real immigration data (provided in a spreadsheet table, Table I), and find trends and interesting phenomena. The worksheet included minimal instructions concerning statistical methods and spreadsheet procedures, to allow students to display their knowledge and abilities.

Figure 1 shows excerpts (freely translated from the Hebrew) from the student's worksheet.

Immigration to Israel (final assessment task)

Research questions and hypotheses

In this activity you are asked to examine real data about the numbers of new immigrants to Israel since the establishment of the state (in 1948). With the help of the spreadsheet you will analyze the data in order to find trends and interesting phenomena.

- Suggest a research question concerning the immigration to Israel. Add your preliminary hypothesis regarding this question.
- The number of immigrants to Israel in 1949, one year after the founding of the state, was almost 240,000 people. The number of immigrants was about 76,000 in 1995. Based on these data, suggest an additional research question and a related hypothesis.

Open the '*Immigration*' file, which holds the data of the numbers of new immigrants to Israel during the years 1949-1995. The data was obtained from *the Statistical Abstract of Israel* (1995).

Trends

In this part you are requested to describe the trends in the immigration data.

- Use the computer to graphically display the immigration data.
- What do you learn from the graph about the immigration to Israel?
- Based on the graph, explain the meaning of the notion 'immigration waves'. Suggest a graphical method that highlights this feature.

Frequencies

The Ministry of Immigrant Absorption characterizes the number of immigrants per year as follows: 'barren' years (0-30,000 immigrants), 'regular' years (30,001 - 60,000), and 'blessed' years (more than 60,000).

- Organize the data in a frequency table which represents these groups.
- Calculate the absolute and relative frequencies of the three categories.
- Display these groups in a graph. What do you learn from it?

Synthesis

Describe briefly what you have learnt about the numbers of immigrants who arrived at Israel since its establishment.

Figure 1. Excerpts from the '*The Immigration to Israel*' assessment task.

TABLE I

The table of immigrants and population in Israel by year

Year	Immigrants during year	Population at end of year	Year	Immigrants during year	Population at end of year
1949	239,954	1,173,900	1973	54,886	3,338,200
1950	170,563	1,370,100	1974	31,981	3,421,600
1951	175,279	1,577,800	1975	20,028	3,493,200
1952	24,610	1,629,500	1976	19,754	3,575,400
1953	11,575	1,669,400	1977	21,429	3,653,200
1954	18,491	1,717,800	1978	26,394	3,737,600
1955	37,528	1,789,100	1979	37,222	3,836,200
1956	56,330	1,872,400	1980	20,428	3,921,700
1957	72,634	1,976,000	1981	12,599	3,977,700
1958	27,290	2,031,700	1982	13,723	4,063,600
1959	23,988	2,088,700	1983	16,906	4,118,600
1960	24,692	2,150,400	1984	19,981	4,199,700
1961	47,735	2,234,200	1985	10,642	4,266,200
1962	61,533	2,331,800	1986	9,505	4,331,300
1963	64,489	2,430,100	1987	12,965	4,406,500
1964	55,036	2,525,600	1988	13,034	4,476,800
1965	31,115	2,598,400	1989	24,050	4,559,600
1966	15,957	2,657,400	1990	199,516	4,821,700
1967	14,469	2,776,300	1991	176,100	5,058,800
1968	20,703	2,841,100	1992	77,057	5,195,900
1969	38,111	2,929,500	1993	76,805	5,327,600
1970	36,750	3,022,100	1994	79,844	5,471,500
1971	41,930	3,120,700	1995	76,361	5,619,000
1972	55,888	3,225,000			

METHODOLOGY

Students

The students in this study were seventh graders (13-year-old girls and boys from three classes) in a progressive experimental school in Tel-Aviv, taught by skillful and experienced teachers, who were aware of the spirit and goals of the *SC* Project. The mixed ability students were used to work collaboratively in small groups in mathematics classes.

Prior knowledge. In primary school, students had learned only about the mean and the uses of some diagrams. Thus, when they started to learn the *SC*, they had limited in-school statistical experience. However, they had some informal ideas and positive dispositions towards statistics, mostly through exposure to statistics jargon in the media. The *SC* course introduced them to basic statistical methods and concepts, statistical reasoning, and the 'art and culture' of EDA.

The experiment and the data collection

The study took place two weeks after the end of the *SC* course. The students worked on the '*Immigration to Israel*' assessment task during 90 minutes (two lesson periods) in the school's computer labs. They were told to work in pairs using a worksheet (Figure 1) and spreadsheets, and that the assessment task would not affect their final grade in statistics. Thus, a familiar learning setting was created that resembled to a regular *SC* classroom activity. At the end, the students furnished their worksheets and computer files for analysis. Additional sources of data were interviews with the teachers before and after the activity and a videotape of one pair's work.

The teachers reported that most students engaged seriously with the task, were independent in their work, and followed their instructions not to ask for teacher's help, except in a case of technical difficulty with hardware or software.

Methods

The data were systematically categorized and then quantitatively analyzed taking into account the type of explanations associated with them, and possible links with other stages of the investigative cycle. The goals of the analysis were to infer the degree of: (a) students' statistical knowledge and reasoning and their ability to manage meaningful, realistic situations, as *interpreters* of data and findings; and (b) students' statistical dispositions, in particular, their use of local / global views of data and their representations, at the end of the *SC*.

In order to evaluate students' responses, I relied on their written answers and explanations in the worksheet and on their 'products' in the computer files: graphs, tables, and numerical summaries. I first organized these sources of information by item for all 40 pairs. Then, categorization scales were carefully created, tested and verified for each item. The categorization scales were not *pre-determined*, but rather the result of an evaluation of students' responses, in order to authentically reflect students' types of reasoning and dispositions. Finally, responses were carefully graded according to these scales, summarized and presented. In cases of doubts about an interpretation of a response, it was assigned to a 'lower' level category.

In order to validate the analysis, parts of the data were presented to expert researchers for analysis, in order to check for alternative interpretations. The researchers discussed, presented, advanced and/or rejected my hypotheses, interpretations, and inferences about students' responses to reach a consensus. Additional specific methodological details will be provided in the appropriate sections below.

ANALYSIS OF DATA

In the following sections I chronologically analyze how seventh graders responded[17] to the following tasks:

1. formulating research questions and related hypotheses,
2. choosing and designing data representations,
3. interpreting data representations,
4. dealing with cycles in the data through conceptualization of 'immigration waves', and
5. handling frequency distributions and their representations.

Each section opens with the question from the student's assessment worksheet, followed by rationale and purpose. I then describe the item's specific categorization scales, identified on the basis of students' responses. Finally, I present interpretations and implications of the analysis' results.

[17] Students' quotes are freely translated from Hebrew, therefore they may not sound as authentic as in the original.

1. FORMULATING RESEARCH QUESTIONS AND HYPOTHESES

1. Suggest a research question concerning the immigration to Israel. Add your preliminary hypothesis regarding this question.
2. The number of immigrants to Israel in 1949, one year after the founding of the state, was almost 240,000 people. The number of immigrants was about 76,000 in 1995. Based on these data, suggest an additional research question and a related hypothesis.

Figure 2. First two questions of the '*Immigration to Israel'* task.

After a brief introduction of the investigation context, the students were asked to formulate research questions and hypotheses (Figure 2), without using the data set at this stage. The purpose was to assess students' ability to formulate sensible and relevant research questions and hypotheses in a given situation, based on their previous knowledge. The students were familiar with the task, since during the learning of the *SC*, the teacher encouraged them to propose research questions and hypotheses concerning a given 'real' context and discuss their proposals. As a result of those discussions, students sometimes investigated issues of their own interest, rather than those suggested by the textbook.

The presentation of two questions in the assessment worksheet (Figure 2) is purposeful. The first question is open-ended and general. The repeating question is more directive, yet open, aimed to evaluate the effectiveness of a *'focusing' technique* that we frequently used in the *SC* student's textbook. By the use of directive leading questions, we 'gently' guided students to focus on the task's meaning and target. In the above, the guidance is provided by two 'special' cases of the data set (the immigrant numbers in the first and last years).

The analysis of responses to this item identified the following hierarchical categories of research questions (and hypotheses).

- *(Statistically) irrelevant* research question - focuses on (a) irrelevant features of the data in terms of the investigation theme, e.g., "*Can the number of immigrants be controlled in order to avoid a population explosion*?"; (b) variables that do not exist in the given data set, e.g., "*What is the origin of most of the immigrants to Israel?*"; or (c) causes for an hypothesized phenomenon ('*why*' questions), e.g., "*Why the immigration rate decreases* *with time?*". Although some of these responses are sensible, the irrelevancy is considered from a statistical perspective. Thus, questions about population explosion or causes for a decreasing trend, which were natural and interesting in students' view, were categorized (statistically) irrelevant for the purpose of this analysis.

- *Local* research question - focuses on an individual value (or a few of them) within
  a group of data. For example, "*In which year was the largest immigration to Israel?*"
- *Global* research question - focuses on general patterns in a set of data, such as change over time or trends, e.g., "*What is the trend in the immigrants' numbers over the years?*"; "*Is the number of immigrants associated with the number of Israeli citizens?*"

The three categories are not fully distinct. Irrelevant responses sometimes indicate global reasoning (e.g., a causal question about trends), as well as raising contextually relevant issues (e.g., population explosion). Moreover, some of the global responses may include a combination of local and global utterances. Naturally, the *SC* course aimed at 'pushing' students towards global reasoning (intertwined with relevant local observations) and away from irrelevant and local reasoning. Table II summarizes students' responses to the first two questions of the assessment task.

TABLE II

Types of research questions (and hypotheses) formulated by students.

Type of Research Question	**Question 1 (Non-guided)**		**Question 2 (Guided)**		**Change in Response**
	N	%	N	%	%
Statistically irrelevant	9	22.5	11	27.5	+ 5.0
Local	8	20.0	2	5.0	-15.0
Global	23	57.5	27	67.5	+10.0
Total	40	100.0%	40	100.0%	

A majority (57.5% of the students) formulated *global* research questions in response to Question 1, focusing mainly on trends and association in the data. One fifth of the responses were *local*, focusing on the number of immigrants in a specific year, or on maximal yearly number. Although all the *(statistically) irrelevant* research questions (22.5%) dealt with immigration issues, they were related to variables not given in the data set, general irrelevant issues (such as, absorption and population explosion), or asking about causes for the hypothesized phenomenon.

When prompted by two data entries (number of immigrants in 1949 and 1995) in Question 2 (see Figure 2), more students (67.5% compared to 57.5%) were able to formulate *global* research questions. The *local* responses almost vanished (only 5%). The increase (by 5%) in the third category - the *(statistically) irrelevant* questions - was surprising, since I had speculated that the 'focusing' questioning technique would direct students' reasoning to focus on relevant variables and global view of data. However, a more careful analysis of theses responses provided a possible explanation.

Most of the students who formulated *(statistically) irrelevant* responses to Question 2 were global reasoners in Question 1, and focused on the *causes* for their previous global hypothesis. For example, a pair of students, who hypothesized in Question 1 that "*the number of immigrants decrease with time*", wrote in Question 2, "*Why do the numbers of immigrants decrease since the establishment of the state?*" I suggest that this pair: (a) sensed that their first global formulation 'exhausted' the key features of the data, and therefore advanced to finding causes for their hypothesis; and (b) perceived the given data in the 'focusing' question as a confirmation of their previous hypothesis, and thus felt free to speculate about causes. Although I categorized such '*why*' questions as *(statistically) irrelevant*, all of them dealt with relevant variables (number of immigrants and year) and some of them reflected global reasoning about the immigration data.

Nevertheless, almost half of the students (49%) in Question 2 were more *global*. The types of changes were: (a) focusing on the two relevant variables, i.e., number of immigrants and year (50%), (b) transferring from a non-global (irrelevant or local) to a global question (37.5%), and (c) re-wording the first global question (12.5%). Therefore, these data suggest that: (a) in the absence of data in Question 1, it was harder for students to formulate appropriate questions and hypotheses; and (b) the 'focusing' data in Question 2 supported students' reasoning towards global view of the data.

In formulating research questions and hypotheses, students used several key statistical concepts, such as trends and association. Although many responses were accurate and articulate (e.g., "*Does the data show an association between the year and the number of immigrants?*"), several types of difficulties were evident in handling basic statistical concepts. For example, one pair asked, "*How many immigrants arrive at Israel every year?*" - refereeing to the immigrants' yearly average, or possibly to trends in the data. Their hypothesis, "*Every year, a larger number of new immigrants arrive at Israel*", reveals that they intended to ask about trends. This example, as well as other 'wording' difficulties, indicates that at this stage, although students may understand key statistical concepts, their expression may be incomplete. Therefore, any evaluation of student's statistical reasoning must include cautious measures to uncover student's understandings that are not yet fully 'expressible'. The related pair of prompts (research question and hypothesis) and the 'focusing' questioning technique are evaluation tools that may assist in this direction.

Additional linguistic/conceptual confusions were associated with the dual meaning of the Hebrew word '*Aliyah'*, which denotes both 'immigration' and 'increase'. For example, Pair 3 asked, "*What is the trend of* ***increase*** ['Aliyah'] *or decrease*[18] *in the number of immigrants to Israel every year?*" and hypothesized, "*In our view, there is a trend of* ***immigration*** ['Aliyah'] *of approximately 50,000 people per year*" [emphasis added]. They seem to have asked whether there was an increasing (or decreasing) trend, but their hypothesis reveals that they were more concerned with their wild guess of the yearly average. Similar linguistic/conceptual obstacles were found in other parts of this thesis (see, for example, Ben-Zvi and Arcavi, in press). They seem to be an important component in the development of meanings in handling data, sometimes hindering progress, but in other cases supporting it.

[18] The Hebrew word '*yerida*' also has dual meaning: 'decrease' and 'emigration'.

2. CHOOSING AND DESIGNING DATA REPRESENTATIONS

Trends

In this part you are requested to describe the trends in the immigration data.

Use the computer to graphically display the immigration data.

Figure 3. The graph-plotting question.

In the next task students were asked to graphically display the trends in the immigration data (Figure 3). Producing a graph in a spreadsheet is a rather straightforward process, which consists of selecting the cells that contain the data to appear in the chart and using a 'Chart Wizard' to step through the process of choosing the chart type and the various chart options. Nevertheless, choosing the appropriate variables to display, the graph's type and its design (scales, titles, etc.) can be quite a demanding activity for students, even in a supportive, computer-assisted environment. They have to 'navigate' through many representational options and *choose* the appropriate one based on their partial statistical and contextual understanding. However, the immediate feedback provided by the computer supports students' experimentation with data representations.

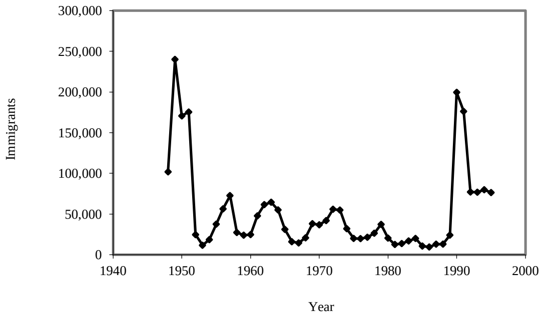
The purpose of this task was to assess what the students' graphical choices were, and whether they took advantage of the spreadsheet facilities to design graphs, and for what purposes. The graphical choice and use of that choice may reveal the extent to which students saw meaning in data and were able to formulate sensible questions, hypotheses, and interpretations.

The teachers reported in their interviews that most students were fluent in using the computer to produce graphs and did not ask for their assistance. The teachers also reported that several pairs were experimenting with different graphs, before they made their final choice. The distribution of students' graphical choices is presented in Table III. The various types of graphs used by the students are exemplified in Figures 4 and 5.

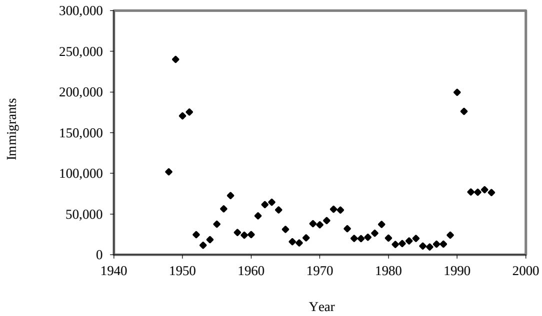
TABLE III

Types of graphs chosen to display trends in the immigration data.

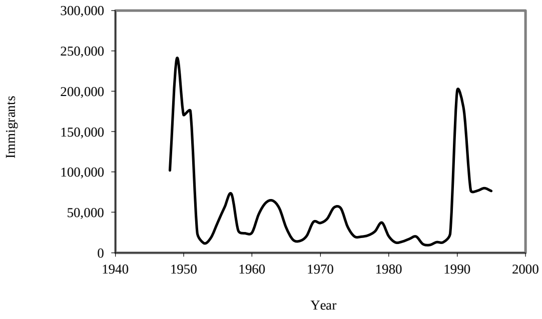
	Graph type	N	%	Total (%)
Time plot	A connected dot plot	8	22	
	A dot plot	6	16	
	A smoothed connected plot	7	19	
	A smoothed connected plot, shaded area underneath	2	5	62
Bar chart		14	38	38
Total		37[19]	100%	100%



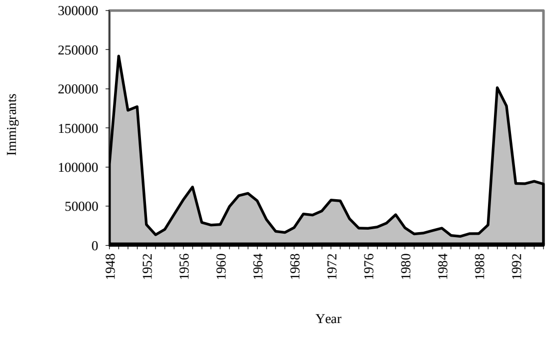
A connected symbol plot



A symbol plot



A smoothed connected plot



A smoothed connected plot with shaded area

Figure 4. Types of time plots used by students to display the immigration data.

[19] Graphs of three pairs of students were not found in their files. Thus the precentages were adjusted to the new total of 37 students.

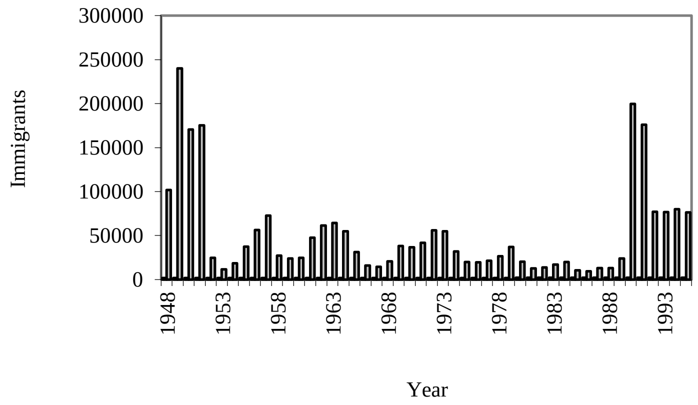


Figure 5. Bar chart used by students to display the immigration data.

The data show that all of the students were able to appropriately display the immigration data: 62% used time plots (Figure 4) and 38% used bar charts[20] (Figure 5). Although the students were encouraged in the *SC* to use a scatter plot to display time series, their choices in the assessment task were dispersed between five different kinds of graphs (Table III). Further, 46% of the students used a line plot (consists of connecting segments or a smoothed line), which may indicate that they conceived of the line as an artifact that could help them to observe and present patterns in the data. This is different from the initial phenomena identified in the case study, in which *A* and *D* did not conceive of the line as an artifact to support a global view, but as something foreign and disruptive. Thus, students seem to be fluent in choosing a variety of graphs to display global features of data.

Furthermore, one-third of them made substantial changes in the default graph provided by the computer in order to investigate trends in the data. The changes included scaling, adding a regression line, and using colors to emphasize 'blessed' immigration periods. Thus, students chose to transform between many different representatives (Schwarz & Dreyfus, 1995) of the same statistical object (i.e., different graphs of the same data). I consider these actions as an indication of students' '*graphicacy'*, that is, skillful and flexible manipulation of graphical representations of data, and awareness of their role in conveying ideas by the visual impression they may induce.

[20] Although a bar chart displays the data properly (since the intervals on the x axis are equal), the more adequate graph in this case is a time plot, which is used by experts to display change over time and highlights features such as trends, rates of change, fluctuations, and cycles.

In the following section I further explore the meanings of students' choices and design actions by looking at their interpretations of those actions.

3. INTERPRETING DATA REPRESENTATIONS

Trends *[continued]*

What do you learn from the graph about the immigration to Israel?

Figure 6. The graph interpretation question.

In the next task (Figure 6), the students were asked to interpret the graph they had previously formed, which was displayed on the computer screen in front of them. The purpose of this task was to investigate the ways they understood graphical data representations in the context of the immigration data enquiry, and to assess what were their views on data in making those interpretations.

The analysis identified that students' interpretations of data representations were generally of two major types - *local* and *global*, similar to the split between local and global research questions that was observed above (in the analysis of the first task). In EDA, *local interpretation* involves focusing on an individual value (or a few of them) within a data representation (e.g., a single point in a graph). *Global interpretation* involves focusing on general patterns in a data representation (e.g., change over time, trends). EDA experts are able to flexibly shift between local and global interpretations, and intertwine both perspectives to search for, recognize, describe and explain general patterns in a set of data.

The data show that the local-global dichotomy represents only a partial description of students' interpretations. Within each category (local, global) there was a broad range of interpretation levels. Most notably, students' *global* interpretations were divided into the following non-hierarchical categories.

- *Three-period interpretation* - noticing, grouping and describing three immigration periods: 'high, 'low' and then 'high' again. For example, "*There were few periods in which the immigration thrived: the foundation of the state and 1990 onwards. In the rest - there was a stable, average immigration.*"

- *Four-period interpretation* - noticing, grouping and describing four immigration periods in the data: ‘high’, ‘low’, ‘high’, and then ‘moderate’. For example, “*In the late forties, there was a mass immigration. Since the fifties there was a significant decrease; and in the beginning of the nineties there was again a large immigration, which later became moderate*”.
- *Focus on variability* - interpretation that emphasizes the existence of variability in the data, without indicating any specific pattern in them, e.g., “*There are many increases and decreases in the number of immigrants during the years*”; or “*The number of immigrants is not constant nor have any trends, but is rather variable*”.
- *Cyclic interpretation* - focuses on the cycles in the data. This category is discussed in the analysis of the ‘*immigration waves*’ task (Section 4 below).
- *General patterns interpretation* - noticing and describing general patterns (trends or association), for example, “*There is no association between the year and the number of immigrants. However, except of several outliers - it is a decreasing line*”.

The data show that all the students understood the intention of the question and its formulation, and answered it in a sensible way. The distribution of interpretations is presented in Table IV. The major finding here is that most of them (87.5%) were able to interpret the graph in a *global* way, attending to general (rather than local) features of the data. These global interpretations were at different level of sophistication focusing on various groupings (three or four subintervals), the existence of variability, cycles, trends or association in the data. In particular, the data show that students at the end of the *SC*: (a) provided a wealth of global interpretations; (b) exhibited different levels of global reasoning (such as, *“the number of immigrants is not constant”* or identifying and describing trends and cycles); and (c) combined local and global views of data (as illustrated below).

TABLE IV

Types of graph's interpretations.

Type of Interpretation		**N**	**%**	**Total**
Global	Three periods	16	40.0	
	Four periods	4	10.0	
	Variability	3	7.5	
	Cyclic	4	10.0	
	General patterns (trends, association)	8	20.0	87.5
Local		5	12.5	12.5
Total		40	100%	100%

These results are different than our previous observations (Ben-Zvi and Arcavi, in press) on students' interpretations at the beginning of the *SC*. Then, their remarks indicated that questions like "*what do you learn from this graph?*" may encounter an initial inability to focus attention on relevant (even informal) views of the data representation. Novices seem to be either at a loss (when asked this kind of questions), or their perceptions of what is relevant are very different from the experts' view. Their initial relevant views of data representations were mostly local, and only later slowly changed and evolved towards an experts' perspective that includes global points of view of data sets in different representations.

It is also notable that the rate of global interpretations in this task is higher than the rate of global research questions and hypotheses observed in the first task. It could be related to the fact that students were acting in the presence of a data graph, which supports the construction of global meanings, while the first task was accomplished without any data at hand.

The co-constructive interactions between local and global views of data are illustrated in *A* and *D*'s sophisticated interpretation of their graph:

> "*The number of immigrants per year is not constant. During the first years of the state of Israel - the number of immigrants was relatively high. Then, there was a long period in which the number of immigrants increased and*

decreased - but did not arrive at the number of immigrants, which were in the first years of the state - and ranged between 50,000 and 70,000. Later on - in recent years, the number of immigrants soared, then became moderated - but still was higher than the previous years, but not as high as the first years of the state."

This description consists of a mix of semi-local and various types of global observations, in a 'narrative' and 'graphical' style. They first focus on the variability in the data ("*the number of immigrants per year is not constant*"), then divide the data to four subintervals and characterize each one of them, emphasize variability in the data (e.g., "*there was a long period in which the number of immigrants increased and decreased*"), provide a specific range ("*between 50,000 and 70,000*"), and compare between periods (e.g., "*but still* [the immigration in recent years] *was higher than the previous years, but not as high as the first years of the state*").

Finally, in the graph interpretation task, only five responses (12.5%) were *local*. Like global reasoning, local reasoning was also non-monolithic. It varied between 'pointwise' observations - focusing on single value (arbitrary or 'special', such as, maximum, outlier), e.g., "*The number of immigrants in 1986 was the smallest*"; and focusing on a subinterval (few individual values with a common characteristic) within the data set, e.g., "*The largest number of immigrants arrived between 1948 to 1951, about 240,000 immigrants* [the approximate number of immigrants in 1949]".

4. DEALING WITH CYCLES ('IMMIGRATION WAVES')

Based on the graph, explain the meaning of the notion '*immigration waves*'. Suggest a graphical method that highlights this feature.

Figure 7. The 'immigration waves' question.

In the previous tasks, students worked with statistical tools and concepts with which they were used in the *SC* (such as, tables, graphs, trends, variation), in order to make sense of data. The assessment worksheet also introduced a new idea - '*immigration waves*'. The purpose was to assess how students were able to handle it. The students were asked to explain what 'immigration waves' might mean, and to suggest a graphical method to highlight the phenomenon. On the computer screen, they had the graph from the previous task (a time plot in most cases).

Before I turn to analyze students' responses, I draw the reader's attention to the distinct *cycles* in the immigration data during the years 1952-1989. The cyclic phenomenon is evident in Figure 8, which is a smoothed (by a three point moving average) and properly scaled graph of the data. In this interval, the cycles are non-regular, and can be described as an average seven-year cycle, with decreasing amplitude. Therefore, this may give rise to students' statistical meaning making, even though they did not deal with such phenomenon in the curriculum.

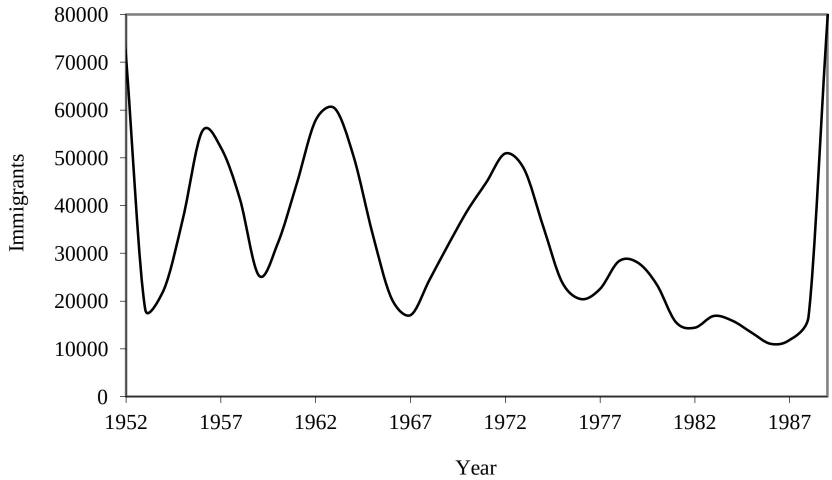


Figure 8. The immigration data displayed in a smoothed graph emphasizing cycles.

Students' responses are presented in Table V. I consider the first response, which described in one way or another a succession of influxes of immigrants, or successive ups and downs (27.5% of the students), as an indication to students' understanding of the global essence of the circulatory feature of the data. For example, "*Immigration waves indicate that there are downs and ups in the immigration to the state - a periodical phenomenon, for example, 1954-56 - an increase, 1956-58 - a decrease*". The second response (17.5%) is similar in nature, but focuses only on one complete cycle of an immigration influx. These two types of self-explanations (45% in total) seem to indicate a *global* understanding of the cycle phenomenon, i.e., the ability to recognize and describe cycles in a set of data on the basis of the variability in the data. By attending to how a collection of values periodically change over time, these students dealt with features not inherent to individual elements, but to the aggregate that they comprise.

TABLE V

Students' types of explanation of 'immigration waves'

#	Type	Interpretation of 'Immigration Waves'	N	%	Total
1	Global	A *succession* of influxes of people immigrating, or successive ups and downs	11	27.5	
2		The number of immigrants swells and dies away (a *ridge*) or vice versa	7	17.5	45.0
3	Local	A sudden rapid *increase* in the number of immigrants	12	30.0	
4		Marked *change* in the number of immigrants	4	10.0	40.0
5		Unclear or incorrect answer	4	10.0	
6		Did not answer the question	2	5.0	15.0
		Total	40	100.0%	100.0%

Other responses focused differently on the variation in the data set. In the third (30%) and fourth (10%) types of responses, students explained that 'immigration waves' refer to a sudden rapid increase in the number of immigrants, like an ocean wave, or a marked change in the number of immigrants, without stating explicitly the direction of the change. These responses may indicate a partial understanding of the cycles, and a greater focus on local data. In sum, 45% of the students made sense of 'immigration waves' by using *global* explanations, and additional 40% generated incomplete but sensible *local* interpretations.

Representations of cycles. Students were also asked to suggest a graphical method to highlight the 'immigration waves' phenomenon. There were two types of suggestions: (a) changes of the current type of graph, mostly to line plot, offered by 46% of the pairs; and (b) changes within the graph (54%). The latter kind included changes of scales, coloring parts of the graph, grouping of data, and connecting points by lines. One example is displayed in Figure 9, in which the graph was changed from a scatter plot (a dot plot) to a smoothed line plot, displaying only a partial interval (1952-1989) on the x axis, and 'minimized' vertical scales (0-80,000), in order to visually emphasize the cycles. I consider such purposeful actions as indications of students' ability to make sense of, use and manipulate data representations.

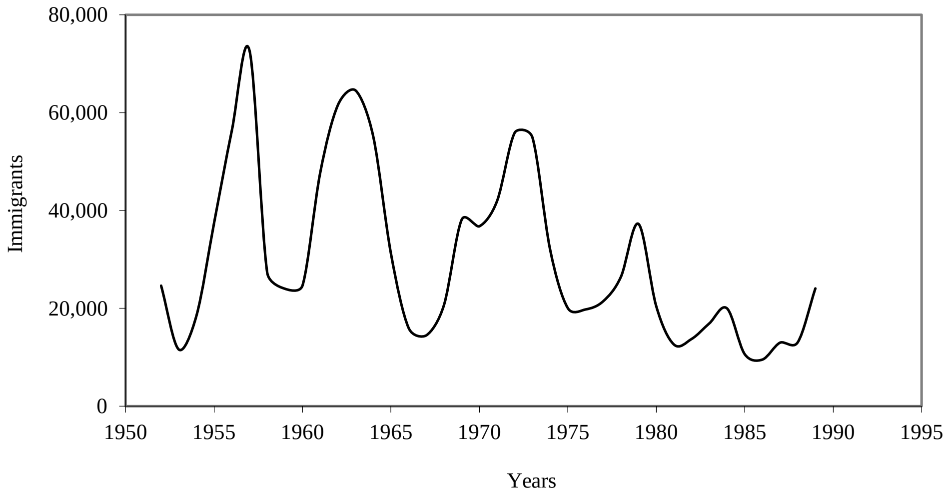


Figure 9. A smoothed and properly scaled line graph emphasizing the 'immigration waves'.

5. HANDLING FREQUENCY DISTRIBUTIONS

Frequencies

The Ministry of Immigrant Absorption characterizes the number of immigrants per year as follows: 'barren' years (0-30,000 immigrants), 'regular' years (30,001 - 60,000), and 'blessed' years (more than 60,000).

- Organize the data in a frequency table which represents these groups.
- Calculate the absolute and relative frequencies of the three categories.
- Display these groups in a graph. What do you learn from it?

Figure 10. The frequency question.

In this task, students were required to handle a frequency distribution of the immigrants data, based on a given grouping to three unequal subintervals (Figure 10). Students were asked first to organize the data in a frequency table, by dividing it to the given subintervals, and calculate the corresponding absolute and relative frequencies. Students were also asked to display the frequency distributions in a graph and answer the interpretation question, "*what do you learn from this graph?*" Even though these questions were familiar to students, they required rather sophisticated statistical reasoning on frequencies and relative frequencies, and spreadsheets skills.

Most of the students (36 pairs, 90%) answered the first part of the task[21]. All of them managed to create a correct frequency table, group the data to equal subintervals of 30,000 immigrants each, and calculate absolute frequencies[22]. Most of the pairs (94%) succeeded in calculating correctly relative frequencies, a task which required them to write a spreadsheet formula, copy it to corresponding cells in the frequency table, and choose the kind of fractions' inscription. Table VI presents the distribution of the inscriptions of relative frequencies chosen by the students.

TABLE VI

Inscriptions of relative frequencies.

Type of Fraction Inscription	**N**	**%**
Percentages	16	47
A decimal fraction	13	38
A simple fraction	5	15
Total	34	100%

Students' responses show a high rate of success in handling frequencies in a variety of inscriptions. Only two pairs had errors in calculating the relative frequencies, and their spreadsheets formulas indicate they misunderstood the concept. On the other hand, only three pairs used the default inscription provided by the spreadsheet (a decimal fraction with eight digits), while the rest (91%) chose to transform the fraction to a meaningful inscription by annexing the number of decimal places, or by changing to percentages or simple fractions. Students also used the computer to calculate the sums of the frequency columns and handled appropriately rounding issues (e.g., changed 99.9% to 100.0% or 48/48 to 1). These intentional actions indicate that students not only knew how to use the computer, but also were able to make sense, judge and transform their computerized numerical artifacts, while having inscriptional personal preferences.

[21] Four pairs did not get to the frequency task and did not finish the assessment worksheet. Responses' requencies were adjusted accordingly in this section.

[22] Dividing to unequal sub-intervals, as required by the task (0-30,000, 30,000-60,000, and 60,000-240,000) is not a straightforward spreadsheets procedure, which was unfamiliar to the students. Only three pairs managed to overcome the *technical* difficulty.

Representation of frequency distributions. Students were later asked to display the frequency distributions in a graph and interpret it. This task occurred towards the end of the activity and therefore only 24 pairs (60%) responded to it. Students' choices of graphical representations are presented in Table VII.

TABLE VII

Students' displays of frequency distribution of the immigration data.

Type of Representation	**N**	**%**
Pie chart	11	46
Bar chart	7	29
Histogram	4	17
Line graph	2	8
Total	24	100%

The data show that students were fluent in using a number of appropriate data representations. The most typical graph chosen was a pie chart, which is a common way to display relative frequency of univariate distributions. In addition, students' written interpretations of their graphs indicate understanding of key features of the frequency distribution, such as, "*Almost half of the years were 'barren', exactly a quarter were 'regular', and a bit more than a quarter were 'blessed'* ".

I suggest that several factors contributed to the abovementioned results: (a) students' prior knowledge, namely, previous and simultaneous experiences with factors and percentages in their algebra lessons; (b) extensive experience with frequencies during the *SC* investigations and the 'research project'; and (c) the situating of the 'numbers' in a meaningful and familiar context.

DISCUSSION

My account is offered as a contribution to understanding the processes of EDA learning by beginners in junior high school. I analyzed students' responses to a 'real' data analysis assessment task, which was administered after the end of a 10-week EDA course (the *SC*). The analysis concentrated on students' emerging 'expertise' in making sense of data and data representations, i.e., skills in statistical analysis, understanding and using basic statistical procedures and concepts, as well as, adopting and exercising aspects of the EDA 'culture'. In particular, I focused on the ways they exercised global views of data and their representations. The goal was also to provide a quantitative indication of the extent and scope of the phenomena identified in a previous study (Ben-Zvi and Arcavi, in press). In the following I discuss the data presented above.

Global views of data and data representations

Current studies in mathematics education show that students with a sound *local* understanding of certain mathematical concepts, struggle to develop *global* views (cf., Monk, 1988). In statistics education, Konold, Pollatsek & Well (1997) observed that high school students - after a yearlong statistics course - still had a tendency to focus on properties of individual cases, rather than about *propensities* of data sets.

My observations at the beginning stage of the *SC* (Ben-Zvi and Arcavi, in press), in which students were novices in terms of their views of data, show that they persistently emphasized local views of data in tables and graphs. They were attentive to the prominence of 'local deviations', which kept them from dealing more freely with global views of data. Only later, the focus on certain pointwise observations, the gradual adoption of the notion of *trend*, and the exercise of scaling, helped them to direct their attention to the shape of the graph as a whole, taking into account the variability in the data. Thus, they slowly began to develop understanding of global view of data and their representations, and later were able to flexibly integrate local and global views in data analysis.

The results of this study show that most of the students at the end of the *SC* held several important aspects of experts' point of view on local-global approaches to data and data representations. In particular, in the context of a real and meaningful data investigation, the majority was able to formulate global research questions and hypotheses (67.5%), interpret graphs globally (87.5%), and handle cycles in data, while independently learning about 'immigration waves' (45%). Students were fluent in choosing a variety of graphs to display global features of data and their frequency distribution. Further, 46% of the students used a line in a graph as an artifact that could help them to observe and present patterns in the data.

Another important aspect of students' adoption of the EDA point of view, are their frequent manipulations of data representations (e.g., change type of graph, scaling), in order to better present their conceptions and understanding of the data. Global expressions were more frequent in the presence of graphical representations of data, and when prompted by 'guiding' (yet open) questions. Even the minority that still remained attached to local views of data (e.g., 12.5% gave local interpretations of graphs) did better in the presence of supporting prompts and data graphs.

I suggest that this relative success in adopting *global view*, that is, the tendency to notice and describe generalities in data was supported by:

(a) the emphasis of the *SC* on *enculturation* processes, i.e., entering and picking up the points of view of a community or culture, through interactions with a teacher, who plays an important role as an '*enculturator*';

(b) the extensive and meaningful learning experiences in handling data within a purposeful context related to complex ideas in a social setting during the classroom activities and the 'research project';

(c) the structure of the assessment worksheet which was similar in contents and style to the textbook's worksheets (but with fewer instructions); and

(d) the support provided by the computerized tool, which removed most of the computational/technical load, to allow students focus on becoming *interpreters* of data and findings (cf., Ben-Zvi, 2000; Paper VI in the thesis).

Intertwining the local and global views of data

The above description may create a false impression that local and global views of data exist in sharp dichotomy, and that students' understanding mainly develops from the 'less sophisticated' local view to the 'more sophisticated' global view. However, this is *not* what this study tells us. The data show that students functioned in various levels of understanding within their local or global views, and also variably combining both views. This was evident in the multiple ways they used their views to make sense of data and their representations, in formulating research questions and hypotheses, handling and interpreting graphs, dealing with new concepts, etc.

I suggest that sophistication in students' understanding of data developed both within each point of view (local and global) and within the dynamic and flexible integration of those views. Thus, I see two trajectories of development, which may occur simultaneously: '*vertical*' - growth in sophistication within a view (local and global); and '*horizontal*' - growth in the webbing of the local and global views. These ideas require further research.

Students' emergent 'expertise' in data based enquiry at the end of the SC

Students' emergent statistical knowledge and dispositions at the 'end point' made the basis for their meaningful engagement in the following:

1. making sense of questions normally asked in EDA, being able to interpret and answer them, even if in an incomplete way;
2. choosing, transforming, designing and interpreting data representations to make sense of 'real' situations;
3. interacting with statistical concepts in complex ways combining statistical and contextual perspectives;
4. intertwining flexibly local and global views on data and their representations to see multiple perspectives in real-world situations; and
5. constructing new knowledge (e.g., about cycles) on the basis of their previous knowledge.

Assessment

Many challenges exist in the assessment of outcomes of students' work in such a complex learning environment: the existence of multiple goals for students, the mishmash between the contextual (real-world) and the statistical, the role of the computer-assisted environment, the group vs. the individual work, etc. (Gal & Garfield, 1997; Garfield, 1993). The assessment employed in this study consists in the use of a performance task in similar settings to those exercised during the learning, i.e., similar design of worksheet with open, semi-structured questions, work in pairs, and use of computers. Although shown beneficial in many respects, this method still needs further investigation, in particular to find efficient ways to evaluate the knowledge of the individual within a group (in the sense of Hershkowitz, 1999), and to assess dispositions and points of views.

Attending to the many issues raised here, the challenges remain overwhelming. Supporting students' becoming familiar not only with statistical techniques, concepts and tools but also with the many nuances, considerations and points of view involved in generating, describing, analyzing and interpreting data and in reporting finings, is one of the greatest challenges in statistics education.

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Chapter 4

Curriculum and Technology

for the Learning of EDA

Paper V **Learning arenas**

Paper VI **The impact of technology**

CHAPTER 4

CURRICULUM AND TECHNOLOGY FOR THE LEARNING OF EDA

Any response to the research questions of this thesis cannot be accomplished in a 'vacuum'. A carefully designed and systematically researched learning environment - imbued with a genuine spirit of EDA - is required. In using the *SC* approach, standards, materials and tools as a model, I experimented with one kind of complex and 'rich' learning environment. Thus, although curriculum-based research is not the main focus of this thesis, some issues concerning the interrelationships between curriculum design, research, technology and learning were however included in its scope. I consider in this chapter what the *SC* has to offer in order to promote changes in the way statistics is now being taught and learned in junior high schools. More specifically, my purpose was to study the following themes.

In the first paper of this chapter - 'learning arenas' (Paper V) - I analyze the interrelationships between curriculum design and research on learning to characterize the nature of the instructional activities. The purpose is to study the type of contexts and the nature of tasks that may support students' meaningful statistics learning, and understand how we can be more aware of their actions, in order to design tasks which offer them opportunities to engage seriously, work and reflect, and advance their statistical knowledge.

In the second paper of this chapter (Paper VI), I study the impact of computerized tools, such as spreadsheets, in enhancing learning and understanding statistics. My assumption is that although software is just a medium (and not the message), its integration, especially when well chosen to fit the course, could introduce deep changes. My aim is thus to study how technological tools, that have the potential to support conceptualization of statistical objects and actions, may help students to gradually become experienced '*data explorers*'.

Paper V

Learning Arenas

Ben-Zvi, D., & Arcavi, A. (1998). Towards a characterization and understanding of students' learning in an interactive statistics environment. In L. Pereira-Mendoza (ed.), *Proceedings of the Fifth International Conference on Teaching of Statistics, II*, (pp. 647-653). Voorburg, The Netherlands: International Statistical Institute.

Paper V

Learning Arenas

In this paper, I take a close look at the interrelationships between curriculum and learning. I observed junior high school students as they engaged in two kinds of action: *taking a stand* in a debate on the basis of data analysis and *designing data representations* to emphasize a point they want to make. The purpose of the analysis is to advance the understanding of (a) how students learn to handle data and data representations in the *SC* educational environment, and (b) how can we be more aware of student actions, in order to design situations in which students engage seriously, work and reflect, and advance their knowledge. The analysis also takes into account the contribution of student interactions (within-pair interactions, group interactions) to their understanding and learning of data representations.

The first example presented in the paper is the '*Work Dispute*' activity (described in Chapter 1). The task is based on a familiar context and provides realistic, manageable and meaningful data, which were 'engineered' to provide points of departure for raising some key statistical concepts, and most importantly - the students are asked to *take sides* in a conflict situation. I observed that central to students' actions (e.g., handling data, choosing statistical measures, creating data displays and arguing) and motives was the stand they had taken. The selection of the stand, if done thoughtfully, has an important value of its own, in that it transforms the pure learning situation into a meaningful position. Concept learning thus takes place in context, using and applying the concepts and through exchange of ideas among peers. Putting themselves in the position of one side in a dispute, motivated the students to find a 'winning' strategy, namely, to have their proposal approved by the majority. Thus, their explanations for choosing representative measures and graphical data displays emerged from their stand in the dispute. Taking a stand also made students check their methods, arguments and conclusions with extreme care. They felt it natural to face criticism and counter-arguments made by peers and teacher, and to answer them. Sometimes, when the results of their work were not in line with their stand, they were forced to persevere and search for more congenial statistical evidence and proof.

These observations suggest that students' actions and interactions were partially due to the design of the problem situation, which includes *taking a stand*. The students showed themselves able to understand and judge the complexities of the situation, engaged in preparing a proposal which in their view was acceptable, rational and just, and were able to defend it.

In the second example - *'The Same Song, with a Different Tune'* activity (described in detail in Paper I) - students go beyond reading, interpreting and understanding data representations - they are encouraged to *design (or re-design) representations*. The task directed students' attention to the problem of scaling in the design and interpretation of a representation. Thus, they developed 'scale sensitivity', became critical of the false impressions that may be induced. This task provides an opportunity for students to represent ideas creatively in various ways, and also understand some meta-representational issues (in the spirit of diSessa, Hammer, Sherin & Kolpakowski, 1991).

In sum, the examples described in this paper illustrate how curriculum design can take into account both: (a) new trends in subject matter (EDA), its needs, values, and tools; and (b) student learning and interactions. By staging and encouraging students - e.g., to *take sides* and *design* (and *re-design*) data representations, we pushed them towards levels of discussion, which I have not observed in most traditional statistics classroom. I suggest that we can usefully characterize *taking sides* and *designing representations* as *learning arenas* and propose a definition of the term. *Learning arenas* are problem situations based on careful research design, aimed at promoting and supporting certain student actions, which encourage meaningful learning of statistics.

TOWARDS A CHARACTERIZATION AND UNDERSTANDING OF STUDENTS' LEARNING IN AN INTERACTIVE STATISTICS ENVIRONMENT

DANI BEN-ZVI and ABRAHAM ARCAVI

ABSTRACT. We shall describe episodes of middle school students working on Exploratory Data Analysis (EDA) developed within an innovative curriculum. We outline the program and its rationale, analyze the design of the tasks, present extracts from students' activities and speculate about their learning processes. Finally, from our observations, we propose a new construct -- *learning arena*, which is suggested as a curriculum design principle, which may also facilitate research.

KEY WORDS: Exploratory Data Analysis, learning and instruction, curriculum development.

THE LEARNING ENVIRONMENT

The teaching of statistics is undergoing major changes (Shaughnessy et al., 1996; Moore, 1997), mostly due to: (a) a shift in content at the introductory stage from descriptive statistics to EDA -- i.e., emphasis on organization, description, representations, and the analysis of data, with a considerable use of visual displays and technology; (b) new pedagogy which abandons an "information transfer" model in favor of a *constructivist* view of learning (Davis et al., 1990; Garfield, 1995) -- i.e., students construct their own knowledge by combining present learning experiences with existing conceptions; and (c) the incorporation of new technologies, which facilitate the handling, change, and use of various data representations, and provide tools for making sense of data (Biehler, 1993).

With these perspectives in mind, we initiated a middle school statistics project, which includes curriculum development (Ben-Zvi & Friedlander, 1997a), classroom trials, teacher courses, research on learning processes and the teacher's role in an investigative technological environment (Ben-Zvi & Friedlander, 1997b). The project is part of an innovative program developed at the Weizmann Institute (Hershkowitz & Schwarz, 1997).

We chose to emphasize *statistical content* and *concepts*: for example, types of data, posing questions and collecting data, statistical measures, handling and interpretation of graphical representations, and intuitive notions of inference and correlation. We also emphasize the elements of *statistical thinking*; e.g. the need for data, proof and certainty in statistics, the existence of variability, and the logic underlying processes and methods.

The form of *instruction* for the course was based on the following main principles: (a) the use of extended *real* (or realistic) problem situations; (b) an emphasis on a community of learners communicating with each other; and (c) a new role for the teacher as "a guide on the side" (Hawkins et al., 1992).

The curriculum content and pedagogy is based on the PCAIC cycle (*pose*, *collect*, *analyze*, *interpret, and communicate*) proposed by Graham (1987) and Kader & Perry (1994), using a spreadsheet. Students work on two parallel strands - (1) classroom activities and (2) a research project.

(1) The *classroom activities* are semi-structured statistical investigations, in which considerations and processes involved are also pursued. The students are encouraged to hypothesize about possible outcomes, choose tools and methods of inquiry, representations, conclusions, and interpretation of results. Most learning is designed to be done in pairs.

(2) The *research project* is an extended activity, in which the students act as independent and responsible learners. Students identify a problem and the question they wish to investigate, suggest hypotheses, design the study, collect and analyze data, interpret the results and draw conclusions. At the end they submit a written report and present their main results to fellow students and parents in a "statistical happening".

During the experimental implementation, the learning materials were field-tested, presented in teachers courses, and then published. In order to study the effects of the new curriculum, we analyzed student behavior using video recordings, classroom observations, interviews, and the assessment of students' notebooks and research projects.

In the following, we "observe" pairs of students, as they engage in two kinds of action: *taking a stand* in a debate on the basis of data analysis, and *designing* (or re-designing) a representation to emphasize a point they want to make. The purpose of the analysis (presented briefly below) is to advance our understanding of (1) how students learn in such an environment, and (2) how can we be more aware of student actions, in order to design "better" tasks. (By "better" we mean, situations in which students engage seriously, work and reflect, and advance their knowledge.)

STUDENTS TAKING A STAND

One activity was the *'Work dispute'* in a printing company: the workers are in dispute with the management, who has agreed to a total increase in the salary bill of 10 percent. How this is to be divided among the employees is a problem - and thereby hangs the dispute. The students are given the salary list of the one hundred employees, and an instruction booklet to guide them in their work. They are also provided with information about the national average and minimum salaries, Internet sites to look for data on salaries, newspaper articles about work disputes and strikes, and a reading list of background material. In the first part of the activity, students are required to take sides in the dispute, and to clarify their arguments. Then, using the computer, they have to describe the distribution of salaries and use measures (e.g. median, mean, mode, and range) guided by their position in the dispute. The students learn the effects of grouping data and the different uses of statistical measures in arguing their case. In the third part, the students suggest alterations to the salary structure without exceeding the 10 percent limit. They produce their proposal to solve the dispute, and design representations to support their position and refute opposing arguments. Finally the class meets for a general debate and votes for the winning proposal. The time spent on the full activity is about seven class periods.

This task context is familiar to students with interesting, realistic and meaningful data (Cobb & Moore, 1997). The data was "engineered" so that the salary list is manageable, and provides points of departure for raising some key concepts. For example, the various central tendency measures are different from each other, to allow students to choose a representative measure to argue their case. We also arranged that the mean salary (5000 IS) is above the real national averages (4350 IS - all employees, 4500 IS - printers only).

Communication of statistical ideas seems to us an essential part of student experiences. We expect students to clarify their thoughts, learn to listen to each other, and try to make sense of each other's ideas. But, most importantly we ask students to *take sides* in the conflict situation. Their actions, e.g. handling data, choosing statistics, creating displays and arguing are all motivated, guided, and targeted by the stand they have taken. Their actions may also cause them to change their original stand, etc. The selection of the stand, if done thoughtfully, has an important value of its own, in that it transforms the pure learning situation into a meaningful position.

Concept learning thus takes place in context, using and applying the concepts and through exchange of ideas among peers. This obviously includes exercises of the "traditional" textbook type, but also much more, as described above.

To illustrate the use of concepts and arguments that the task promoted, we bring the following short transcript from a video recording of one of the experimental classes. It records a group of students who chose to take the side of the workers. After clarifying their arguments, they described the distribution of the current salaries, guided by their position in the dispute. The student pairs prepared various suggested alterations to the salary structure to favor workers (as opposed to management), and then held a series of meetings with fellow student pairs (about 10 students in all), in which they discussed proposals, and designed graphical representations to support their position, and prepared themselves for the general debate.

The following transcript is taken from the second "workers' meeting".

Id. *OK, we have this pie* [chart] *and we plan to use it* [See Figure 1]. *Everybody agrees?*

Students *Yes, yes.*

Id. *Let's see what should we say here? Actually we see that... 60 percent of...*

It. *60 percent of the workers are under the average wage* [4500 IS]. *Now, by adding 12 percent... there are far fewer* [workers under the national average].

S. *OK, but I have a proposal, that brings almost everybody above the average wage. If we add 1000 shekel to the 49 workers, who are under the average...*

Id. *It's impossible. Can't you understand that?*

S. *This* [my proposal] *will leave us with 1000 shekel, that can be divided among the other workers, who are over* [the average].

It. *Then each of them will get exactly five shekel!...*

M. *But we don't have any chance to win this way.*

Id. *What is the matter with you? We'll have a revolt in our own ranks. Do you want that to happen at the final debate?*

S. *Anyway, this is my opinion! If there are no better proposals...*

Id. *Of course there are: a rise of 12 percent on each salary* [excluding the managers]...

Sh. *OK. Show me by how much will your proposal reduce the 60 percent.*

D. *I am printing now an amazing proposal - everybody will be above the* [national] *average: NO WORKER WILL BE UNDER THE AVERAGE WAGE! This needs a considerable cut in the managers' salaries...*

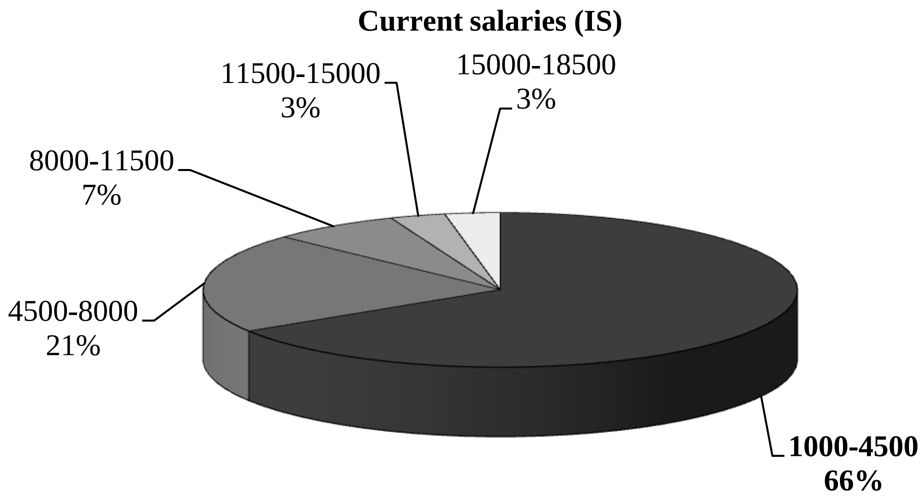

Figure 1. The “workers’” description of the current salary distribution.

In this exchange, three different proposals for the alteration of the salary structure were presented. The first, offered by It. and Id., suggested an increase of 12% for all workers. The managers' salaries remain unchanged. The second proposal, originated by S., suggested an equal (1000 IS) increase for each of the 49 workers earning less than the national average (4350 IS), the small remainder to be divided among the other workers. Again the managers' salaries remain unchanged. The third proposal, presented by D., suggested a considerable cut in managers' salaries, and an increase for all workers under the national average, to bring them above the average.

We observe that central to students’ actions and motives is the stand to be taken by the workers. Putting themselves in the position of workers in a dispute against the managers, motivated the students to find a “winning” strategy, namely to have their proposal approved by the majority. For example, Figure 1 above, is grouped to emphasize the large proportion of salaries below the printers’ national average. Moreover, predicting that the management will boast about the relatively high company average (5000 IS), the workers chose to emphasize the relatively low median and mode. Thus, the workers' explanations for choosing representative measures and graphical displays, emerged from their stand in the dispute.

Taking a stand also made students check their methods, arguments and conclusions with extreme care. They felt it natural to face criticism and counter-arguments made by peers and teacher, and to answer them. Sometimes, when the results of their work were not in line with their stand, they were forced to persevere and search for more congenial evidence and proof. Finally, after much refining they formulated the following proposal.

> *To divide most of the money among the workers, because they currently earn very low salaries; i.e., an increase of 13.35% for workers under the printers' average salary; an increase of 8.85% for the remaining workers. The managers salaries to be increased by 1000, 900, 800, ..., 400 IS, inversely related to the size of their present income. The small remainder to be contributed to orphanage homes in town.*

They also prepared the following arguments for the final debate.

> *If we remove five managers from the salary list, the company average decreases dramatically below the national average. We shall display the significant improvement in the workers' salaries, made by our proposal, using a pie chart* [Figure 2] *and statistical measures* [median and mode]. *We wish to create a "balance" in the company. No longer will 66% of the employees earn below the national average. Most of the employees will earn a salary close to the company average.*

These observations suggest that students' actions and interactions were partially due to the design of the problem situation, which includes *taking a stand*. They

a) dealt with a complex situation and the relevant statistical concepts (averages, percentages, charts, etc.);

b) used critical arguments to confront conflicting alternatives;

c) used statistical procedures and concepts with a purpose and within a context, to solve problems, relying heavily on visual representations and computer;

d) demonstrated involvement, interest, enthusiasm, and motivation in their learning;

e) were able to create their own products (proposals and their representations).

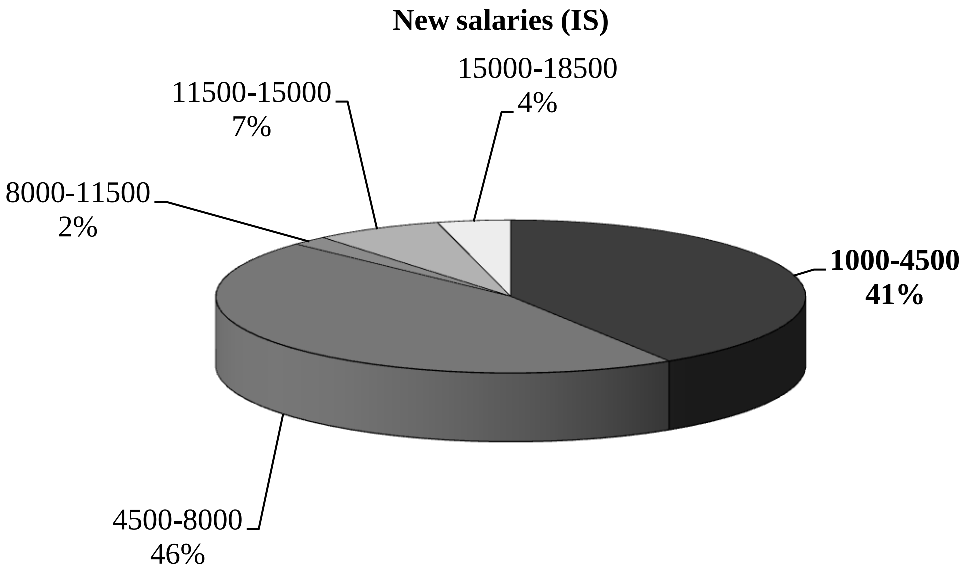

Figure 2. The workers' final proposal for the new salaries.

We turn now to a second brief example from another activity.

STUDENTS AS DESIGNERS (OR RE-DESIGNERS)

The activity - *"The same song, with a different tune*" - occurs early in the curriculum. The problem is presented to students in the following way.

> Two sports journalists argue about the record times in the 100 meters. One of them claims that there seems to be no limit to human ability to improve the record. The other argues that sometime there will be a record, which will never be broken. To support their positions, both journalists use graphs.

One task of this investigation asks students to design a representation, using a computer, to support different statements, such as:

(a) During the years, the times recorded in the Olympic 100 meters improved considerably.

(b) Throughout the years, the changes in the Olympic times for the 100 meters were insignificant.

(c) Between 1948 and 1956, the times in the 100 meters worsened considerably.

This task context is familiar to students with interesting and meaningful data, providing opportunities to search for patterns, centers, variations, and gaps in the data. Students go beyond reading, interpreting and understanding data representations - they are encouraged to *design* (or *re-design*) *representations* (Harel, 1991).

The task directed students attention to the problem of scaling in the design and interpretation of a representation. Thus, they developed "scale sensitivity", became critical of the false impressions that may be induced (see, for example, the graphs in Figures 3-6).

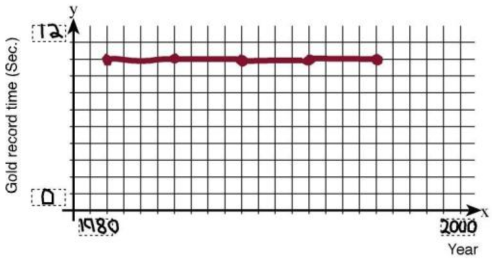

Figure 3. Design to support statement (b) above.

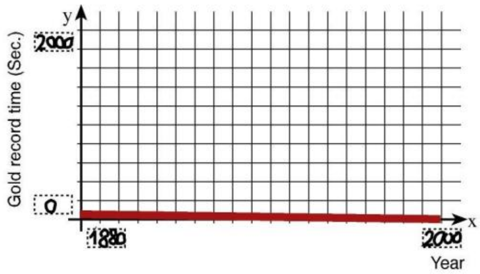

Figure 4. Design to support statement (b) above.

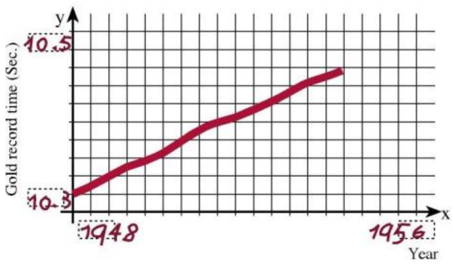

Figure 5. Design to support statement (c) above.

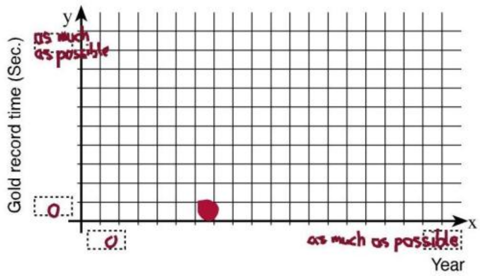

Figure 6. Design to support statement (b) above.

This task provides an opportunity for students to represent ideas creatively in various ways, and also understand some meta-representational issues (diSessa et al., 1991).

DISCUSSION

The examples we have described illustrate how curriculum design can take into account *both*: (1) new trends in subject matter (EDA), its needs, values, and tools; and (2) student learning. By staging and encouraging students - e.g., to *take sides* and *design* (and *re-design*), we pushed them towards levels of discussion, which we have not observed in the traditional statistics classroom. They showed themselves able to understand and judge the complexities of the situation, engaged in preparing a proposal which in their view was acceptable, rational and just, and were able to defend it. The second example involved more than just being able to read and interpret, it focused attention on the crucial role of scales in creating a visual impression for good or ill.

We suggest that we can usefully characterize *taking sides* and *designing* as *learning, arenas* and propose a definition of the term. *Learning arenas are problem situations based on careful research design, aimed at promoting and supporting certain student actions, which encourage meaningful learning of statistics (mathematics).*

We propose that *learning arenas* can be a helpful guide in the development of tasks for student learning. We intend to pursue this work further in order to identify, develop, and research more arenas similar to those above. We hope that the refinement of this idea, and the accumulation of examples, will contribute towards a theory of principled development of learning materials.

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Paper VI

The Impact of Technology

Ben-Zvi, D. (2000). Towards understanding the role of technological tools in statistical learning. *Mathematical Thinking and Learning, 2*(1&2), 127-155.

Paper VI

The Impact of Technology

In the *SC* learning environment, computerized tools are brought into play to assist students in their explorations of the new subject matter. Since most traditional assumptions about teaching and learning are challenged by the use of technology in statistics education, we are forced to think deeply about all aspects of our educational and research work. I described above (Chapter 1) how the choice of the tool played an important role in the preliminary stages of the *SC* development. This study focuses on the practical and theoretical impact of introducing technological tools in statistics education. Drawing on observations in the experimental classes and on theoretical perspectives, this paper discusses the role of technological tools in statistics teaching and learning.

Technological tools especially designed for statistics learning have been developed to support:

1. Students' active knowledge construction, by ‘doing’ statistics (e.g., creating, interpreting and transforming data and data representations) and ‘seeing’ statistics (e.g., using simulations).
2. Opportunities for students to reflect upon observed phenomena.
3. The development of students' metacognitive capabilities, i.e., knowledge about their own thought processes, self-regulation, and control.
4. The renewal of statistics instruction and curriculum on the basis of strong synergies between content, pedagogy and technology (in the spirit of Moore, 1997).

I emphasize in this paper the use of technology as a powerful tool to mediate and support students’ construction of meanings for statistical conceptions and perspectives, rather than as a learning goal in itself. A detailed example of an activity from the *SC* is introduced to illustrate that an attempt at serious integration of computers in teaching and learning statistics brings about a cascade of changes in curriculum materials, classroom praxis, students and teacher's interactions, and students' ways of learning.

The computer changed my assumptions about what can be learned, and, as a result, facilitated the redesigning of the *SC*. Using the computer to investigate data, students acted directly on statistical objects and relations through various linked representations. They produced, interpreted, transformed, and used data representations, and discussed the meanings of their actions and motives. The computer's ability to operate (plot, calculate, sort, etc.) quickly and accurately, dynamically link multiple representations, simplify procedures, provide immediate feedback to the user, and transform a whole representation into a manipulable object, contributed to the students' sense-making processes.

A theoretical discussion follows which underpins the impact of technological tools on teaching and learning statistics, by emphasizing how the computer lends itself to supporting cognitive and socio-cultural processes. This can take the form of amplification and reorganization (Pea, 1987; Dörfler, 1993) and of experiencing a new 'mathematical realism', namely, "*the experience of direct manipulation of mathematical objects and relations*" (Balacheff and Kaput, 1996, p. 470). A powerful tool brings about the reorganization of physical or mental work in at least the following ways: shifting the activity to a higher cognitive level, changing the objects of the activity, focusing the activity on transforming and analyzing representations, supporting a situated cognition mode of thinking and problem solving, accessing statistical conceptions by the use of graphics, constructing meaning of conceptions by the use of the 'representative ambiguity' (see Schwarz and Hershkowitz, in press).

Subsequently, I present a review of educational technologies, which represents the sorts of software that have typically been used in statistics instruction: statistical packages (tools), microworlds, tutorials, resources (including Internet resources), and teacher's meta-tools. This paper ends with a discussion of implications and recommendations for the use of computers in the statistical educational milieu.

Towards Understanding of the Role of Technological Tools

in Statistical Learning

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The purpose of computing is insight, not numbers.

(Hamming, 1977)

Abstract

The paper begins with some context setting on new views of statistics and statistical education. These views are reflected, in particular, in the introduction of Exploratory Data Analysis (EDA) into the statistics curriculum. Then a detailed example of EDA learning activity in middle school is introduced, which makes use of the power of the spreadsheet power to mediate students' construction of meanings for statistical conceptions. Through this example, I endeavour to illustrate that an attempt at serious integration of computers in teaching and learning statistics brings about a cascade of changes in curriculum materials, classroom praxis, and students' ways of learning. A theoretical discussion follows which underpins the impact of technological tools on teaching and learning statistics by emphasizing how the computer lends itself to supporting cognitive and socio-cultural processes. Subsequently, I present a sample of educational technologies, which represents the sorts of software that have typically been used in statistics instruction: statistical packages (tools), microworlds, tutorials, resources (including Internet resources), and teacher's meta-tools. Finally, certain implications and recommendations for the use of computers in the statistical educational milieu are suggested.

Introduction

The practice and research of statistics have been revolutionized by the introduction of modern computing technology. Computers have greatly increased the efficiency and adequacy of practical statisticians' work, and contributed to the range, sophistication and complexity of theoretical statisticians' research (Biehler, 1993). By contrast, the field of educational technology in statistics is relatively young. Only some twenty years have elapsed since personal computers became available to homes and schools. Since then, the rate of technological changes has been amazingly fast. Developments that push the edge of technology may seem outdated immediately after they are introduced.

At first, the leaders of the educational community were carried away by euphoric enthusiasm and voiced prophecies of universal panacea. A slow but steady disenchantment followed, with a message to explore carefully the methods of application and research and the criteria regarding what is important to investigate (Hawkins, Jolliffe, & Glickman, 1992; Kaput, 1992). The statistical educational community was forced to re-examine what statistics should be included in the school curriculum, what technology is appropriate for educational purposes, and what type of research initiatives are needed.

Recently, a more realistic approach to the potential of technological tools (which, in this paper, I take to include computers (systems of hardware and educational software), graphical calculators, the Internet, and interactive multimedia systems) in statistics education has been implemented. Successful experiences are beginning to accumulate, and a better understanding of the role of technology is beginning to emerge. Thus, an abundance of available resources has begun to yield exemplary statistical software, innovative curricular materials that make use of technological tools, and research on the design, use, and impact of them. For example, the last International Association for Statistical Education Roundtable Conference, held in Granada in 1996, focused on the role of technology in teaching and learning statistics and helped identify important issues and needed areas of research (Garfield & Burrill, 1997).

Pedagogically, technological tools especially designed for statistics learning have been developed around the world to support:

1. Students' active knowledge construction, by "doing" and "seeing" statistics.
2. Opportunities for students to reflect upon observed phenomena.
3. The development of students' metacognitive capabilities, i.e., knowledge about their own thought processes, self-regulation, and control.
4. The renewal of statistics instruction and curriculum on the basis of strong synergies between content, pedagogy and technology (Moore, 1997).

Educational technologies allow one to deal with the quantitative complexity inherent to statistics, easily access methods of data analysis, and explore the roles played by chance and probability models. However, more empirical wisdom, blended with theoretical reflection is required. Computers have changed the practice of statistics, but have they changed the practice of teaching statistics? What impact do they really have on the teachability and learnability of statistics, both new and old? What has research shown us about the role of technology and what do we still need to know? The overall goal of this paper is to further the understanding of the role and impact of technological tools in statistics education.

CURRENT TRENDS IN STATISTICS EDUCATION

The Change in Statistics

On the verge of a new millennium, statistics is more pervasive than ever. We live in a society which is ever more dependent on information and technology. Major political, social, economic and scientific decisions are made on the basis of data. Politicians resort to more data-based arguments, often reaching different conclusions from the same data. Statistical reports affecting virtually all aspects of our lives appear regularly in all the news media. Accordingly, statistical literacy is becoming a major goal of the school curriculum, regardless of the professional future of the student (Gal, in press). Statistical thinking offers simple but non-intuitive mental tools for trimming the mass of information, ordering the disorder, separating sense from nonsense, and selecting the relevant few from the irrelevant many.

New views of the disciplines of statistics and statistical education have gradually emerged. Today's leading statistical educators see statistics and mathematics and statistical and mathematical reasoning as quite distinct (Garfield and Gal, 1999). Moore (1998) expanded the view of statistics by suggesting that it should be considered one of the liberal arts. Thinking of statistics as a liberal art balances its essential technical expertise with its flexible and broadly applicable mode of thinking and reasoning about data, variation, and chance. Similar to statistical thinking, the liberal arts, especially in its philosophical tradition, encourage skeptical, analytical thinking, unconstrained by *a priori* standards, and bearing in mind that any conclusions are subject to continuing challenge. The liberal arts image emphasizes that statistics involves distinctive and powerful ways of thinking:

> Statistics is a general intellectual method that applies wherever data, variation, and chance appear. It is a fundamental method because data, variation, and chance are omnipresent in modern life. It is an independent discipline with its own core ideas rather than, for example, a branch of mathematics. (Moore 1998, p. 1254).

The Change in Statistics Education

Classroom practice has been influenced by the new approaches to statistics, the accumulating educational research knowledge that has gradually moved away from mathematics alone back to data and interdisciplinary work (Garfield, 1995), and the changes in technology that have forced statistics educators to focus on what is not automated. These driving forces have led to a shift in introductory courses from traditional views of teaching statistics as a mathematical topic (with an emphasis on computations, formulas, and procedures) to the current emphasis on statistical reasoning, and the ability to interpret, evaluate, and flexibly apply statistical ideas. Thus, teaching and learning statistics should give more attention to big ideas and general strategies for dealing with data, variation, and chance (G. Cobb, 1992). Current recommendations for first course in statistics include at least the following:

1. Incorporate more data and concepts and treat recipes and formal derivations as secondary in importance.
2. Wherever possible, automate computations and graphics by relying on technological tools.
3. Foster active learning, through various alternatives to lecturing.
4. Encourage a broader range of attitudes, including appreciation of the power of statistical processes, chance, randomness, and investigative rigor, and a propensity to become a critical evaluator of statistical claims.
5. Use alternative assessment methods (Gal & Garfield, 1997) to better understand and document student learning.

The Introduction of EDA

One significant manifestation of these ideas and the increasing use of new technological tools has been the incorporation of Exploratory Data Analysis (EDA) (Biehler, 1982, 1993; Shaughnessy, Garfield, & Greer, 1996; Tukey, 1977) into the statistics curriculum. EDA is the discipline of organizing, describing, representing, and analyzing data, with a heavy reliance on visual displays as analytical tools and, in many cases, on technology. In addition to inference, its goal is to make sense of, and gain insight into, data, analogous to the exploration of unknown lands (G. Cobb & Moore, 1997).

EDA studies patterns, centers, clusters, gaps, spreads, and variations in data, and its essence can be captured by the slogans - look at the data (preliminary analysis), look between the data (comparisons), look beyond the data (informal inference) and look behind the data (context) (Curcio, 1989; Shaughnessy et al., 1996). Pedagogically, EDA is an opportunity for open-ended data exploration by students, aided by basic concepts of statistics. EDA is often portrayed as an iterative and interactive process in which the explorer moves, from a question, to collecting and looking at pertinent data, to reformulating and refining the question, to looking at more data, and so on (Graham, 1987; Konold, 1995b). Intuitively, technological tools seem to be ideal in these processes, by empowering students to do data analysis that is interactive and exploratory, using visualization and simulations to understand statistical concepts and methods (Biehler, 1993). In the following section, I introduce two typical EDA activities in a computer-based environment, blended with theoretical reflections on the role of the technological tools.

AN EXEMPLARY, STATISTICAL, COMPUTER-BASED ENVIRONMENT

The purpose of presenting the following example is to show that an attempt at a serious integration of technological tools in teaching and learning statistics brings about a cascade of major changes in curriculum and students' learning. The example is adopted from a sequence of episodes involving interactions among students, teacher, and computer in a spreadsheet environment described in Ben-Zvi (1999).

The Setting

Our studies of students' use of spreadsheets to construct meaning of statistical concepts, such as "trend", take place as part of the ongoing research of the CompuMath Project. In this project, we explore the effects of continuous and immediate access to technological tools on students' construction of meaning in mathematics and statistics (Hershkowitz & Schwarz, 1997).

We have developed an innovative middle school EDA curriculum (Ben-Zvi & Arcavi, 1998; Ben-Zvi & Friedlander, 1997a, 1997b). In this curriculum, statistics learning is seen as a combination of two desirable processes: (a) "Doing" meaningful statistics, namely, posing questions, hypothesizing about possible outcomes, choosing tools and methods of inquiry, designing or changing representations, interpreting results, drawing conclusions and justifying them; and (b) learning as a cognitive and socio-cultural process. The curriculum is organized as sequences of real (or realistic) open-ended data investigations, which allow for many interactions, collaboration and communication in the classroom. The use of technological tools is not considered a learning goal in itself. Their incorporation is intended to contribute to support making sense of data and constructing meanings of basic statistical concepts, and to facilitate the use of multiple data representations.

First Activity: Men's 100 Meters Race

This study originated from a classroom activity. Seventh grade students (aged 13) were presented with a table of the Olympic record times for the men's 100 meters, the years in which they occurred (from 1896, the first modern Olympiad, to 1996), and additional data, such as athletes' names and countries. Their task, working in pairs and using the spreadsheet, was to describe the data graphically and verbally, and discern trends in the data.

Our preliminary observations in experimental classes showed that students were able to engage quickly and with relative ease with the task and the computer-based environment. They were usually able to read the table of results, compare the records of consecutive Olympiads, consider the issue of extreme data, sort the data, and create a prescribed time plot using a spreadsheet (Figure 1).

Students used the table and graph to infer, for example, that (a) the best record is 9.84 sec. and the worst is 12 sec., (b) the greatest improvement is from 12 to 10.8 sec., and (c) the differences between records are not constant. In many instances, students also added explanations for interesting phenomena, such as gaps (World Wars I & II), and change of accuracy of measurement (due to introduction of new digital timing devices).

Students' success in making sense of the data was supported by the familiarity of the context (sports), the use of a familiar graph (time plot), and the presentation of a complete image of a graph by the computer, avoiding decomposition into a series of small plotting techniques. The computer's basic capacity to draw graphs quickly and accurately allowed the students to focus on interpreting the graph rather than on plotting it.

Despite their success in describing the data, students often had difficulties in discerning the overall trend. It seemed (based on data not detailed here) that their difficulty in discerning the general trend in the data was caused by: (a) their lack of experience with the notion of trend; (b) the discrete nature of the graph; and (c) the non-deterministic and disorganized nature of statistical data, which is very different from the deterministic formulae they had met in algebra. Moreover, even students who correctly perceived the trend of the specific data had only a fragmented concept image of the general concept of trend.

We carried these observations with us when we decided to redesign the activity in order to prompt a more serious engagement with the concept of trend, which is a central component in any time series analysis. We realized that the task as originally designed did not make full use of the computer's power to create, manipulate and transform multiple representations. We speculated that students would be more successful in constructing meaning of time plots and trend through intensive production, interpretation, transformation, and use of multiple representations, as contrasted with the use of a limited number of them.

Improved Activity: The Same Song but With a Different Tune

The redesigned investigation, based on the same data, was presented to students. In the first part of the activity, the students were introduced to the context of the investigation and were asked to describe the data graphically and verbally. In the second part, the students were asked to use the spreadsheet to manipulate data graphs, i.e., change scales, delete an outlier, and connect points by lines. In the third part, they were asked to design graphs to support claims, such as:

1. During the years, the times recorded in the Olympic 100 meters improved considerably.
2. Throughout the years, the changes in the Olympic times for the 100 meters were insignificant.
3. Between 1948 and 1956, the times in the 100 meters worsened considerably.

A pair of 13-year-old students (A and D) was videotaped at different stages of their statistics course (resulting in 20 hours of tapes). I concentrate here on short segments of their work with brief teacher interventions. The students were considered by their teacher to be both very able and very verbal. They were asked to talk aloud and explain their actions.

When A and D were requested to describe trends in the data, they did not seem to understand the meaning of the question. With the teacher's help, they concluded correctly that "the record times seem to improve, yet there was occasionally a lower (slower) result than the one achieved in previous Olympiads". Although A and D seemed to notice the general trend of improvement in the records, their view was mostly local and focused on discrete data points, or, at most, on two consecutive records.

Acquiring Manipulation Tools

Changing scales. In this part of the activity, the students acquired tools to support claims based on graphs. For example, the following transcript describes A and D’s comments on the effect of changing the vertical scales of the original graph from 0-12 to 0-40 (Figures 1 & 2):

A. Now, the change is that… that the whole graph stayed the same in shape, but it went down…

D. The same in shape, but much, much lower, because the column [the y-axis] went up higher. Did you understand that? [D uses both hands to signal the down and up movements of the graph and the y-axis respectively.]

A. Because now the 12, which is the worst record, is lower. It used to be once the highest. Therefore the graph started from very high. But now, it [the graph] is already very low.

The students' perception of the change is restricted to the overall relative position of the graph; they considered the shape itself as remaining "the same". Their description includes: global features of the graph ("The whole graph … went down"), an interchange of background and foreground (the graph went down and/or the y-axis went up), and local features (12 as a "starting point" of the graph). These descriptions are linked and complement each other.

In this example, the computer allowed direct teaching about scales to be avoided, and transformed the graph into an object to be manipulated and explored by students as a whole, rather than as the sum of its components. Although the computer assisted the students in this task, work with objects and actions at a higher cognitive level seems to require a great deal of effort. A and D were not able, at this stage, to capitalize on the two different representations of the situation (Figures 1 & 2) to learn about the role of scales in shaping the graph to support claims.

Deleting an outlier. When asked to delete an outlying point (the record of 12 sec. in the first Olympiad, 1896) from the graph (Figure 2), and describe the effect on its shape, they struggled to interpret the effect of the deletion on the graph (Figure 3):

D. The change is not really drastic … Now, however, the graph looks much more tidy and organized.

A. One point simply disappeared. The graph… its general shape didn't change.

They wrote in their notebooks different descriptions of the change: "The graph became straighter" (D); "One point in the graph disappeared" (A). Thus, the students struggled between different views of the effect - global and significant change (the graph is tidy and organized), no change at all (the general shape didn't change), or just a mere description (one point disappeared). Although the dispute about the outlier was not resolved, it served another purpose in that it drew A's attention to a mistake in their conclusion about the greatest improvement in records in the first part of the activity which they corrected.

The computer's dynamic system, which links the various data representations, allows students to change, delete or add data point(s) in the table, and consider the immediate effect on the graph (much statistical multi-representational software supports such direct manipulation of data points on a graph). The emergence of D's global view of the graph seems to be supported by this powerful attribute of multi-representational software. It can enhance the accessibility of statistical concepts (e.g., outlier), and pave the way for students' discussion, reflection and control of their actions (e.g., correction of errors).

Connecting points. In the third task, they were asked to connect the points to obtain a continuous graph. The new graph (Figure 4) elicited many comments from the students, who tried to make sense of what they saw. In their view, the connecting lines (as provided by the spreadsheet) not only did not add any new meaning, but also contradicted the context, as D observed: "Olympiads occur only once in every four years" (namely, there are no data between the points). The students did not see the line as an aid to detect or highlight patterns in the data, which is consistent with their previous difficulties in recognizing data trends.

Our observations indicate that the students' efforts to construct meanings of statistical concepts are supported by making connections between the context of the investigation, the data, and the graph (the study of Ainley, Nardi and Pratt, (1998) offers another perspective on this idea). Technological tools can assist students to switch their discourse interchangeably between the realms of context, graph, and data, by providing an easy access to tabular, numerical, graphical, and textual representations. However, in the above incident, the attention of A and D was drawn to one aspect of the representation, i.e., the contradiction between the new graph and the discreteness implied by the context, rather than to perceiving the connecting line as an aid in representing trend.

Designing Graphs to Support Statements

As reported so far, A and D were practicing manipulations (changing scales, deleting an outlier, and connecting points), and discussing their effect on the graph's shape. The intention was to provide students with the means to design a graph in order to support a particular claim. It was only after a teacher's intervention focusing on the change in the graph's shape that D expressed what appears to be a new understanding:

D. As a matter of fact, we make the graph shape look different, although it is actually the same graph. It will look as if it supports a specific claim.

At this point, D seems to grasp that a change of scales may change the perceptual impressions one may get from the graph. At this point, A and D seemed to understand the purpose of the activity, started to focus on its goal and to direct their attention to the shape of the graph, rather than to its relative position on the screen. Their previous work with multiple representations and their struggle with language seemed to have prepared them for the reinterpretation of what they had done, triggered by their teacher's comments. After many attempts to design graphs, supported by the computer's powerful facilities, they developed "scale sensitivity" and became critical of the false impressions that may be induced (see, for example, the graphs in Figures 5-7).

Thus, their concept image of trend became much richer and linked to change over time, situations, variability in data, and time plots. The following elements seem to have contributed to the construction of students' enhanced understanding of time plots and trend: the instructional engineering, the students' ways of making sense (descriptions, self-explanations, questions to a colleague and the teacher, transfer of ideas, etc.), the computer's mediation, the teacher's intervention, and the use students made of these elements.

In the computer-based learning environment described in the examples above, students acted directly on statistical objects and relations through various linked representations. They produced, interpreted, transformed, and used data representations. Several attributes of the computer seem to contribute to the students' sense-making process, namely its ability to operate (plot, calculate, sort, etc.) quickly and accurately, dynamically link multiple representations, simplify procedures, provide immediate feedback to the user, and transform a whole representation into a manipulable object. In addition, the computer changed assumptions about what can be learned and the statistics curriculum was accordingly redesigned.

Balacheff and Kaput (1996) characterized this epistemological impact of technological tools on mathematics education as "a new experiential mathematical realism", namely, "the experience of direct manipulation of mathematical objects and relations" (p. 470). In the following section, I introduce a more systematic view of the impact of technological tools on statistical realism.

THE IMPACT OF TECHNOLOGICAL TOOLS IN STATISTICS EDUCATION

For the purpose of my deliberation, I adopt an approach based on empirical research and theoretical analysis that views computers as cognitive tools, i.e., means for human cognitive activity (Dörfler, 1993). A cognitive technology has been described by Pea (1987) as "any medium that helps transcend the limitations of the mind" (p. 91).

This approach is based on a specific conception of the human cognition (Dörfler, 1993), of which the following are key aspects:

- Cognitive processes have a concrete and imagistic basis and are not (only) organized by formal/general rules, nor are they mainly of linguistic and prepositional character.
- Cognition depends heavily on the available tools and means: it is to be viewed as being distributed (Pea, 1992) over the system made up by the individual, her social context, and the various available cognitive means (including technological tools). Cognitive development is understood not merely as development of the individual mind, but also as a social development of the available means and tools and their acquisition by the individual through socially structured activities (e.g., P. Cobb, 1998).
- Cognition tends to be context-bound in the sense that context-specific objects, properties, relations etc. are used and exploited, and can even be viewed as systemic elements and parts of the cognitive system. This is expressed by the term “situated cognition”. Situated thinking exploits as much as possible the specific quality, relations and elements of the respective situation.

This conception of cognition leads to specific ways of using computers in statistics education (which can be viewed as a specific case of mathematics education), and a view on how the computer lends itself to supporting cognitive and socio-cultural activities.

The Amplifier Metaphor

In environments that are not based on technological tools, representations produced and used during classroom activities are limited in number. Graphs or tables are often presented to students or constructed according to prescriptive instructions (Kaput, 1992). Representations are then often the same for students learning in the same class.

Instruction often concentrates simply on translation skills between representations, and mastery of these skills tends to become the central goal of teaching. The use of multi-representational technological tools turns many of the manipulations of representations into automatic operations. It is then not surprising that students produce a variety of representations. Figures 1-7 display seven representations as visible outcomes of actions produced by students in the activity described in the previous section.

Pea (1987) suggested the amplifier metaphor to portray the fact that with a computer we can carry out many more calculations in a much shorter time and with a much higher accuracy, but with minimal change in the quality of what we do. Thus, the tool enables us to do what we already do, but faster, more often, more accurately, better, or to a larger extent or degree, with fewer errors. In this sense, the use of a computer in the examples above amplified students' ability to produce many graphs quickly and easily. However, the amplifier metaphor falls short of adequately capturing the effects of using technological tools in statistics education.

The Reorganization Metaphor

An appropriate usage of technological tools has the potential to bring about structural changes in the system of the students' cognitive and socio-cultural activities, rather than just to amplify human capabilities. A powerful tool brings about the reorganization of physical or mental work in at least the following ways.

Shifting the Activity to a Higher Cognitive Level

Working with a powerful technological tool may shift parts of the activity to a higher cognitive level. For example, students have to integrate and focus their interest on detailed planning, and anticipating possible results (instead of calculating or drawing), before executing. In turn, this implies a shift of the main problems, difficulties and tasks of the activity. In the examples above, the computer made it possible to shift the students' attention to the problems of scaling and designing graphs to support statistical claims. Thus, computer usage entails the emergence of different levels of operations and objects, e.g., plotting of graphs might be considered as one level, and various manipulations and transformations of them as another. Several empirical studies point out that working with a powerful technological tool may shift parts of the activity to the metacognitive level as well (Hershkowitz & Schwartz, 1999).

Computers support actions on a higher level by condensing and curtailing complex processes to easily manipulable units. The software commands encapsulate sequences of actions formerly disposable only in unfolded process form. This power makes possible an effective and goal-oriented reflection on the organization, the sequencing, the consequences and other aspects of the whole activity.

Changing the Objects of the Activity

Using technological tools causes a change in the objects to be worked with and upon. Therefore, not only the structure and form of the activity but also its content are changed. For example, by using a statistical software package in the teaching of elementary statistics, the field of objects is enlarged to include statistical tables, data, graphs and numerical results. These representations can become the objects of the cognitive activity, e.g., by varying the given data, sample size, number of repetitions, and are no longer just the products of the drawing or calculating activity. Representations as a whole can be edited, manipulated, transformed, combined, separated into parts, stored, recalled, etc. This capability entails a far-reaching reorganization of cognitive activity and a shift of the focus of attention to a higher cognitive level. The products of such activity with the technological tool can become the objects of reorganized and extended activity and reflection.

Focusing the Activity on Transforming and Analyzing Representations

Kaput (1992) made the distinction between an “action notation system”, that involves calculations and transformations (e.g., the algebraic representation of a function), and a “display notation system”, whereby the activity of the user is generally confined to interpretation (e.g., graphical and tabular representations of a function). This theoretical distinction between action and display notation systems holds when the material tools at disposal are limited to paper, pencil and ruler. It does not hold any more when one uses tools providing, in addition to the representations themselves, passage among representations and user-based manipulations, or what is often called multi-representational software. In this case, all representations are action notation systems - it is possible to stretch graphs (by scaling, changing aspect ratio, etc.), to rearrange a table according to a particular criterion, and so on.

Cognitive processes can often be successfully guided and organized by tangible representations of the given situation. The thinking process then consists essentially of transformations and manipulations of these representations, with the aim of reaching a solution or solving a problem. To assist these processes the computer offers the user a great variety of graphical and symbolic elements for the construction and manipulation of representations. Thereby the user can build on the screen representations of many situations, work with them, and analyze them. Thus, a computer system has many valuable advantages over the traditional technology of paper and pencil, including easy and interactive changes in the representations, saving, recalling, and editing representations and their construction, unlimited repeatability, and documentation of all the actions carried out with the representations, for later reflection and analysis.

Statistics education should strive to enhance students' flexibility in using representations, and should prevent sticking to a single restricted one. It is important to develop the ability to switch between representations, to translate one into another, or even to disregard a specific one. Some statistical packages for introductory statistics are good example of tools that may enhance such flexibility in the multiple-window representations they offer (e.g., Data Desk, Fathom, MEDASS Light, TableTop, StatView). They go far beyond the role of mere means for visualization. They permit the construction and flexible usage of various representations for analyzing data, moving beyond a means of display to become a tool for thinking and problem solving.

Supporting the Situated Cognition Mode of Thinking and Problem Solving

Situated thinking exploits as much as possible the specific quality, relations and elements of the respective situation. The computer can support such a mode of thinking and of problem solving through its simulative power. Thus, technological tools can assist students "to bridge statistics and 'real' life by opening access to modelling of concrete situations and real data" (Balacheff & Kaput, 1996, p. 478).

For example, Pratt (1994) reports on a new computer-based pedagogic approach, termed "active graphing", which may help young students to develop interpretative skills. In an active graphing task, students enter data directly into a spreadsheet, and produce scatterplots as part of an ongoing experiment. The physical experiment, the tabulated data, and the graph are brought into close proximity. The ability to produce graphs during the course of an experiment enables the graph to be used as an analytical tool; the students make decisions about future trials in their experiment on the basis of their interpretation of the graph.

Accessing Statistical Conceptions by the Use of Graphics

Technological tools support enhanced accessibility of many statistical conceptions, by permitting the transformation of purely symbolic presentations into spatial-geometric ones, which are easier to grasp and build cognitive models on. For example, some statistical software packages present a distribution's mean, mode or median as geometrical markers on histograms. Thereby, these concepts may become objects for manipulation and reflection.

In addition, the unique dynamic features of technological tools support linkages between data alterations and their geometrical representations, examples being a graphical "slider" to change the bin width and observe the effects on a histogram's shape, or a program which allows one to add an outlier point to a scatterplot and observe consequent changes in the correlation coefficient and the position of the regression line.

Constructing Meaning of Conceptions by the Use of the Representative Ambiguity

The most common visible outcomes of actions mediated by multi-representational software are the "representatives" (Schwartz & Dreyfus, 1995; Schwartz & Hershkowitz, in press) that are displays within the representations - for instance, specific windows for graphs, or specific tables of values through which problems on functions are solved. Studies of the function concept (Schwartz & Dreyfus, 1995; Schwartz & Hershkowitz, 1999) report that the gains of students using technological tools originate from the fact that such tools enable them to produce more examples of the mathematical entity (function) and that students could act on representatives and translate among them. Schwarz and Hershkowitz (submitted for publication) claim that such gains by students using multi-representational software may originate from the fact that teachers often take advantage of "representative ambiguity" to create situations of social interactions that boost shared social construction of mathematical meaning. “Representative ambiguity” refers to the partial nature of representatives, as parts of concept examples, which often make them ambiguous in the sense that only some of the critical properties of the entity are displayed in them. The study of Schwartz and Hershkowitz has shown that representatives produced via the mediation of computers helped in the social construction of meaning of the function concept.

Similarly, Figures 5 – 7 show three graphical representatives of the same situation, namely, the men’s 100 meters Olympic race. Each of them shows a different aspect of the change over time, which may be difficult or impossible to infer from another representation. For example, Figure 7 shows a linear increase of record time, Figure 6 shows a slight decrease, while Figure 5 shows a non-uniform decrease. Intuitively, it might seem that such ambiguity is detrimental to learning and or development. However, our examples suggest quite the contrary, namely, that students gained from the representative ambiguity through its promotion of shared social construction of statistical meaning. Research is needed to establish the role of representatives in the construction of meaning in statistics.

TYPICAL TECHNOLOGICAL TOOLS IN STATISTICS EDUCATION

In this section, I present a sample of educational technologies that have typically been used in statistics instruction. They fall into at least one of the following categories (distinguished by their educational functions): statistical packages (tools), microworlds, tutorials, resources (including Internet resources), and teacher’s meta-tools (Ben-Zvi, 1997; Biehler, 1997).

Statistical Packages

Statistical packages (tools) include software for computing statistics and constructing visual representations of data, often based on a spreadsheet format to enter and store data. These packages create a computing environment that is mainly used to prepare students to become professional statisticians. Professional statistical software is often very complex and not suitable for all students. Therefore, an adaptation to adjust the software to the classroom is often required.

MEDASS Light (Biehler, 1995; 1997) is a good example of an "educationally modified" statistical package. It is an elementary tool for interactive data analysis for high school students (grade 7 and higher) in a first statistics course. MEDASS Light is intended to be an easy to use elementary tool, and thus fill the gap between the common, too simple, and insufficiently flexible tools designed for high school use, and the professional tools that are too complex for high school introductory courses. It is designed to support flexible interactive data analysis with multiple analyses and results. A spreadsheet-like data table is used for data input, editing and display. The graphical methods include boxplots, histograms, bar graphs, dot plots, scatterplots, and line plots. All the plots can be used for single variables or together with a grouping variable that will produce composite or multi-window plots. Graphs can be enriched by further statistical information, such as lines for the mean or median, regression lines or curves from fitting polynomials, and exponential functions and simple smoothers. Numerical summaries and frequency information are also available for analyses with grouping variables. Numerical results are displayed in data tables that can be further analyzed as new data with the available tools. Selection of subsets, transformation of variables and exclusion of points from an analysis are available on two levels: graphical selection "by hand", and in a more formal way, with the support of a menu system. The system supports numerical, categorical, text, and nominal variable types. Generic commands adapt to the variable types and the roles chosen for the variables (such as grouping variable, x or y variable).

Fathom is an innovative computer-based learning environment for exploratory data analysis and algebra, which is intended for use in high school and college introductory courses. Fathom's features include: dynamic manipulation - instantaneous updating of every representation and calculation while dragging data points, axes, attributes, or bars; formulas to calculate values, plot functions, and control simulations; 'sliders' as part of function plotting, attribute definition, and filters; simple simulation and sampling tools; and direct import of data from the Internet.

Spreadsheet packages (e.g., Excel) are widely used for introductory statistical instruction, and have many compelling capabilities. However, their educational use has been disputable (Hunt, 1996).

Microworlds

These consist of software programs to demonstrate statistical concepts and methods, including interactive experiments, exploratory visualizations, and simulations. In microworlds students can conceptualize statistics by manipulating graphs, parameters, and methods. Typical examples are microworlds that allow the investigation of the effects of changing data on its graphical representation or the value of a correlation coefficient, the effects of manipulating the shape of a distribution on its numerical summaries, the effects of manipulating a fitted line on the quality of the fit, and the effects of changing sample size on the distribution of the mean. Prob Sim and Sampling Distributions are good examples of computer simulation microworlds.

Prob Sim (Konold, 1994, 1995a) is a simulation tool intended for high school and introductory college courses where the major emphasis is on modelling real situations. To model a probabilistic situation with Prob Sim, a "mixer" is constructed containing the elementary events of interest. The mixer is sampled from, after specifying replacement options, sample size, and number of repetitions. Events of interest in a sample are specified and counted. Specified events are counted in new random samples. Prob Sim makes the last step especially easy. Once analyses have been conducted on one sample, the user can press a button to see the results of the same analyses performed on a new sample.

Sampling Distributions is intended for secondary students and undergraduates in introductory statistics courses. Its purpose is to teach the Central Limit Theorem and the behavior of sampling distributions. Sampling Distributions allows the user to create a population and then draw random samples from that population. The populations are created graphically using up and down arrows that "push" an outline of the distribution to change its shape. Populations can be simulated in one of three modes: binomial, discrete, or continuous. In drawing samples, the user determines the sample size and number of samples to be drawn. The sampling distributions of sample means and sample medians can be displayed, and summary statistics are also provided (e.g., mean of sample means, mean of sample standard deviations, standard deviation of sample means). Users can visually compare the sampling distribution to the population, and visually witness the effects on the sampling distribution caused by changing the shape of the population and sample size (Chance, Garfield, & delMas, 1999; delMas, Garfield, & Chance, 1998).

Tutorials

These include programs developed to teach or tutor students on specific statistical skills, or to test their knowledge of these skills. The tutorial program is designed to take over parts of the role of the teacher and textbooks, by supplying demonstrations and explanations, setting tasks for the students, analyzing and evaluating student responses, and providing feedback. Some tutorials function as an interface with other statistical software, when one purpose of the tutorial is to demonstrate use of that software.

ActivStats (Currall, Young, & Bowman, 1997) is a rich learning environment for an introductory statistics course in a multimedia, computer-based format. The material is provided on a CD, which includes video clips, simulation, animation, narration, text and interactive experiments. A compelling feature is the extensive use of video and animated illustrations to present concepts and techniques. The user can launch associated pieces of software, such as the accompanying DataDesk package or a Web browser, independently of the activity being pursued in ActivStats. Options are also available to customize the operation and appearance of the material.

ConStatS (Cohen & Chechile, 1997; Brewer, 1999) is a good example of a computer-based tutorial. It is intended to allow students experiment with statistical concepts taught in introductory college statistics courses. The emphasis of ConStatS is on gaining conceptual understanding of statistics by dealing with real and interesting data. It consists of 12 programs, grouped into five distinct parts: representing data (displaying data, descriptive statistics, transformations, bivariate data), probability (probability measurement, probability distribution), sampling (sampling distribution, sampling error, sampling problem), inference (introductory estimation, hypothesis testing), and experiments. Each program is divided into a large number of "screens", no one of which confronts the student with more than a small number of closely related decisions. The choices the student has to make on each screen lead to an active style of learning. WHY and HELP buttons are available on every screen. Students are required to perform a series of experiments on the same data, which are provided in the program. Once they have worked through the experiments and become familiar with concepts, they can use a data analysis package to explore data on their own. Data sets from different disciplines are included. New data sets can be added readily by students and teachers.

The Authentic Statistics Stack (ASP) (Lajoie, 1997) is a good example of a tutorial designed to model assessment standards. Intended for middle school students in introductory statistics course, the software demonstrates performance criteria for the statistical investigation process. ASP consists of three components, Discovering Statistics (a descriptive statistics HyperCard stack), Authentic Statistics (a library of examples developed to assist students in their learning of descriptive statistics), and Critiquing Statistics (students assess peer projects). The examples demonstrate student performance standards for the statistical investigation process. Thus, the performance standards are made clear and open to learners. Students can see concrete examples by watching video tapes and computer screen recordings of student performance (average and above average), accompanied by textual descriptions of the program scoring criteria. There are six assessment categories: quality of research question, data collection, data presentation, data analysis and interpretation, presentation style, and creativity. Students can access information about each category to see a textual and visual demonstration of average and above average performances. After viewing the information, they can develop their own project and align their performance to the criteria.

Resources

These consist of various resources to support teaching statistics. The development of the World Wide Web has produced unprecedented global means for teachers to easily share their ideas on ways to improve the teaching of statistics (Lock, 1998). Although the volume of on-line material may seem daunting, and the process of searching for worthwhile information can be frustrating, the rewards, both for teachers and students, can be quite substantial. If current trends continue, universal access to the Web should become easier and more common, on-line applications should become even more sophisticated, and useful resources should continue to appear at a steady rate. Typically, the sorts of Web resources are as follows (Lock, 1998):

On-line Course Materials

More and more instructors are providing course materials to their students through web-sites. In addition to providing convenient access for students, these pages can be perused by teachers at other institutions looking for hints and ideas to improving their own courses. For example, the Chance database contains materials designed to help teach a course on chance or a more standard introductory probability or statistics course. The Chance course is a quantitative literacy course based on case studies, with the aim of making students more informed and critical readers of current news that uses probability and statistics, as reported in daily newspapers.

On-line Texts

Several individuals and groups have undertaken ambitious projects to develop statistics textbooks that can be accessed via the Web.

JAVA demonstrations

These are interactive programs that can be accessed over the Web, based on the JAVA programming language, which are simple self-run demonstrations of statistical concepts. For example, Guessing Correlations is a game to show the relationship between correlations and scatterplots, Regression demonstrates the effects of adding an outlier, and Histogram checks the effect of bin size on histograms showing data on the "Old Faithful" geyser in Yellowstone National Park.

Electronic Journals and Newsletters

Examples include Journal of Statistics Education (JSE) which publishes refereed articles, datasets, reviews, and tips related to all aspects of teaching statistics at the post secondary level and Chance News which includes references to statistical issues in current news media.

Electronic Discussion List

These e-lists allow instructors to share questions, ideas and announcements related to teaching statistics, practicing statistics, and statistical computing. The web-sites which archive the messages also provide good resources for searching through past discussions.

Data

Many data sources, often in downloadable formats, are available in the Web from different types of resources: dataset archives, government and official agencies, data about the web, and textbook data. For example, The Data and Story Library (DASL) is an online library of data files and stories that illustrate the use of basic statistical methods which is available freely via Internet. It provides data from a wide variety of topics, can be used to demonstrate simple statistical ideas using real data, and may be searched by data subjects or by statistical techniques.

General Links

There are many web-sites with general links related to teaching statistics, such as materials to support a course, examinations, statistical software providers, and statistics textbooks. Examples include Exploring Data, CTI Statistics.

Other resources are available in a non-Internet format, for example, teacher's interactive movies (e.g., Moore, 1989), and multimedia books (e.g., ActivStats).

Teachers' Meta-tools

These create an interface that enables teachers to adopt software to their specific audience and educational goals.

The categories listed above are not necessarily distinct, and in many cases a specific software falls in more than one category. In some cases, professional tools support the construction of simulations or tutorial shells within the system itself. For example, spreadsheets have a command language that serves to build small programs called "macros". It makes the software extensible and adaptable, in order to build educational microworlds, such as DISCUS (Gray, 1996). However, the use of a command language system in secondary school is problematic. Instead, programming in Excel takes place in a simple form, by entering a formula, a function, creating a chart, or operating a procedure. Once the "program" is set, any change in data affects its outcomes (Carr, 1999).

CONCLUSION

Over the last decade, we have seen huge progress in technological tools for statistics education, which have become more powerful, flexible and efficient, with friendlier interfaces, and better connectivity via the Internet. Despite all this progress and promise, the penetration of these technologies in educational practice proves to be very slow and with great disparity from place to place. The shortage in technology in schools is surely one reason, though with the recent introduction of smaller and powerful machines, like graphing calculators and palm-top computers, tools will soon become common and cheap.

However, the limited commitment of teachers and curriculum developers, and our great ignorance about teaching and learning in computer-based environments are at least as important factors contributing to the scarcity of actual use in classrooms.

Educators should be encouraged to view technological tools as legitimate extensions of cognitive systems and partners in the socio-cultural arena of the statistics classroom. Using these technologies as a cognitive tool and a medium will not weaken cognition. The opposite is true. It opens up the opportunity for the development of a much richer, powerful and flexible learning environment in which students are active learners of statistics. This is a demanding task for students and teachers. Like any other symbolic technology the use of the computer must be learned and integrated into the learning environment through considered use and extended experience. Many considerations should be taken into account to deal with the deep changes in learning and teaching that are emerging.

Choosing Appropriate Tools

How can teachers and curriculum developers of introductory statistics courses navigate within the rich universe of existing software? Biehler (1997) suggests the following list of priorities for software supporting introductory statistics education:

1. Student tools whose learning and use can be integrated into an introductory course
2. Resources of data
3. Microworlds (a good but limited selection)
4. Further resources (Internet, electronic books, multimedia resources)
5. Tutorial shells. (pp. 168-169)

In addition, teachers' meta-tool functions should be included at all levels to allow adaptations for the specific class.

Critical Evaluation of Software

Biehler (1997) stresses the need of the statistics education community to develop a perspective, guidelines, and a specification of ideal requirements, in order to critically evaluate existing software and to produce better software in the future. The main problems of existing statistical software are: (a) the complexity of professional systems, which cause a high cognitive entry cost, (b) the constraints of microworlds, which limit their adaptability, and (c) the uncoordinated interfaces, concepts and notations of various microworlds and tools, which limit the possibility of combining a few tools in the same course. What we ideally need is an integrated system of coordinated tools for educational purposes which is adaptable, extensible, and simple. For example, tools for doing statistics should be adaptable to be used also as a host system for defining and modifying microworlds. Simplicity can be realized by a coherent system of collaborating (modular) tools with well-defined interfaces, minimal functional overlap, adequate data transfer, and object linkage facilities between the components - for example, a system that includes (a) a modelling, simulation and data analysis tools; and (b) dynamic-interactive linkages between them. The MEDASS Light and Fathom projects represent efforts to design educational software in these directions.

Educational Use of Technological Tools

Obviously, it is not enough to provide students with a carefully designed, powerful tool, with nice graphical representations and friendly interface, to ensure that meaningful learning will occur. In rich computer-based learning environments students should be encouraged to:

1. Practice data analysis with an exploratory, interactive, open-ended working style, and combine exploratory and inferential methods, graphical and numerical methods.
2. Extensively use multiple linked representations and simulations to construct meanings for statistical concepts and ideas.
3. Construct models for simple and multistage random experiments, and use computer simulation to study them.

Professional Development

For teachers, traditional professional knowledge is not sufficient to deal with the complexities in the teaching/learning situation created by the technological revolution. We need more experimental research and theoretical analysis to identify the differences between traditional teaching and computer-based teaching, to explore how teachers can prepare themselves to function in the new instructional environments, and to develop appropriate forms of professional development and support systems.

Curriculum Design

Ehrmann (1995) claims that one of the clearest lessons from attempts to apply technology to teaching and learning is this: if you use technology to simply carry the same old thing, you get the same old results. To get different results, you must add new thinking to new technology. This principle implies that learning activities have to be transformed so that students can develop their usage of tools in line with the reorganization metaphor. This goal necessitates new kinds of tasks, supported by powerful tools which permit activities on higher cognitive levels. Such a statistics curriculum, which takes advantage of the technology, can stress conceptual understanding, mathematical modelling and problem solving, real-world applications, and new methods of analyzing data.

Finally, our knowledge base regarding how to exploit the power of technology in statistics education is sparse, as we are in the first steps of its appearance. The task is enormous: "In challenging most traditional assumptions about teaching and learning, technology forces us to think deeply about all aspects of our work, including the forms of the research that need to be undertaken to use it to best advantage. Clearly, our most important work lies ahead of us." (Balacheff & Kaput, 1996, p. 495). Time will show if we can succeed in this adventurous challenge.

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Chapter 5

Discussion and Conclusions

Children as ‘*Data Explorers*’: What was Accomplished?

Part 1 **Review and Discussion**

Part 2 **Conclusions**

PART 1

REVIEW AND DISCUSSION

The overall goal of this study is to advance the understanding of learning and teaching statistics (EDA) by junior high school students in a computer-assisted, carefully designed learning environment (the *SC*). In order to construct a holistic picture of the learning, I aimed at integrating the socio-cultural (cf., Resnick, 1988; Schoenfeld, 1992; Yackel and Cobb, 1996) and the social aspects of learning (cf., Vygotsky, 1987; Perkins, 1986), with the individualist and constructivist aspects of learning and cognitive growth (cf., Piaget, 1976; Davis, Maher & Noddings, 1990). I wanted to investigate what it meant for a student to be deeply involved in open-ended, complex, and long problem-solving processes related to the learning of basic statistics (EDA), interacting with peers and teachers, while using diverse tools, such as relevant computer software (spreadsheets). I asked what it meant for a student to become a '*data explorer*', who is able to meaningfully investigate interesting questions, and using data to answer them.

Purpose

More specifically, the main purpose was to investigate the following themes in the context of statistical problem solving processes.

Firstly, student understanding of data representations, namely, how junior high school students learn to choose, interpret, design, transform and use data representations in the context of open-ended EDA problem solving situations with computerized tools. By giving specific attention to the handling of data representations, I investigated whether or not the students' involvement with creating, designing, transforming and using representations of data, influenced their thinking and learning about the basic concepts underlying statistics (such as, trends, distribution, and association).

Secondly, the contribution of student interactions to their understanding of data representations. The interactions - peer interactions (within-pair or group interactions), and student-teacher interactions -- were used to characterize student understanding and learning processes. I studied in what ways students' experiences of discussing and explaining to other students, resolving conflicts, providing critique of others' work, presenting reports, etc. helped them to develop meaning of data representations. I also studied the nature of student-teacher interactions, and how they promote student work on EDA problem situations.

Thirdly, students' adoption and exercise of the habits and points of view that are common among EDA experts (Wild and Pfannkuch, 1999), in particular the adoption of experts' point of view on local-global considerations of data and their representations. That is, a focus on socialization processes into the culture and values of 'doing statistics' (*enculturation* in the sense of Resnick, 1988). I studied how students become competent in the new and complex domain, by considering learning as acquiring the habits, language and dispositions of interpretation and sense making, in addition to acquiring the particular set of statistical skills, strategies, or knowledge.

Furthermore, in using the *Statistics Curriculum* (*SC*) materials and tools as a model, I experimented with one kind of complex learning environment, in which students are given significant opportunity to take charge of their own learning and interpretation. Thus, I also considered what it has to offer in order to promote changes in the way statistics is now being taught and learned in junior high schools. More specifically, my purpose was to study the following themes.

Firstly, the lessons which can be learnt from the very long, multi-phase curriculum design process to advance our understanding of how we can be more aware of student actions, in order to design 'better' tasks. (By 'better' I mean, situations which offer students opportunities to engage seriously, work and reflect, and advance their knowledge.) I aimed at studying the type of contexts and the nature of tasks that support students' meaningful statistics learning, including the integration of 'research projects' within the *SC*.

Secondly, the influence of technological tools, such as spreadsheets, on students learning processes in the *SC* educational environment. My assumption was that although software is just a medium (and not the message), its integration, especially when well chosen to fit the course, could introduce deep changes. My aim was to study how the technological tool, that have the potential to support conceptualization of statistical objects and actions, helped students to explore and conjecture about data.

Methodology

The data collection and analysis consisted in conducting several kinds of investigations.

- Gathering 'complete' and systematic data on the microevolution of one pair of students entering a new domain and picking up its language, 'culture', concepts and skills, that is, investigation on an *individual level* (N=2).
- Gathering data on the process of the whole three experimental classes participating in the project of the *SC*, that is, investigation on a *classroom 'cultural' level*.
- Conducting larger scale assessment of: (a) students' statistical reasoning by analyzing their 'research projects' (in several schools); and (b) students' 'achievements' at the end of the *SC* (in the three experimental classes), that is, *a quantitative summative evaluation.* My purpose in the latter study was not to prove the effectiveness of the *SC* learning activities (compared, for example, to other statistics courses or instructional methods), but rather to check for coherence within the various sources when appropriate and relevant, and study the scope of several key phenomena identified in my research.

Accomplishment of goals

In the papers presented in this thesis, I described my work towards the accomplishment of the above-mentioned goals by:

- Analyzing at a very fine level of detail using *interpretive microanalysis* (see, for example, Meira, 1991), the ways in which two students (*A* and *D*) began to make sense of data and data representations, as well as the process of adopting and exercising the habits and points of view that are common among experts (Papers I and II). The focus is on the ways they started to develop global views (and tools to support them) of data and their representations on the basis of their previous knowledge and different kinds of local observations. I realized how knowledge was gradually constructed through complex cognitive and socio-cognitive processes, which included their interactions with each other, the teacher, the materials and the computerized tool. I analyzed the ways in which the same 'piece' of students' prior knowledge not only hindered, but ultimately also supported the construction of meaning for data and data representations.

- Analyzing students' 'research projects' to study typical junior high school students' *statistical thinking modes*, that is, an initial taxonomy of novices' reasoning in the domain of EDA with an emphasis on handling data representations (Paper III).
- Assessing students' knowledge of statistics after the end of the *SC* in 'real' data-based inquiry settings (Paper IV). The summative evaluation focused on: the ways students formulated research questions and hypotheses; the ways they created, designed, transformed, and used data representations; their handling of frequency distributions; their interpretations of data and diverse findings; and if and how they adopted the point of view of EDA experts on local-global conceptions of data and data representations.
- Analyzing the interrelationships between curriculum design and research on learning in order to characterize the nature of the instructional activities, which promoted meaningful learning of statistics (Paper V; Hershkowitz, et al., in press).
- Studying the impact of computerized tools in enhancing learning and understanding statistics (Paper VI).

The case study, the classroom observations, the analysis of 'research projects' and the results from the summative final assessment, taken together, demonstrated that seventh-grade students, placed in pairs to work on data-based activities and on projects over a long period of time (30 hours in class and 30 hours out of class), were strongly motivated and in charge of their own learning. The sequences of extended statistical problem-solving enterprises including the long-term project work, coupled with short, more traditional exercises, created the cognitive trajectories of their learning. The high levels of discussion, self-explanations, argumentation, and reasoning (which are not frequent in the majority of traditional mathematics or statistics classroom), together with students' interactions with the ‘enculturator’ (the teacher who represents the domain’s ‘culture’ in class, cf., Voigt, 1995), mainly in the form of *appropriation* processes (in the sense of Moschkovich, 1989), formed the socio-cognitive, cultural aspects of students’ learning and development. The use of spreadsheets, a semi-professional data-tool, with its straightforward, dynamic and interactive procedures, was an effective way of supporting student understanding of representations of data and the links among them. Students deep involvement in creating, designing, interpreting and using multiple representations for data, enhanced their understanding of statistics, and the ‘big ideas’ that establish the foundations of it (such as, distribution, trend, and center). The students’ ongoing autonomous work during the ‘research project’ also enhanced their motivation and, more importantly, their metacognitive awareness (e.g., monitoring and control, in the sense of Schoenfeld, 1992).

The Learning that Took Place in the *SC*

The simplest description of the *SC* experiment may read like a ‘treatment’ type of experiment: these subjects (seventh graders) did something particular (explored ‘real’ data) for many hours (close to 60 hours, including the time spent on the project out of class) in a particular computer-assisted learning environment (the *SC*). In fact, because there were too many particulars involved, the situation is vastly more complex than anything that could be described as ‘changing one variable while keeping everything else constant’.

The *SC* experiment approach has a broader context: it is as an attempt to restructure classrooms at many levels including designing EDA curricula, introducing new roles for teachers, students, and researchers, and reconceptualizing technology and assessment. As such, it can be considered as an example of a *design experiment* (Brown, 1992), i.e., engineering innovative educational environments and simultaneously conducting experimental studies of those innovations. This involves orchestrating all the synergetic aspects of daily life in classrooms, such as, curriculum selection, teacher training, and assessment, which are part of a systemic whole.

The learning processes described in this thesis took place in a carefully designed learning environment that included many complex ‘particulars’, which were:

- the curriculum built on the basis a sequence of semi-structured (yet open) leading questions within the context of extended meaningful problem situations;
- timely and non-directive interventions by the teacher as representative of the discipline in the classroom; and
- computerized tools that enable students to handle complex actions (change of representations, scaling, deletions, restructuring of tables, etc.) and to conduct conceptual discussions without having to engage in extended technical work.

One can formulate innumerable conjectures about the 'real' source of students' learning about statistics. For example: Did the simple fact of spending some sixty hours exploring data with spreadsheets contribute? Was the 'socio-cultural' and 'socio-statistical' climate in class (in the sense of Yackel and Cobb, 1996) largely responsible? Or, was the fact that the teacher was experienced and skilled, and felt part of something important and significant? Studying control groups may check some such conjectures, or aspects of such conjectures, but there are far too many!

What can be said here with some certainty is that we created an *innovative learning environment* in which some impressive learning took place. Teasing out the contributions of particular parts of the environment is not a reasonable goal for any single well-defined experiment. In fact, it was not part of my theoretical and practical agenda. Instead, I suggest that understanding of statistics learning will come through the process of accumulation of many projects and a great deal of theory building (which is emerging since this study started, e.g., Wild & Pfannkuch, 1999; Jones, Langrall & Thornton, 1997; Jones, Thornton, Langrall, Mooney, Perry & Putt, 2000; Konold & Higgins, in press), and discussions in international research fora (such as, SRTL-1, 1999; SRTL-2, 2001). What I can do here is to share intuitions, based on carefully, systematic, and detailed analysis, as part of a larger scholarly enterprise, formulate and discuss some conjectures concerning these intuitions (many of these intuitions were developed through discussions and re-analysis of the data with my Ph.D. advisors and other colleagues in Israel and abroad). I can also offer a detailed description of my data (including transcripts and video segments) for judgments by the statistics and mathematics education communities (as proposed by Schoenfeld, 1994).

I shall speculate that meaningful learning of EDA took place through complex cognitive and socio-cognitive processes of *enculturation*, the processes of teacher-student (and possibly, student-student) *appropriation*, the students' exposure to carefully designed *learning arenas* and relevant *computerized tools*, the students' deep involvement in constructing a '*research project*', and students' becoming '*data explorers*', that is, 'doing' statistics by solving 'real' data problems. However, each one of those conjectures, when considered alone, would give only partial information about the learning that took place in the experimental classes. Only by considering them together, and by speculating about their interrelationships, can we take a step towards understanding the holistic character of the learning that took place in the *SC*.

PART 2

CONCLUSIONS

Structure of this part

Summaries of five major conclusions are presented (not in an hierarchical order) in this part of the summary. The conclusions are based on evidence provided in the papers of this thesis, and also several other papers, which are not included and published elsewhere (Ben-Zvi, 1997a; 1997b; 1999b; Ben-Zvi & Arcavi, 2001).

Each conclusion is first presented, followed by, a discussion of theoretical and practical implications for education.

Conclusion 1

Learning EDA

A core idea I used in this study is that of *enculturation* (cf., Schoenfeld, 1992; Bishop, 1988; Resnick, 1988). The learning processes were interpreted in the context of students' entering the statistics practice and picking up experts' points of view. In the learning environment of the *SC*, students met and worked from the beginning, in pairs and small groups, with ideas and dispositions related to the 'culture' of EDA (making hypotheses, summarizing data, recognizing trends, identifying interesting phenomena, and handling data representations, and the related dispositions). Skills, procedures and strategies (e.g., reading graphs and tables, rescaling, calculating statistical measures) were learned as integrated in the context and at the service of the main ideas of EDA.

Thus, the case study of *A* and *D* focused on socio-cultural and cognitive processes of their learning EDA and becoming '*data explorers*'. It concentrated on the way they started to develop views (and tools to support them) that are consistent with those of EDA experts.

I propose that the analysis of data illustrated the following 'mechanisms' in the process of learning EDA.

The role of previous knowledge in learning EDA

The data illustrated the multifaceted and sometimes unexpected roles prior knowledge may play, some of which may hinder progress, but others may advance it in interesting ways. In *A* and *D*'s case I found many examples of this phenomenon.

One of the strongest visible 'pieces' of knowledge *A* and *D* applied and referred to repeatedly was the 'difference' between single pairs of data, which came from their practices in the algebra curriculum. This background knowledge played several roles. On the one hand, it provided them with the 'differences' lens, which conditioned most of what they were able to conclude for quite a while. On the other hand, looking at differences helped them to refocus their attention from 'pure' pointwise observations towards more global conclusions, such as, "*the differences are not constant*". Also, I suggested that looking at differences helped them in implicit and subtle ways, to start getting accustomed to a new domain, in which data do not behave in the deterministic way in which they were used to in algebra, in which regularities are captured in a single exact formula.

I stress that *A* and *D*'s focus on the 'differences' served more than one function in their learning. It was invoked and applied not only when they were asked to look for patterns in the data, but also in a very fruitful way when they spontaneously evaluated the results of re-scaling the graph. There, they used the 'differences' in order to judge the extent to which the re-scaled graph matched their goal of designing a graph to support a certain claim about trends.

Thus *A* and *D*'s previous knowledge not only conditioned what they saw, sometimes limiting them, but also on other occasions, empowered them. Moreover, their previous knowledge served new emerging purposes, as it evolved in light of new contextual experiences.

In the data, I found other instances of how previous knowledge played out. For example, *A* and *D* were adamant in their rejection of the line segments connecting the discrete data in the graph because of their (mathematical) knowledge of how a single data entry is represented in the 'Cartesian-like' representation. The points 'in between' have no representational meaning, since "*there were no Olympiads in between*". In this case, their sophisticated prior knowledge did not allow them to focus on the connecting segments as a mere artifact of the new domain, used by experts to help them notice or highlight a trend, rather than to claim interpolated values which do not exist.

In brief, this is a 'story' of how strong 'pieces' of prior knowledge, which constrained what the students saw and did and in some sense hindered their progress at the beginning of their work, evolved and turned into a main tool with which they made sense of trends. As such, it can be considered as an example of the process described in Smith, diSessa & Roschelle (1993): 'misconceptions' are not replaced by 'correct knowledge', rather they are the basis upon which knowledge undergoes a process of refinement and broadening of applicability contexts.

From a local-pointwise view towards a flexible combination of local and global views

Learning to look globally at data can be a complex process. Studies in mathematics education show that students with a sound local understanding of certain mathematical concepts, struggle to develop global views (cf., Monk, 1988). Konold, Pollatsek & Well (1997) observed that high school students - after a yearlong statistics course - still had a tendency to focus on properties of individual cases, rather than about propensities of data sets.

In *A* and *D*'s case, they also very persistently emphasized local points and adjacent differences. Their views were related to their 'history', i.e., previous background knowledge about regularities with linear relationships in algebra. The absence of a precise regularity in a set of statistical data was their first difficulty. When they started to adopt the notion of trend (instead of the regular algebraic pattern expected) they were still attentive to the prominence of 'local deviations'. These deviations kept them from dealing more freely with global views of data. Later on, it was precisely the focus on certain pointwise observations (for example the place and deletion of one outlying point) that helped them to direct their attention to the shape of the (remaining) graph as a whole. During the scaling process, *A* and *D* looked at the graph as a whole, but rather than focusing on the trends they discussed its relative locations under different scales. Finally, when they used the scaling, and had to relate to the purpose of the question (support of claims in the '*journalists' debate*'), they seemed to begin to make better sense of trends.

It is interesting to note: (a) the local pointwise view of data sometimes restrained the students from 'seeing globally', but in other occasions it served as a basis upon which the students started to see globally; (b) in a certain context, even looking globally may indicate different meanings for the students than for an expert (position of the graph vs. trend).

The results of final assessment study (Paper IV), at the end of the *SC* course, show that most of the students held several important aspects of experts' point of view on local-global approaches to data and data representations in the context of a real and meaningful data investigation. In particular, the majority of them were able to formulate global research questions and hypotheses, interpret graphs and tables globally, and handle cycles in data, while independently learning about 'immigration waves'. Students were fluent in choosing a variety of graphs to display global features of data and their frequency distribution. Several students used a line in a graph as an artifact that could help them to observe and present patterns in the data. In addition, students frequently manipulated and designed data representations (e.g., change type of graph, scaling), in order to better present their conceptions and understanding of the data.

I suggest that this relative success in adopting *global view*, that is, the tendency to notice and describe generalities in data was supported by: (a) the emphasis of the *SC* on *enculturation* processes, i.e., entering and picking up the points of view of a community or culture, through interactions with a teacher, who plays an important role as an '*enculturator*'; (b) the extensive and meaningful learning experiences in handling data within a purposeful context related to complex ideas in a social setting during the classroom activities and the 'research project'; (c) the structure of the assessment worksheet which was similar in contents and style to the textbook's worksheets (but with fewer instructions); and (d) the support provided by the computerized tool, which removed most of the computational/technical load, to allow students focus on becoming *interpreters* of data and findings (cf., Ben-Zvi, 2000; Paper VI in the thesis).

I suggest that sophistication in students' understanding of data developed both within each point of view (local and global) and within the dynamic and flexible integration of those views. Thus, I see two trajectories of development, which may occur simultaneously: '*vertical*' - growth in sophistication within a view (local and global); and '*horizontal*' - growth in the webbing of the local and global views. These ideas require further research.

Theoretical implications

If beginning students were to work in environments such as the above, the learning would involve the following:

1. their prior knowledge will be (and should be) engaged in interesting and surprising ways - possibly hindering progress in some instances but making the basis for construction of new knowledge in others;
2. many questions will either make little sense to them, or, alternatively, will be re-interpreted and answered in different ways than intended; and

3. their work will inevitably be based on partial understandings, which will grow and evolve.

It can be expected that students will have difficulties (of the type described) when confronting the problem situations of the curriculum. However, these difficulties and experiences are an integral and inevitable component of their meaningful learning process with long lasting effects, which were evidenced in the summative assessment (Paper IV, and in additional data collected one year and 2.5 years after the end of the *SC*, which are now being analyzed). My study confirmed that even if students do not make more than partial sense of that with which they engage, appropriate teacher guidance, in-class discussions, peer work and interactions, and more importantly, ongoing cycles of experiences with realistic problem situations, will slowly support the building of meanings.

The account of this thesis on making sense of data show how students generated some apparently wrong, sometimes unacceptable or irrelevant interpretations, held novices' local view on data and their representations, and initially used everyday or ambiguous language. While student conceptions may look wrong to experts, an account of their transformation cannot be reduced to moving from the wrong conceptions to the right ones. Instead I suggest that research in statistics education focus on any of the processes listed above to incorporate broader and crucial aspects of learning by 'doing statistics'. Such research must examine how learning involves not only correct knowledge, but also the gradual growth of conceptions, dispositions and language, which are inherent to the nature of learning.

Teachers as 'enculturators'

It is important for teachers of a complex subject matter (such as, statistics) to become aware of the changes that enculturation processes would bring to learning. Whereas the student in the traditional statistics class is expected to 'absorb' doses of prescribed amounts of information and the teacher is the 'disseminator', the student in this view learns to participate in a certain cognitive and cultural practice, where the teacher has the important role of a mentor, mediator, and 'enculturator'. This is especially the case with regard to statistical thinking, with its own values and belief systems, and habits of questioning, representing, concluding and communicating. Thus, for statistical enculturation to occur, specific thinking tools are to be developed alongside collaborative and communicative processes taking place in the classroom.

Similarly, Brown (1992) suggests to "*transform grade-school classrooms from work sites where students perform assigned tasks under the management of teachers into communities of learning and interpretation, where students are given significant opportunity to take charge of their own learning*" (p. 141). In these learning environments, teachers cease to be the 'dispensers' of a curriculum. Instead they must respond to a wide range of unpredictable events, and can play a significant role in their interactions with students: encouraging them to employ critical thinking strategies and use data representations to search for patterns and convey ideas; directing them to a potentially stimulating investigation context; expanding and enriching the scope of their proposed work; and providing reflective feedback on their performance.

Educational implications

Learning by explaining

It has been observed in this thesis that one of the best mechanisms of learning statistics was to explain it to others. In Chapter 2, the intellectual benefit of generating explanations within the pair has been stressed. Many other researchers (e.g., Brown, 1988) elucidate the ways in which explanatory processes, as part of reciprocal teaching activities, motivate learners and encourage the search for deeper levels of understanding and subject mastery. Brown characterizes these explanatory-based interactive learning environments as ones that push the learners to explain and represent knowledge in multiple ways and therefore, in the process, to comprehend the subject more fully themselves.

In the *SC* comprehension and interest was enhanced when students had to explain their views, clarify their positions, and argue their data-based cases to others. It provided students with opportunities to persuade and explain to others, which made them think about their own knowledge and thinking, internally organize and present what they know, and also invite them to commit themselves to some ideas (for example, in the '*Work Dispute*' activity). Peer teaching can be used to give students such opportunities to learn, in similar ways tried in the *SC*.

Integrating the problem-solving skills and meta-skills

The *SC* offers a complete integration of the learning of statistical problem solving skills, including relevant thinking skills and control strategies (meta-skills), into the subject matter involved. In fact, they were integrated to such an extent that it was difficult for the researcher to study and document these skills as separate from the content knowledge involved. Other current computer assisted learning environments, in other subjects (such as, the *CompuMath* in mathematics), integrate the learning of problem solving skills into the learning of subjects, to empower the active use of knowledge for critical and creative thinking.

Conclusion 2

Learning through Appropriation

One of the premises of this study is that the analysis of cognitive change cannot be isolated from the social interactions within which it is embedded. The data show that most of the learning took place through dialogs between the students themselves and in conversations with the teacher.

Of special interest to me were the teacher's interventions, at the students' request. These interventions, which though short and not necessarily directive had catalytic effects, can be characterized in general as ‘negotiations of meanings’ (in the sense of Yackel and Cobb, 1996). More specifically, I regard them as interesting instances of *appropriation*, as a non-symmetrical two-way process (in the sense of Moschkovich, 1989). This process takes place, in the *zone of proximal development* (Vygotsky, 1978, p. 86), when individuals (expert and novices, or teacher and students) engage in a joint activity, each with their own understanding of the task. Students take actions that are shaped by their understanding; the teacher ‘appropriates’ those actions - into her own framework - and provides feedback in terms of her understandings, views of relevance, and pedagogical agenda. Through the teacher's feedback, the students start to review their actions and create new understandings for what they do.

In the data, we see the teacher appropriating students’ utterances with several objectives: to legitimize their directions, to redirect their attention, to encourage certain initiatives and also implicitly to discourage others (by not referring to certain remarks). The students appropriate from the teacher a reinterpretation of the meaning of what they do. For example, they appropriate from her answers to their inquiries (e.g., what ‘trend’ or ‘interesting phenomena’ may mean), from her unexpected reactions to their request for explanation, and from inferring purpose from the teacher's answers to their questions.

Appropriation by the teacher (in order to support learning) or by the students (in order to change the sense they make of what they do) seems to be a central mechanism of *enculturation*. As shown in this thesis (see Paper II), this mechanism is especially salient when students learn the dispositions of the subject matter rather than its skills and procedures.

Within-pair interactions

The data of the case study, as well as the classroom observations, show close collaboration between the pairs of students in all stages of their work. The students:

1. verbalized their understandings of the problem-situations. At times this spontaneous verbalization produced mere descriptions, but later served as stepping stones towards a new understanding, and at times, it served as self-explanation (Chi et al., 1989) to reinforce ideas;
2. complemented and extended each other's comments and ideas, which seems to have ‘replaced’ some of the teacher's role in guiding their evolution. Sometimes, these interactions took a similar form to the above-mentioned teacher-student appropriation processes, with one student temporarily taking the role of the ‘tutor’; and
3. decided to request the teacher's help when faced with a difficulty, which could not be resolved among themselves.

Students’ collaborative work also included use of the computer, division of roles, resolution of conflicts, reflection, critique of others' work, and presentation of reports. One of the explicit ways in which those interactions contributed to develop meanings of data, was through the important role of conversations in the processes of learning. Because they were intensively involved in thinking about explaining to each other, arguing their case, thinking about communicating their findings to other students, etc. through the use of statistical language, tools, representations, etc., they developed cognitively during their work. This intensive use of language and their interactions with peers and teachers facilitated in many ways their ability to gain control over their actions and process, and adopt the *discourse* and the *meta-discursive rules* typically used in the new domain (Sfard, 2000). As Vygotsky suggests, speech acts as an organizer, unifier, and integrator of many disparate aspects of student’s behavior, such as, perception, memory, and problem solving. Words provide learners with ways to become more efficient in their adaptive and problem-solving efforts (Vygotsky, 1978, Chapter 4).

Theoretical implications

The notion of (teacher-student) *appropriation* focuses on how children are able to learn from experts, who are providing them with an alternative interpretation for their actions. Based on my observations and analysis, I propose that appropriation can possibly be extended to similar processes between peers, which may be termed *peer appropriation*. My data suggest that appropriation may characterize *both* tutor - learner activity, and learner - learner activity, and that these interactions can be described as cycles of repeated two-way processes. However, further study is required to extend the concept of appropriation beyond a non-symmetrical two way process, to also describe how students learn to carry out problem-solving activities from each other. Such enterprise will be based on the existing work on peer interactions (cf., Azmitia & Perlmutter, 1989; Damon, 1984; and Davidson's review, 1985[23]), which uses (Neo-) Piagetian's notions to look at how peers collaborate, and will have to extend beyond that.

Educational implications

Teachers should be made aware of the importance of social interactions in the classroom, in particular the role of teacher-student appropriation processes and peer-collaboration. The latter is especially significant in a computerized-assisted learning environment in which the student inevitably receive limited amount of teacher's attention. In my data, students frequently 'replaced' the teacher, by explaining their thoughts results to peers.

In order to become an active participant in the process of teacher-student appropriation, the teacher is required to: (1) reflect on her own understanding of the task the students are engaged in; (2) be attentive to student action arising out of her own understanding, even if it indicates minimal participation in a task; (3) appropriate that

[23] To the best of my knowledge (and surprise) there are no newer studies on this topic.

action - into the her framework - and provides feedback about the action in terms of her understanding of the task (goals, relevancy, etc.), and so on. Teacher appropriation is not necessarily focused on direct instruction. Instead, it reveals the role of tacit knowledge in the teaching and learning process.

Teachers also have to become attentive to peer appropriation processes in classrooms, and aware of their constructive role in students' learning. In this regards the teacher is required to:

1. legitimate and encourage within-students conversations;
2. promote students' 'decision making' processes in argumentative discourses; and
3. know how, when and why to intervene (or refrain from intervening), when asked by students to help.

Conclusion 3

An Initial Framework for Statistical Reasoning

In my study of students' 'research projects' (Paper III), I observed the following statistical reasoning modes in the context of solving statistical problems:

- *Uncritical thinking*, in which the technological power and statistical methods are used randomly rather than 'targeted'.
- *Meaningful use of a representation*, in which students use an appropriate graphical representation or measure in order to answer their research question and interpret their results.
- *Meaningful handling of multiple representations*, in which students are involved in an ongoing search for meaning and interpretation to achieve sensible results, and in monitoring their processes. They are able to: (a) critically choose statistical graphs and measures; (b) consider their contribution to the investigation; and (c) perform corresponding modifications and transformations of those representations in order to answer the research questions and justify findings.
- *Creative thinking*, in which students decide that an uncommon method would best express their thoughts, and they manage to produce an innovative graphical representation, or self-invented measure, or method of analysis.

This initial framework presents the reasoning modes in a *discrete* manner and does not allude to how and when students move from one mode to another. However, other parts of this thesis (Papers I, II and IV) show that although this framework is not complete, during their engagement with the *SC*, students have gradually developed their statistical reasoning through some of these modes. The following preliminary and simplified description is offered to present a possible trajectory along which reasoning about the use of data representations in the service of statistical investigation may develop.

The maturation of students' statistical reasoning typically started at the *uncritical thinking* mode. Given students' limited experience with statistics, their initial encounters with statistical problem situations focused on familiarization with statistical tools, methods and 'culture', in an *uncritical* mode of reasoning. Students gradually became familiar with rudimentary statistical methods, ideas, and points of view, and although frequently generated imprecise or irrelevant interpretations and, were able gradually to *meaningfully use a representation* in the context of a data problem situation. Later, students became capable of performing modifications and transformations of representations and monitoring their processes, in order to answer and justify their research questions and interpret their results. Thus, they were able to reason about data representations at a higher mode, while *meaningfully handling multiple representations*, choosing between representations and purposefully modifying them. Naturally, creative behavior occurred less frequently.

Theoretical implications

Research into *statistical reasoning* (or thinking) requires further orientation. Garfield and Ben-Zvi (SRTL-2) use *statistical reasoning* to refer to decision-making, prediction, judgments and arguments as they apply to the use of statistical information. More specifically, statistical reasoning involves conceptual understanding of important statistical ideas and the ability to recognize and not be misled by misconceptions and biases. In the statistics education literature, it is not always possible to discern whether the research tasks and the students' responses to these tasks involved statistical reasoning or more basic forms of statistical knowledge.

Thus, further research is needed to: (a) refine the understanding of different aspects of and factors related to statistical reasoning (e.g., by distinguishing it from mathematical and other types of reasoning); (b) generate assessment protocols based on reasoning frameworks, such as the above-mentioned framework; (c) refine and validate those frameworks using assessment protocols and case-study analysis, with particular attention to developmental aspects; and (d) evaluate and compare emergent statistical thinking frameworks (cf., Mooney, 1999; Jones et al., 1997 and 2000; Garfield, delMas, and Chance, 1999).

Educational implications

Frameworks of statistical reasoning have implications for curriculum design, instruction, and assessment. Because these models provide a picture of students' reasoning, curriculum designers and teachers can use them to establish goals and tasks that are within the scope of students' statistical reasoning, or even in their *zone of proximal development* (Vygotsky, 1978, p. 86). More specifically, in providing research-based cognitive knowledge to inform instruction (Fennema et al., 1996), these models offer an accessible means for teachers to build instructional sequences or hypothetical learning trajectories (Simon, 1995). That is, the model provide a valuable tool for helping the teachers and curriculum developers in planning learning goals, designing learning tasks, and predicting the kind of learning and thinking that will occur as those tasks are played out in the classroom. The models also indicate structural and conceptual aspects of students' statistical reasoning that need to be evaluated through assessment.

Conclusion 4

Designing Curricula to Enhance the Learning of EDA

Even though curriculum development is not the main focus of this thesis, some issues concerning the relationships between curriculum design, research and learning were however in its scope, and are considered below. (The interrelationships between the development and research of the *SC* are described in Hershkowitz, et al., in press.)

The *SC* is an attempt to create a learning environment, consists of a coherent sequence of learning situations, together with appropriate materials and tools, in which students are engaged in meaningful statistics. By meaningful statistics I mean that students' main concerns are statistical processes and perspectives rather than ready-made algorithms. These processes arise for the students in familiar situations as natural means for investigating and solving 'real' data-based problems, rather than as ritual procedures that are imposed by the teacher or the textbook.

The curricular examples I have described illustrate how curriculum design can take into account: (a) new trends in subject matter (EDA), its needs, values, and tools; (b) lessons learnt from research about student learning; (c) relevant computerized tools; and (d) integration of cognitive and socio-cultural views about statistics and the learning of statistics. My purpose in the design of the *SC* and in the subsequent research was to make the curriculum more attuned to student interests, motivation, capacities and interactions, and more calibrated to the appropriate uses of technology. The following aspects of statistics curricula were discussed in the thesis.

Representations as means of expression

In the *SC*, the various data representations are not just didactic means to convey statistical ideas to students (as in many traditional textbooks), but rather *means of expression* by which the students present their points of view, or try to convince opponents. For example, in the '*Work Dispute*' activity, students showed themselves able to understand and judge the complexities of the situation, engaged in talking and arguing about it, preparing a data-based proposal which in their view was acceptable, rational and just, and were able to defend it. Thus, students' learning involved more than displaying, but rather manipulating data and their representations for *rhetorical* use. This constitutes a jump comparable to between *learning to speak* a language and *speaking to learn*.

Learning arenas

The data show that by staging and encouraging students - e.g., to *take sides* (in the '*Work Dispute*'), and *design* and *re-design* data representations (in '*The Same Song with a Different Tune*' activity and in other activities of the *SC*), we 'pushed' them towards levels of discussion and reflection, which are not frequent in most traditional statistics classrooms. These discussions had a catalytic effect on students' construction of meanings for data and data representations.

Taking a stand in the '*Work Dispute*', made students check their methods, arguments and conclusions with extreme care. Criticism and counter-arguments by peers and teacher were a natural part of the activity. When the results of their work were not in line with their position, students were forced to persevere and search for more evidence and convincing arguments. Similarly, in the - '*The Same Song with a Different Tune*' - students became involved more than in reading and interpreting data representation, but rather in focusing attention on the crucial role of scales in creating a visual impression for good or ill. Thus, *Learning arenas*, such as *taking sides* and *designing*, promoted and supported certain student actions, which encouraged meaningful learning of statistics.

Cross-fertilization between project work and classroom activities

The project work consisted of: (1) cooperative work in small group (preferably in pairs); (2) students are in charge of the full cycle of investigation, from the question formulation to the presentation of results; (3) the teacher guides the students in their work, assigns dates for intermediate phases, and provides ongoing feedback; (4) alternative assessment methods are used, and assessment criteria are discussed to assure transparency to students (Garfield and Gal, 1999; Ben-Zvi, 1999b); and (5) project work and classroom activities are carried out simultaneously.

The data show that the simultaneous work on the self-propelled 'research project' and the semi-structured, open-ended classroom activities was beneficial to student learning. The classroom activities supplied some of the basic statistical approaches, concepts and skills needed for the project design. The project motivated the students to take responsibility for their work and methods of inquiry, and gave them a sense of relevance, enthusiasm, and ownership. Students evaluated and applied new concepts and methods that were introduced in the classroom activities, not only in the given context of the different activities, but also in their own project. Thus, the project work gave them an added opportunity to experiment with the new concepts and methods, become practiced in communicating their ideas, and versed in the use of statistical software, and often raised new statistical issues to explore, which were not originally part of the curriculum.

Theoretical implications

Research is a necessary and integral part of curriculum development. In following the learners and investigating their learning in their own environment, research directs the redesign of the learning activities in such a way that the intended curricular goals are realized. Future curriculum based research in statistics education is likely to combine cognitive and socio-cultural approaches, due to the current trend toward a socio-cultural outlook in the field of mathematics education. These developments in the scholarly discipline of statistics education may lead to investigate particular issues within the curriculum. I suggest that curriculum-based research efforts will be aimed at characterizing and studying the following issues:

1. *Learning arenas*, such as, *taking sides* and *designing.* Such research can be a helpful guide in the development of tasks for student learning, aimed at promoting and supporting certain student actions, which encourage meaningful learning of statistics (and possibly mathematics).
2. *The incorporation of project learning in statistics*. Some issues have already been addressed in this area, such as, assessing student projects (Starkings, 1997; Holmes, 1997), but others still wait an adequate address. Among the challenging issues are: the effects of cooperative group work, student's choice of the project

topic

(vs. teacher setting it), report-writing on the learning of statistics, class and out of class meeting management, etc.

3. *The place of EDA in the mathematics curricula*. Current views (cf., Moore, 1992) distinguish between mathematics and statistics as separate disciplines, and depart from traditional views of teaching statistics as a mathematical topic. Moore (1998) expanded the view of statistics by suggesting that it should be considered one of the liberal arts. Thinking of statistics as a liberal art balances its essential technical expertise with its flexible and broadly applicable mode of thinking and reasoning about data, variation, and chance. Similar to statistical thinking, the liberal arts, especially in its philosophical tradition, encourage skeptical, analytical thinking, unconstrained by a priori standards, and bearing in mind that any conclusions are subject to continuing challenge. The liberal arts image emphasizes that statistics involves distinctive and powerful ways of thinking:

   > "*Statistics is a general intellectual method that applies wherever data, variation, and chance appear. It is a fundamental method because data, variation, and chance are omnipresent in modern life. It is an independent discipline with its own core ideas rather than, for example, a branch of mathematics.*" (Moore 1998, p. 1254).

   However, in most cases, statistics is taught in school within the mathematics curricula. My data show that linking mathematics and data handling can be beneficial to learning in complex and multifaceted ways. The students I observed gained fundamental understanding of data and their representations, while shifting, albeit with great difficulty, from the precision of mathematical models to the exploratory nature of statistical reasoning and modeling. Further research is required to carefully investigate learning processes in diverse educational environments at all age levels, in which statistics and mathematics are integrated.

4. *Teacher's knowledge and dispositions*. The professional development needs of statistics teachers at school level, as well as those in other subject areas with statistics literacy requirements, have slowly been recognized and are only now beginning to be met comprehensively (c.f., Hawkins, 1990; Hawkins, Jolliffe, &

> Glickman, 1992; Watson, 1998; Gal 2000). I certainly need to understand in more detail what knowledge and dispositions teachers bring to their study of statistics and what materials and approaches will best help them (and, therefore, their students) to expand and deepen their knowledge.

The abovementioned research themes, as well as others, ought to become a collaborative effort of statistics and mathematics educators, mathematicians and statisticians, researchers of various disciplines, technology specialists, and curriculum developers, to offer diverse and interdisciplinary perspectives on statistics education and literacy.

Educational implications

The *SC* project offers major changes for educational practice in general, and for the teaching of statistics (EDA) in particular. The learning environment and techniques developed in this study could be used for teaching statistics, as well as other subjects (firstly, mathematics). However, this learning environment must be implemented within an educational philosophy that is child centered, open ended, dynamic, and oriented towards cooperative working on extended problem solving activities and projects and on students' construction of meaningful products. It should also be oriented toward connecting various aspects of students' knowledge and individuality (such as, socialization, affectivity, culture, interests) and integrating them into an environment that is rich in computerized tools that are integrated into the school's curriculum and culture. And finally, it should include 'enculturators', namely, experienced and skillful teachers who act as representatives of a community of expert statisticians and its 'culture'. All these criteria were established in the *SC* experimental schools.

The holistic nature of the learning environment must not discourage statistics educators from experimenting with the *SC* and similar statistics learning environments in their classes. Even if only parts of the learning environment can be implemented in school reality, it would make sense to expose teachers and students to them. At a time when statistical knowledge is increasingly being considered essential for effective citizenship and as a part of required workforce preparation, I must seek new strategies that can support the development and implementation of statistics education and literacy and thus help fulfill the promise of informed citizenship for all. Several practical issues in that direction are considered below.

Multi-disciplinary learning

The *SC* offered students an opportunity to integrate different subject matters (mathematics, statistics, and subjects related to their projects, such as, geography and biology) and skills (statistical, mathematical, writing and presenting, computer literacy, etc.). My data show that the experimental students did not find learning different skills or subject matters at the same time difficult or confusing; in fact, they benefited from this pedagogy, which resulted in the learning of one skill's contributing to the learning of another. It is important to note that the very nature of data, which is well phrased by Moore (2000) as being "*numbers with a context*" (P. XXV), and the integration of technological tools, imply that principle of multi-disciplinary teaching of statistics.

However, there are certain problems with integrating subjects in school. Conventional schooling pays little heed to integrating subjects, teachers are not prepared for such integration, and there are few programs aimed at learning a topic in a multi-disciplinary approach. Statistics is one example of a 'natural bridge builder' between subjects in school. By providing basic means to handle data, which is pervasive everywhere (even in school subjects…), statistics could build these connections. In fact, in one of the junior high schools, which currently implement the *SC*, collaboration between the mathematics (statistics), computers, and science teachers was found supportive to the learning of all three subjects.

Learning through open-ended tasks

The open-ended (or semi-structured) nature of tasks in the *SC* made students learn through producing and re-producing various solutions, which emphasized for them that in statistics there could be several strategies and solutions to solve a data problem. Instead of producing short 'right' answers, students were involved in producing a variety of products, in terms of their complexity, scale (from simple answers of calculation, through short survey, to extended 'research project'), completeness, and creativity. This focus of students enabled the teacher to concentrate on students' processes of learning, providing guidance and feedback, and preparing for the whole class discussion, in which major issues encountered in students' work were synthesized.

The integration of project work in EDA courses

The incorporation of project work in statistics courses is becoming a common practice at all age levels. The *SC* offers ways to assist students in their project data analysis without turning assignments into cookbook analyses; effective formats for class meetings that balance the development of the project and theoretical material; out-of-class teacher-students interactions to support group work; assistance for students to develop effective writing and data presentation skills; fair, informative, and time efficient assessment of student reports and analyses (Ben-Zvi & Friedlander, 1997a, Appendix III).

Still, incorporating project work in a statistics course presents complex challenges for the teacher. Therefore, a systematic and extended training is required to prepare teachers for the task. Such training must include hands on experience in conducting and writing a statistical project, and reporting its findings. In addition, various aspects of implementation must be taught, such as, group work management, effective techniques to assist students in data analysis and use of technology, and alternative assessment methods (Garfield & Gal, 1999).

Professional development of teachers

The teachers in the *SC* experimental classes were skillful and experienced, fluent with the contents and the pedagogical approach, and fairly versed with the use of computers. They played a very important role throughout the experiment, by constantly discussing various issues of implementation, as well as the various steps in the students' progress. Can we educate teachers to such levels of knowledge and skills?

The lack of appropriate statistical background of teachers at all levels is the first obstacle to large-scale implementation of projects like the *SC* in schools. My eight-year experience in conducting intensive in-service teacher workshops has showed me the magnitude of difficulties and challenges of effective professional development. Teachers at the school level are mostly immersed in a mathematical culture which is, for the most part, antithetical to the processes involved in Exploratory Data Analysis. At the very heart of EDA are processes, such as, description, prediction, interpretation, and handling uncertainty, estimation, and error, processes which are not part of the school mathematics culture. Further, teachers have been typically teaching mathematics as a subject in which there is, for any particular problem, a clear sequence of steps, leading in a straight line from problem to solution. It goes without saying that this sequence rarely requires the collection of real, original data or questioning of the information in hand.

In light of the above, and based on observations in *SC* teachers' workshops[24], the following approaches seem to be useful in the professional development of statistics educators.

Learning by 'doing statistics'

In the area of EDA, teacher education should consist largely of teacher investigating statistical problems that interest them, collecting data, analyzing it, and drawing conclusion. While the topics and data sets may differ for teachers and students, the fundamental goals and methods of teacher and student education, should be the same; to learn by 'doing statistics'. This is especially crucial in preparing for the teaching statistical projects (see previous section). Teachers must carry through full investigations from question formulation through experimental design, data collection, and data analysis. Only through carrying out their own investigations will teachers gain the understanding and confidence that will enable them to cope with the uncertainties of supporting students in their work.

Teachers involved in 'doing statistics', in the same way statisticians would, is a powerful way for them to gain expertise in statistical 'culture', and get accustomed to their role as 'enculturators', who will represent the 'culture' of EDA in their classrooms. Bishop (1988, Chapter 7) put the spotlight in mathematics education on the preparation of mathematical 'enculturators', on the creation of a community of mathematical 'enculturators', and the development of mathematics education community. Further theory and concrete plan is required to apply that vision in statistics education.

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[24] The *SC* teachers' workshops are inservice courses of various structures: from a one-day seminar, to a two-week intensive summer course and continuing support meetings during the academic year.

Learning pedagogical knowledge

In addition to participating in ‘doing statistics’, in the same way statisticians would, teachers have to consider a wider context of factors associated with teacher performance: knowledge of content, pedagogy, curriculum, learners, education context, and education ends, purposes and values (Shulman, 1987, p. 8). In a learning environment similar to that of the classroom environment, I incorporate the following activities in teachers’ workshops.

- I use segments from *A* and *D*’s videos, to educate teachers about students’ knowledge and dispositions, development, difficulties, and the like.
- I analyze student-teacher interactions using transcripts from this study, to draw their attention to the different role of the teachers in this educational environment.
- I gradually assist teachers to become computer literate, and gain basic knowledge and confidence in using spreadsheets to analyze data.
- Skillful teachers who taught in the experimental classes reinforce the workshop’s staff. They share their experiences, success stories, mistakes and failures, which are most instructive in the applied arena.
- Finally, and most importantly, I provide ongoing support to teachers during the initial implementation period in their classes. It included periodical meetings to share experiences, discuss theoretical and practical issues, and ongoing support via phone or email.

Conclusion 5

Use of Computerized Tools in Teaching and Learning EDA

The computer in the *SC* approach is a powerful tool to mediate and support students' construction of meanings for statistical conceptions and perspectives, rather than a learning goal in itself. It can support learning in the form of amplification and reorganization (Pea, 1987; Dörfler, 1993) and experiencing a new 'mathematical (statistical) realism', namely, "*the experience of direct manipulation of mathematical objects and relations*" (Balacheff and Kaput, 1996, p. 470).

The data show that our attempt at serious integration of relevant computerized tools (an educational statistical package first, and then, spreadsheets) in the *SC* brought about a cascade of changes in the instructional materials, classroom praxis, students and teacher's interactions, and students' ways of learning. The computer changed assumptions about what can be learned, and, as a result, facilitated the re-designing of the curriculum. Using the computer to investigate data, students acted directly on statistical objects and relations through various linked representations. They produced, interpreted, transformed, and used data representations, and discussed the meanings of their actions and motives. The computer's ability to operate (plot, calculate, sort, etc.) quickly and accurately, dynamically link multiple representations, simplify procedures, provide immediate feedback to the user, and transform a whole representation into a manipulable object, contributed to the students' sense-making processes.

Theoretical implications

One of the greatest obstacles in integrating technology in statistics education is related to teachers' professional knowledge. For teachers, traditional professional knowledge is not sufficient to deal with the complexities in the teaching/learning situation created by the technological revolution. We need more experimental research and theoretical analysis to identify the differences between traditional teaching and computer-based teaching, to explore how statistics teachers can prepare themselves to function in the new instructional environments, and to develop appropriate forms of professional development and support systems.

The knowledge base regarding how to exploit the power of technology in statistics education is rather sparse. Clearly, enormous work lies ahead of us.

> "*In challenging most traditional assumptions about teaching and learning, technology forces us to think deeply about all aspects of our work, including the forms of the research that need to be undertaken to use it to best advantage.*" (Balacheff & Kaput, 1996, p. 495).

Educational implications

Over the last two decades, we have seen significant progress in technological tools for statistics education, which have become more powerful, flexible and efficient, with friendlier interfaces, and better connectivity via the Internet. Despite all this progress and promise, the penetration of these technologies in educational practice proves to be very slow and with great disparity from place to place. The shortage in technology in schools is surely one reason, though with the recent introduction of smaller and powerful machines, like graphing calculators and palm-top computers, tools will soon become common and cheap. However, the limited commitment of teachers and curriculum developers, and the great ignorance about teaching and learning in computer-based environments are at least as important factors contributing to the scarcity of actual use in classrooms.

Educators should be encouraged to view technological tools as legitimate extensions of cognitive systems and partners in the socio-cultural arena of the statistics classroom. Using these technologies as a cognitive tool and a medium will not weaken cognition. The opposite is true. It opens up the opportunity for the development of a much richer, powerful and flexible learning environment in which students are active learners of statistics. This is a demanding task for students and teachers. Like any other symbolic technology the use of the computer must be learned and integrated into the learning environment through considered integration and extended experience. The following considerations should be taken into account to deal with the changes in learning and teaching that are emerging.

Choosing appropriate tools

How can teachers and curriculum developers of introductory statistics courses navigate within the rich universe of existing software? Biehler (1997) suggests the following list of priorities for software supporting introductory statistics education, starting with:

1. student tools whose learning and use can be integrated into an introductory course;
2. resources of data;
3. microworlds;
4. further resources (Internet, electronic books, multimedia resources); and
5. tutorial shells (pp. 168-169).

In addition, teachers' meta-tool functions should be included at all levels to allow adaptations for the specific class.

Critical evaluation of software

The statistics education community needs to critically evaluate existing software and to produce better software in the future. The main problems of existing statistical software are:

1. the complexity of professional systems, which cause a high cognitive entry cost;
2. the constraints of microworlds, which limit their adaptability; and
3. the uncoordinated interfaces, concepts and notations of various microworlds and tools, which limit the possibility of combining a few tools in the same course.

What we ideally need is an integrated system of coordinated tools for educational purposes which is adaptable, extensible, and simple. The *Fathom* project (Finzer, Erickson, & Binker, 2000) represent efforts to design educational software in these directions that include a modelling, simulation and data analysis tools, and dynamic-interactive linkages between them.

Educational use of technological tools

Obviously, it is not enough to provide students with a powerful tool, with nice graphical representations and friendly interface, to ensure that meaningful learning will occur. In rich computer-based learning environments students should be encouraged to at least:

1. Practice data analysis with an exploratory, interactive, open-ended working style, and combine exploratory and inferential methods, graphical and numerical methods.
2. Extensively use multiple linked representations and simulations to construct meanings for statistical concepts and ideas.

Curriculum design

Learning activities supported by powerful computerized tools have to be designed so that students can develop their usage of tools to promote statistics learning on higher cognitive and socio-cognitive levels. Such a statistics curriculum, which takes advantage of the technology, should stress conceptual understanding, statistical modelling and problem solving, real-world applications, and new methods of analyzing data.

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APPENDICES

Appendix I **Sequence and types of data collection activities**

Appendix II ***A* and *D*'s statistical explorations sequence**

Appendix III **The *Statistics Curriculum* student textbook**

Appendix I

Sequence and Types of Data Collection Activities

1. Pre experiment data:

- 1.1 Whole class observations: Video protocols - Lessons 1-9.
- 1.2 Guided teacher interviews (included in video protocols).
- 1.3 Videotape of other students' mathematical work previous to their learning the *Statistics Curriculum* - Lessons 1-9 (Sep. - Dec., 1996).
- 1.4 Portfolios of *A* and *D* previous to their learning the *Statistics Curriculum* course, summarizing their algebra studies (Sep. - Dec. 1996).

Duration: about four months.

2. Statistics prior-knowledge data (Warm-up session):

- Protocol of class work on an introductory EDA task (27/12/96).

Duration: 1 lesson.

3. Statistical exploration data:

- 3.1 Videotapes of *A* and *D*'s work on the *Statistics Curriculum* in the computer lab, and whole class discussions at the beginning and end of some of the lessons.

Videotape list

Tape No.	Date	Problem-situation Content
1	31/12/96	Unit 2/1: *The Same Song with a Different Tune*
2	7/1/97	Unit 2/2: *The Same Song with a Different Tune*
3	14/1/97	Unit 3/1: *Names*
5	21/1/97	Unit 3/2-3: *Names*
8	28/1/97	Unit 4/1: *Work Dispute*
11	4/2/97	Unit 4/2: *Work Dispute*
12	11/2/97	Unit 5: *Cars*
17	18/2/97	'Research project'
18	25/2/97	'Research project'
20	11/3/97	Final assessment: *Immigration to Israel*

- 3.2 *A* and *D*'s statistics notebooks.
- 3.3 Protocols of interviews with the three mathematics teachers before and after the lessons.
- 3.4 Protocols of the meetings between the teachers and the researcher after the lessons.
- 3.5 Classroom observers' notes.
- 3.6 Researcher's notes.

Duration: about 25 lessons.

4. Post experiment data:

- 4.1 Students' 'research projects': About 30 projects of three experimental classes, including *A* and *D*'s project.
- 4.2 Final assessment task (*Immigration to Israel*) - 11/3/97: Students' reports and spreadsheet files.
- 4.3 Students' evaluation forms.

Duration: about 6 lessons.

5. Delayed post data

- 5.1 Videotape of *D*'s interview regarding their work on the 'research project' (27/3/98)
- 5.2 Videotape of *A* and *D* observing selected video recordings of protocols of their own work (one year and 2.5 years after the end of the *SC*).

Videotape list

Tape No.	Date	Discussion Content
18	13/8/97	*Work Dispute* (delayed post I)
29	29/8/99	*The Same Song with a Different Tune* (delayed post II)

6. Additional data

- 6.1 Students' 'research projects's from *Be'eri* (May 1996), *Ma'ale-Habesor* (1997), and *Leyada* (1997) schools.

- 6.2 Videotape of students' work on the *Statistics Curriculum* in other schools.

Videotape list

Tape No.	Date	Content
04-046	4-5/94	First experiment in *Be'eri'*s school (20 hours).
052-057	5-6/95	Second experiment in *Ma'ale Habesor* school: Focus on one pair of students (*K* and *S*) (20 hours).

- 6.3 Videotape of 'special' whole class sessions:

Videotape list

Tape No.	Date	Content
062	27/3/95	*Work Dispute* - final debate: Chanah's class
071	16/6/95	Statistical Happening in *Leyada* + teacher's interview (Gilead Amir)
072	1/96	*Work Dispute* - final debate: Chanah's class (1996)
10	3/2/97	*Work Dispute* - final debate: Gilah's class
11	3/2/97	*Work Dispute* - final debate: Chanah's class (1997)
19	4/2/98	*Work Dispute* - final debate: Michal's class
22	27/3/98	*Statistical happening*: Chanah's students presenting their 'research project'
23	27/3/98	*Statistical happening*: General presentation and Gila's students presenting their 'research project'

Appendix II

A and *D*'s Statistical Explorations Sequence

Lesson Number	Computer	Problem-Situation	Statistical Concepts and Skills[26]
1		1: Introduction to statistics	What is statistics? The PCAIC Cycle, experience a small-scale investigation.
2-3	√	$2/_1$: *The Same Song with a Different Tune* (a) $2/_2$: *The Same Song with a Different Tune* (b)	Hypothesis, data table, case, time plot, trend, gaps, interpret table and graph, sort data. Manipulate graph to support a claim, scaling, line and symbol time plot, and outlier.
4-5		$2/_{A3}$: *Rain*	Bar chart, compare distributions.
6-7	√	$2/_{A1}$: *Olympic Records*	Phrase research questions and hypotheses, and interpret a table and a graph.
8		*Shoe Color*	Categorical variable, mode, collect data.
9-10	√	$3/_1$: *Names* - Frequencies	Frequency, relative frequency, frequency table, grouping, percentage, compare distributions.
11-12		*Shoe Size* and *Student Height*	Quantitative (discrete and continuous) variable, statistical measures (mean, median, mode and range).
13-14	√	$3/_2$: *Names* (cont.) - Statistical measures $3/_3$: *Names* (cont.) - Graphical analysis	Mean, median, range, and outliers. Bivariate frequency table, pie and bar charts, compare data representations.
15		3: *Names* (cont.) - Summary	Communicate conclusions, compare data representations, and choose the appropriate average.
16-17	√	$4/_1$: *Work Dispute* - Current situation	Distribution, grouped frequency table, histogram, interval width, use of averages, synthesis of methods.

[26] Not all the concepts and skills in this table were explicitly presented in class, but rather served the teacher and the curriculum developers as the underlying statistical knowledge.

Lesson Number	Computer	Problem-Situation	Statistical Concepts and Skills[27]
18		4/$_1$: *Work Dispute* (cont.): Preparations	Take a stand in a debate.
19-20		3/$_{A4+A5}$: *Weights* and additional tasks 'Research project': Introduction	Averages. Plan an investigation, project assessment criteria.
21-22	√	4/$_{2+3}$: *Work Dispute* (cont.) - Preparing a proposal of a new salary distribution	Consider different proposals to distribute the money, design representations to introduce the chosen proposal.
23		4: *Work Dispute* (cont.) - Final debate	Communicate views based on statistical methods; present and argue conclusions.
24-25	√	5: *Cars*	Relationship, positive and negative association, (least square) regression, fit a line to data, regression line, prediction, extrapolation, scatterplot, the correlation coefficient, and causation vs. association.
26-27	√	*'Research project'*	Identify a problem and a question to investigate, suggest hypotheses, design the investigation, collect and analyze data, interpret the results and draw conclusions.
28-29	√	*'Research project'* (cont.)	
30-31		*Statistical 'Happening'*	Present the results of the 'research projects'.

Appendix III

The *Statistics Curriculum* Student Textbook

Ben-Zvi, D., & Friedlander, A. (1997a). *Statistical Investigations with Spreadsheets* (student textbook, in Hebrew). Rehovot, Israel: Weizmann Institute of Science.

Note: The book is enclosed with this thesis.

[27] Not all the concepts and skills in this table were explicitly presented in class, but rather served the teacher and the curriculum developers as the underlying statistical knowledge.