PONTIFICAL CATHOLIC UNIVERSITY OF VALPARAÍSO

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DISSERTATION

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THE TABLE OBJECT: AN EPISTEMOLOGICAL, COGNITIVE AND DIDACTIC STUDY

Thesis directed by

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INTRODUCTION

This dissertation focuses on the table as a learning goal in the first years of schooling.

The author, based on professional motivation and experience, addresses the difficulties in learning and teaching tables by first entering into a historical epistemological study of the issue, a substantial task in the Pontifical Catholic University of Valparaiso Doctorate Program in Didactics of Mathematics, and is able to unravel the constituent elements of the table as a rectangular network and its social uses as regards types of interpretations. Additionally, the author examines the status of the table in a school context according to recollection and exploratory analysis of data.

The dissertation then establishes the context of demands that society currently places on the curriculum, and in this context, based on currently dominant tendencies in statistics education, provides guidelines for dealing with tables in the classroom and maintaining high cognitive demands during the process -Stein and Schmidt-, characterizing in detail the underlying semiotic representations and theoretical conceptual structures -Vergnaud- that emerge in the process of conceptualization of the table in the first years of school. The learning method for statistical reasoning -Garfield and Ben-Zvi-, and in particular *transnumeration* -Wild and Pfannkuch- emerge as paradigms that accommodate the teaching proposal, meeting the social demand for curricular renovation and the international tendency to exceed the national status quo of arithmetization in statistics education.

This dissertation makes evident the *paramathematical* status of tables in the national curriculum, which sets tables aside as a tool and does not consider them a teaching object. The dissertation also elucidates the cognitive demands that studying tables makes on students, exploring in light of this an *ad hoc* table taxonomy and a proposal for critical education aimed at developing competence in representation and analysis of data for decision making.

SUMMARY

Chapter one presents the author's interests in statistical education and the research problem. It also presents the research questions and their goals.

Chapter two presents a review of the specialized literature pertinent to tables in statistics, focused on the learning difficulties that the table format presents and the implications for teaching. Additionally, it introduces the theoretical framework for the cognitive and didactic aspects of the research.

Chapter three offers a panoramic vision of the historic process of evolution of ideas about tables, their connotations as a human tool, and their emergence and development in diverse cultures and various moments in history, issues that contribute to understanding this object and its didactic reach. This chapter shows us the table and its presence as a storage tool, a calculation tool in numbering and meteorological systems, an analysis tool in scientific and mathematical fields, and its relation with the creation of numbers and the concept of a function.

Chapter four provides the scholastic status of tables. It addresses an epistemological goal regarding tables as a significant element in the analysis of the circulation of knowledge and its normalization. Going into greater depth in characteristics of the table format based on information science and statistics, a generic model for the table is proposed. This chapter investigates cognitive aspects, as it studies tables as representations that support the construction of meanings for data and identifies subjects' roles and cognitive processes associated with statistical tables. The chapter ends with a study of the role of tables in the list items of an international primary school test and its status in the current curriculum in Chile and three other countries.

Chapter five presents four studies that give us an approximation of the understanding of table learning at school level. It begins with an analysis of the evidence that emerges from student productions and instructor management in task demands, in a data analysis situation. It continues with the characterization of the types of interpretation that dealing with tables

demands, proposing categories to create a hierarchy of understanding specific to tables, which finally will be tested.

Chapter six completes the work of the dissertation with the conclusions and findings for a first exploration of the progressive dominance of the conceptualization of tables by students in the first years of school, supported by the theoretical references and collected results.

In order to offer the reader a panoramic view, each chapter begins with an ordered list of the contents that we have titled "Chapter Summary".