The Preparedness of Preservice Secondary Mathematics Teachers to Teach Statistics: A Cross-Institutional Mixed Methods Study

by
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A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Mathematics Education

Raleigh, North Carolina

2016

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BIOGRAPHY

Jennifer Nickell Lovett was born in Hamilton, OH on August 21, 1983. She is the daughter of Gary and Judy Nickell. She graduated from Fairfield High School and attended Miami University. She graduated from Miami University in May 2005 with a Bachelor of Science in mathematics education and a Bachelor of Arts in mathematics and statistics. In 2005 she was hired at Edgewood Middle School in Trenton, Ohio. During this time, Jennifer continued teaching full time while taking graduate courses in the summer. She completed her Master of Arts of Teaching in mathematics at Miami University in August 2007. That August she was hired at Lakota West Freshman School, where she served as the associate department chair for mathematics and met her husband, a fellow teacher.

In August 2012, she moved from Ohio to Raleigh to pursue her doctoral degree at NC State. During this time, Jennifer served as a teaching/research assistant, teaching the *Teaching Mathematics with Technology* and *Introduction to Mathematics Education* courses, serving as a teaching assistant for the *Introductory Statistics* course, and working with the Preparing to Teach Mathematics with Technology (PTMT) project under the direction of Dr. Hollylynne Lee, Dr. Karen Hollebrands, and Dr. Allison McCulloch and the Teaching Statistics Through Data Investigations MOOC-Ed team under the direction of Dr. Hollylynne Lee. In 2015, Jennifer married Kyle Lovett.

Upon receiving her degree Jennifer plans to continue examining effective ways to prepare preservice teachers to teach statistics and effectively implement technology into their classrooms. Jennifer has accepted an Assistant Professor position in the Department of Mathematical Sciences at Middle Tennessee State University in Murfreesboro, Tennessee to begin in the fall of 2016.

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude and appreciate to many people that have supported and encouraged me during this journey:

To my advisor and committee chair, Dr. Hollylynne Lee, thank you for the countless hours of guidance, support, and mentorship helping me grow into a mathematics teacher educator and researcher. Thank you for always setting high standards for me and not accepting anything other than my best. I appreciate all the time you have invested in me over the last four years and many of my accomplishments would not have been possible without your help. I look forward to us continuing to work together as colleagues.

To the members of my committee, Dr. Allison McCulloch, Dr. Webster West, and Dr. Temple Walkowiak, thank you for all of your feedback and help during this process. Webster, thank you for allowing me to be your teaching assistant for introductory statistics. It was an invaluable experience in my development as a statistics educator. Allison, thank you for all the support and advice you have provided me over the years. I am glad I had the opportunity to work with you closely on the algebra module and look forward to continuing our research on it.

To Dr. Helen Doerr, an honorary committee member, thank you for the hours you spent (that you did not have to) helping me strengthen my research skills. What I was able to accomplish with these three articles is largely due to your time and patience teaching me about analyzing qualitative data, formulating a "story" from the data, and preparing a research manuscript. I learned so much from working with you!

Thank you to all my professors and classmates that have helped me grow as a mathematics teacher educator. A special thanks to Emily Thrasher for the numerous

conversations helping me in organizing my thoughts and Ashley Whitehead and Nick Fortune for the endless laughs that kept me sane during this process.

Thank you to my family for their love and support. And finally to my husband, Kyle, thank you for moving to North Carolina so we could be together while I pursued my dream. I am very grateful for all the sacrifices you have made for us. I have been lucky to have your patience, your encouragement, and confidence in me, even when I did not believe in myself, and, most importantly, your love, during this time. I cannot wait for our next adventure!

The work in this dissertation was partially supported by the *Preparing to Teach Mathematics with Technology* project, funded by the National Science Foundation with grants to NC State University (DUE 0442319, DUE 0817253, and DUE 1123001). The opinions, findings, and conclusions or recommendations in this dissertation are my own, and do not necessarily reflect the views of the National Science Foundation.

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Chapter 1: Introduction

As of 2015, most states have either adopted the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010) or modified their previously adopted standards to align with CCSSM (Academic Benchmarks, 2015). While the National Council of Teachers of Mathematics has long advocated for the inclusion of statistics and probability in middle and high school mathematics curricula (1989, 2000), the CCSSM has increased the emphasis on statistics in these grade levels. However research has shown that inservice teachers are not prepared to teach statistics (e.g., Burrill & Biehler, 2011; Makar & Fielding-Wells, 2011) and struggle understanding how the statistical content should progress across grade levels (Jones & Tarr, 2010). When teachers do teach statistics, it is often taught procedurally, focusing on computations of statistical measures (Makar & Confrey, 2004) and creating graphical representations (Sorto, 2006).

One of the fundamental recommendations from both the K-12 and College level *Guidelines for Assessment and Instruction in Statistics Education* (Franklin et al, 2007; Garfield et al, 2007) is that conceptual understanding of statistical ideas should be emphasized over procedures and computations. In order for teachers to change their practice and enact this recommendation, they themselves must develop a conceptual understanding of the statistical content they are expected to teach (Franklin et al., 2015). If teachers do not have a conceptual understanding of statistics, they are not likely to have the knowledge they need to help students learn statistics content (Groth, 2013). Since students who enroll in secondary mathematics teacher education programs in the U.S. have likely had minimal experience with statistics in their own K-12 education, they have not had many opportunities

to develop strong statistics understanding and the confidence to teach the expected topics in high school as compared to other areas of mathematics. Thus the aim of this study is to understand better the preparedness of preservice secondary mathematics teachers to teach statistics so as to inform future research and teacher education programs.

Even with the increased emphasis of statistics in the K-12 curriculum over the last 25 years, there has been little research about the statistical knowledge of PSMTs or the misconceptions they develop (Batanero, Burrill, & Reading, 2011; Shaughnessy, 2007), even recently with the increased emphasis on statistics in high school mathematics with the adoption of CCSSM. The majority of research on preservice teachers' statistical knowledge has focused on elementary teachers (e.g., Browning, Goss, & Smith, 2014; Groth & Bergner, 2006; Hu, 2015; Leavy, 2010; Leavy & O'Loughlin, 2006; Santos & da Ponte, 2014). The limited research conducted on PSMTs' statistical knowledge has been small-scale studies, from a small number of institutions on specific statistical content (e.g., Doerr & Jacob, 2011; Lesser, Wagler, & Abormegah, 2014; Makar & Confrey, 2005). For example, a recent study by Casey and Wasserman (2015) examined 11 preservice teachers' statistical knowledge of informal lines of best fit from three universities. From these studies research has shown that preservice secondary teachers focus on procedures, computations, and algorithms lack statistical reasoning skills, and have difficulty interpreting graphical representations.

Along with this statistical knowledge, teachers need pedagogical statistical knowledge "to assess students' level of understanding and plan next steps in the development of their statistical thinking" (Franklin et al. 2015, p. 3). This type of knowledge refers to a teacher's knowledge of potential student difficulties with statistics, developing strategies to support student's learning, teaching strategies to engage students in the statistical

investigative cycle, and vertical and horizontal knowledge of the statistics and mathematics curricula (Ball, Thames, & Phelps, 2008; Groth, 2013).

While examining knowledge needed to teach is important, researchers should also consider the non-cognitive aspects that teachers draw upon and how these are related to a teacher's preparedness to teach statistics (Ball et al., 2008; Fennema & Franke, 1992). Teachers' affect plays a crucial role in the pedagogical approaches they use and the time spent on a subject and thus can impact students' learning (e.g., Love & Kruger, 2005; Pajares, 1992; Wilkins, 2008). Affect includes a teacher's beliefs, attitudes, and emotions towards statistics. However there is a lack of research on secondary teachers' affect in regards to teaching statistics (Batanero et al., 2011). Again the limited research that has been conducted has been with elementary teachers. Researchers have found that a teacher's beliefs and attitudes towards statistics were related to their prior experiences with statistics and impact the choice of instructional tasks and students' attitudes and beliefs towards statistics (Begg & Edwards, 1999; Eichler, 2008; Lancaster, 2008). Since a teacher's beliefs and attitudes play a large role, it is crucial when considering PSMTs' preparedness to teach statistics that PSMTs' affect as well as statistical knowledge is examined.

Significance of the Study and Research Questions

To prepare PSMTs to succeed in teaching statistics standards in CCSSM, the *Mathematics Education of Teachers II* report (Conference Board of the Mathematical Sciences, 2012) and the recent *Statistical Education of Teachers* (SET) report (Franklin et al., 2015) presented recommendations for courses that should be included in secondary mathematics teacher education programs to develop statistical knowledge and pedagogy needed to teach statistics. In addition, the last decade has included recommendations to assist

college faculty to reform statistics courses, particularly at the introductory statistics level (e.g., Cobb, 2015; Garfield et al., 2007). Therefore mathematics teacher education programs are faced with the enormous challenge of implementing these recommendations in an already crowded curriculum of preparing teachers for the myriad of responsibilities in teaching secondary mathematics.

To assist mathematics teacher education programs in implementing these recommendations, information about the current state of PSMTs' statistical knowledge, the nuanced nature of their understandings, including strengths and weaknesses, and their beliefs and confidence in their abilities to teach statistics is needed. Insight into PSMTs' understandings and confidence in teaching statistics can inform strategic changes to teacher education programs.

This study aims to describe the preparedness of PSMTs at the conclusion of their teacher education program before entering student teaching. More specifically, the purpose of this study is to examine preservice mathematics teachers' statistical knowledge and statistics teaching efficacy and determine which factors and experiences influence them. The following research questions will be addressed:

1. What is the preparedness of preservice secondary mathematics teachers [PSMT] to teach statistics as they enter student teaching?
    a. What is PSMTs' statistical knowledge of the high school content they are expected to teach using the phases of a statistical investigation?
    b. What is PSMTs' statistics teaching efficacy for high school? What particular topics or levels of statistics do they feel more or less prepared to teach?

c. What self-reported factors (e.g., gender, number of statistics courses taken, whether or not they took AP Statistics in high school) influence PSMTs' statistical knowledge and statistics teaching efficacy?

d. What is the relationship between PSMTs' statistical knowledge and statistics teaching efficacy?

2. From the perspectives of PSMTs, what factors and experiences influence their preparedness to teach statistics?

Through analyzing the data to answer these research questions, the main goals of this study are to describe the current state of preparedness of PSMTs to teach statistics and investigate the factors and experiences that they feel influenced their preparedness.

Overview of Methodological Approach

The purpose of this two-phase, explanatory sequential mixed methods study was to obtain descriptive quantitative results from a large, cross-institutional sample of PSMTs and then follow up with a subset of individuals to explain those results in greater depth. In the first quantitative phase of the study, participants' statistical knowledge and statistics teaching efficacy was examined through two instruments. The second, qualitative phase was conducted as a follow up to help explain the quantitative results. In this follow-up semi-structured interviews and open-ended survey responses were analyzed to explore factors and experiences that influence PSMTs' preparedness to teach high school statistics.

Definition of Terms

It is worthwhile to define several terms used in this study so they are interpreted consistently throughout.

Preservice teacher – student in a teacher education program at an institution pursuing a teaching license

Preservice secondary mathematics teacher (PSMT) – student in a mathematics teacher education program at an institution pursing a teaching license that includes grades 9 – 12

Statistical knowledge – knowledge of statistics across three levels of development and four phases of the statistical investigative cycle

Pedagogical statistical knowledge – teacher's knowledge of potential student difficulties with statistics, developing strategies to support student's learning, teaching strategies to engage students in the statistical investigative cycle, and vertical and horizontal knowledge of the statistics and mathematics curricula (Ball et al., 2008; Groth, 2013)

Statistical knowledge for teaching – statistical knowledge and pedagogical statistical knowledge teachers need to teach statistics (Groth, 2007)

Statistics teaching efficacy – a teacher's belief in his/her ability to teach statistics to bring about student learning

Organization of the Document

In Chapter 2 a review of relevant literature will be discussed. This includes frameworks and literature on teachers' knowledge needed to teach mathematics and statistics, literature on teacher's beliefs including: self-efficacy and teacher efficacy, and literature of context specific teacher efficacy. Finally, in Chapter 2 the conceptual framework that will guide the study will be presented and discussed. Chapter 3 will present the research questions linked to the conceptual framework and a detailed discussion of the methodology including: the research design, methods for data collection, and data analysis. Chapters 4, 5, and 6

contain journal-ready articles that answer the research questions. Finally, Chapter 7 provides the overall answers to the research questions and implications for future research.

Chapter 2: Literature Review and Conceptual Framework

This study aimed to examine PSMTs' preparedness to teach statistics. In order to examine preparedness to teach statistics, this chapter reviews literature on what is needed for teachers to be prepared to teach statistics and what we may already know about teachers' preparedness for teaching statistics. To be prepared to teach, teachers draw upon their knowledge for teaching. To teach statistics, PSMTs need to develop mathematical knowledge for teaching and statistical knowledge for teaching, therefore frameworks for these types of knowledge as well as literature about teachers' knowledge of statistics will be reviewed. Teachers' beliefs, specifically their self-efficacy, are intertwined with teachers' knowledge and influence their feeling of preparedness (Thompson, 1992). Therefore a review of teachers' beliefs and self-efficacy is provided. Finally, the conceptual framework that is used to guide the study is described.

Mathematical Knowledge for Teaching

Over the last 40 years, teachers' knowledge for teaching mathematics has been a hot topic in research. An early focus on teachers' knowledge was on content knowledge and the number of courses teachers completed at the university level. However research showed that there was little evidence that teachers' content knowledge impacted student achievement (Begle, 1972). An explanation for this was the imprecise definition of teachers' knowledge and how it was being measured (Hill, Rowan, & Ball, 2005). Thus a shift began focusing on the impact of a teacher's pedagogical knowledge on student achievement. This research paradigm was missing a key component, what Shulman called the "missing paradigm" (Shulman, 1986, p. 6). The missing paradigm referred to a teacher's subject matter knowledge.

During the 1980s, emphasis shifted to blending the ideas of content and pedagogical knowledge as knowledge needed to teach (Shulman, 1986). In his 1986 presidential address delivered to the American Education Research Association, Shulman proposed a framework for teacher subject matter knowledge comprised of three components: content knowledge, pedagogical content knowledge (PCK), and curricular knowledge. Shulman (1986) defined content knowledge as "the amount and organization of knowledge per se in the mind of the teacher" (p.9). The second component, which was a new construct for the field of teacher knowledge, pedagogical content knowledge was defined as knowledge "which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching" (p.9). Finally, the third component curricular knowledge represented horizontal and vertical knowledge of the curriculum for a topic.

With this framework, Shulman directed researchers to focus again on teachers' content knowledge. This framework was foundational for frameworks and research in teacher knowledge for the last thirty years (Hill et al., 2005). Building off Shulman's work, Ball and her colleagues introduced the construct of mathematical knowledge for teaching (MKT; Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Hill et al., 2005). Through their qualitative work, Ball et al. (2008) identified six components of MKT (Figure 1).

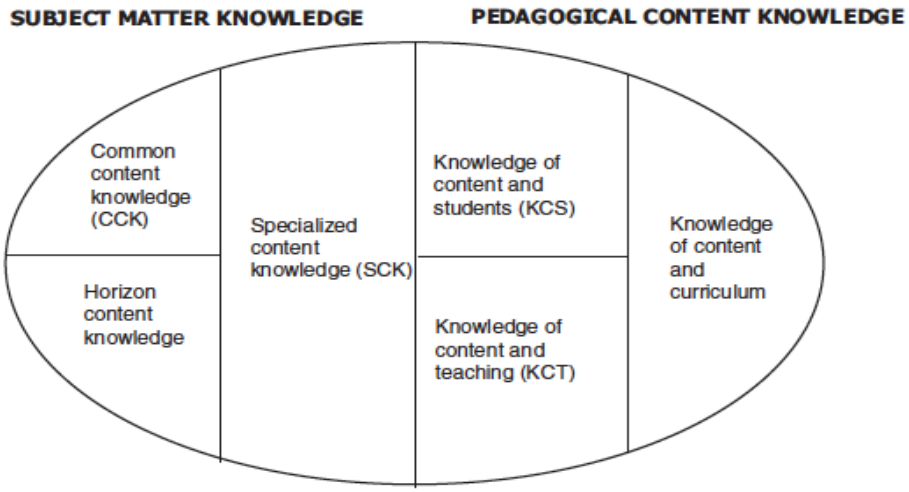

Figure 1. Ball et al.'s (2008) framework for mathematical knowledge for teaching.

In this framework teachers' knowledge is considered to have two parts: subject matter knowledge and pedagogical content knowledge. Subject matter knowledge consists of three types: common content knowledge, horizon content knowledge and specialized content knowledge. Pedagogical content knowledge also consists of three types: knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. However, unlike Shulman, Ball and colleagues refer to specialized content knowledge as mathematical knowledge and not pedagogy (Hill et al., 2005). Table 1 defines Ball and colleagues' (2008) six components of teachers' knowledge.

Table 1. *Six components of Ball et al. (2008) framework*

Component	Definition
Common content knowledge	Mathematical knowledge and skills used in settings other than teaching
Specialized content knowledge	Mathematical knowledge unique to teaching (e.g., looking for patterns in students errors, examining a nonstandard approach)
Horizon knowledge	Knowledge of how the mathematical topics are related over the span of mathematics and statistics
Knowledge of content and students	Knowledge to anticipate students' thinking and misconceptions in mathematics
Knowledge of content and teaching	Knowledge need to sequence instruction and appropriate teaching practices for mathematics
Knowledge of content and curriculum	Vertical and horizontal knowledge of mathematics content and curriculum

Statistical knowledge for teaching. With the increased emphasis on statistics in the K-12 curriculum (National Council of Teachers of Mathematics, 1989, 2000; National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010), mathematics teachers need to be prepared to teach statistical concepts as well. Many researchers have discussed the differences between mathematics and statistics (Cobb & Moore, 1997; delMas, 2004; Gal & Garfield, 1997; Moore, 1988; Rossman, Chance, & Medina, 2006). Since mathematics and statistics are distinct disciplines, MKT differs from statistical knowledge for teaching (SKT). Thus it is crucial to acknowledge the difference between MKT and SKT when preparing teachers (Groth, 2007).

Within the last ten years researchers have begun to examine the SKT that mathematics teachers need to develop to teach statistics effectively (Burgess, 2006; Godino, Ortiz, Roa, & Wilhelmi, 2011; Groth, 2007, 2013; Lee & Hollebrands, 2011; Pfannkuch & Ben-Zvi, 2011; Ponte, 2011; Watson, Callingham, & Nathan, 2009). Frameworks that have emerged have built off Ball and colleagues' (2008) components of MKT since statistics

utilizes mathematics and is traditionally taught by mathematics teachers at the K-12 level (Groth, 2007).

An initial step towards developing a framework for SKT was proposed by Burgess (2006). His framework described components of SKT and elements of statistical thinking needed to teach statistics through investigations (Figure 2). Burgess' (2006) framework is a two-dimensional matrix where the columns are the four types of knowledge identified by Ball and Hill (2005): common knowledge of content, specialized knowledge of content, knowledge of content and students, and knowledge of content and teaching. The rows are elements of statistical thinking and empirical inquiry described by Wild and Pfannkuch (1999): need for data, transnumeration, variation, reasoning with models, integration of statistical and contextual thinking, investigative cycle, interrogative cycle, and dispositions. Research conducted in secondary classrooms on Burgess' framework has shown that all aspects of knowledge included in his framework are needed in the classroom (Burgess, 2009).

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration				
	Variation				
	Reasoning with models				
	Integration of statistical and contextual				
Investigative cycle					
Interrogative cycle					
Dispositions					

Figure 2. Components of statistical knowledge for teaching according to Burgess (2006).

Groth (2007) hypothesized a framework that acknowledges the differences between mathematical knowledge for teaching and statistical knowledge for teaching from ideas of Hill, Schilling, and Ball (2004) using the components common content and specialized content knowledge (Figure 3). In this framework, Groth uses the four phases of a statistical investigation: formulating questions, collecting data, analyzing data, and interpreting results, to identify components of common and specialized knowledge for statistical problem solving at the high school level. For each phase of the investigative cycle, Groth describes common mathematical knowledge, common statistical knowledge, specialized mathematical knowledge, and specialized statistical knowledge. For example, in terms of interpreting results, common mathematical knowledge would be correctly interpreting the mathematical meaning of the p-value. The common statistical knowledge would be judging the appropriateness of the significance level chosen by the researcher. However this knowledge is not enough for teachers to help their students understand how to interpret the results of a hypothesis test. Teachers also need specialized mathematical and specialized statistical knowledge. In this case specialized mathematical knowledge refers to understandings students' interpretations of the concept of the p-value and the specialized statistical knowledge would be anticipating students' over generalizations of the term "significant."

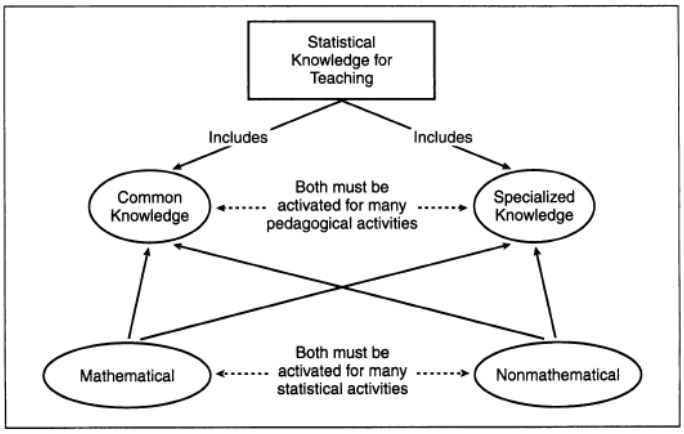

Figure 3. Groth's (2007) hypothesized structure of statistical knowledge for teaching.

Groth (2013) expanded his 2007 framework for statistical knowledge for teaching to include aspects of pedagogical content knowledge from Ball et al. (2008). He also included the idea of key developmental understandings (Simon, 2006), and the notion of pedagogically powerful ideas (Silverman & Thompson, 2008) to create a developmental structure to the framework. In this framework, Groth has re-categorized knowledge of content and students from subject matter knowledge to pedagogical content knowledge and include aspects of Ball's et al. (2008) framework not previously included in his 2007 framework (Figure 4). Groth also proposed a developmental structure of statistical knowledge for teaching. Developing key understandings of statistical knowledge is foundational for developing statistical knowledge of teaching. The framework shows that once a teacher has personal experience with key developmental understandings, he/she can then develop pedagogical content knowledge in terms of pedagogically powerful ideas. This hypothetical developmental structure adds to the understanding of statistical knowledge for teaching and provides a development structure.

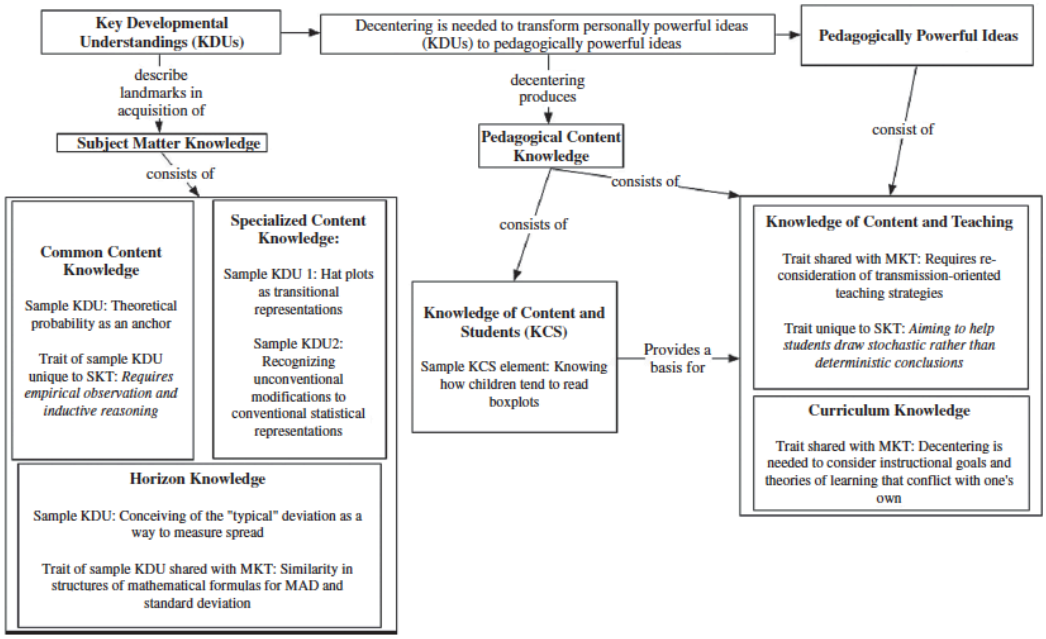

Figure 4. Hypothetical statistical knowledge for teaching framework Groth (2013).

Key differences in MKT and SKT emerge from these frameworks. To teach statistics, mathematics teachers need to develop mathematical as well as statistical common and specialized content knowledge. They also need to develop specific pedagogical approaches to teach statistics such as the statistical investigative cycle (Wild & Pfannkuch, 1999). Teachers also need to see the benefits in teaching mathematics and statistics together. Statistics can provide real-world examples and real data that can strengthen a mathematics class by supporting and illustrating mathematical topics (Scheaffer, 2006). Thus mathematics teachers need to develop MKT and SKT to be prepared to teach statistics.

Statistical knowledge. As previously mentioned, Groth's (2013) framework proposes a development structure for statistical knowledge for teaching. This core of this framework is that teachers need to develop key developmental understandings and subject matter knowledge (common content knowledge) before they can develop rich pedagogical statistical knowledge. The content knowledge that teachers will be teaching and they need to develop is

rooted in the *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) K-12 report (Franklin et al., 2007).

To complement the NCTM (2000) *Principles and Standards*, the American Statistical Association (ASA) commissioned the GAISE K-12 report. The GAISE report provides a framework for the statistics education curriculum in K-12. This framework was needed because from *Principles and Standards,* many teachers did not see statistics as a "cohesive and coherent curriculum strand" or "a developmental sequence of learning experiences" (Franklin et al., 2007, p. 5). The GAISE framework consists of three levels A, B, and C. Although there are not explicit definitions given for each level, the levels increase in statistical sophistication and become more abstract. Each level is aligned to specific content and the content in level A represents topics for early or novice learners of statistics (no matter what grade level, but often introduced in elementary and middle school), level B represents slightly more complex statistical content (often taught in middle school or early high school), and level C represents more advanced content (typically taught in high school or introductory college courses) (Franklin et al., 2007). Table 2 provides a description of the objectives for each level (Franklin & Kader, 2006).

Table 2. *Objectives for each GAISE level (Franklin & Kader, 2006)*

GAISE Level	Objectives
A	• Students should have opportunities to generate questions about a particular context (such as their classroom) and determine what data might be collected to answer these questions. • Students should learn how to use basic statistical tools to analyze the data and make informal or casual inferences in answering the posed questions. • Students should develop basic ideas of probability in order to support their later use of probability in drawing inferences at Levels B and C.
B	• Students become more aware of the statistical question distinction (a question with an answer based on data that vary versus a question with a deterministic answer). • Students make decisions about what variables to measure and how to measure them in order to address the question posed. • Students use and expand the graphical, tabular and numerical summaries introduced at Level A to investigate more sophisticated problems. • Students develop a basic understanding of the role that probability plays in random selection when selecting a sample and in random assignment when conducting an experiment. • Students investigate problems with more emphasis placed on possible associations among two or more variables and understand how a more sophisticated collection of graphical, tabular and numerical summaries is used to address these questions.
C	• Students should be able to formulate questions and determine how data can be collected and analyzed to provide an answer. • Students should be able to design and implement a data collection plan for statistical studies, including observational studies, sample surveys, and simple comparative experiments. • Students should be able to summarize numerical and categorical data using tables, graphical displays, and numerical summary statistics such as the mean and standard deviation. • Students should understand how sampling distributions (developed through simulation) are used to describe sample-to-sample variability. • Students should be able to describe relationships between two numerical variables using linear regression and the correlation coefficient. • Students should understand the meaning of statistical significance and the difference between statistical significance and practical significance, understand the role of P-values in determining statistical significance and be able to interpret the margin of error associated with an estimate of a population characteristic.

Embedded in each GAISE level, students engage with statistical topics throughout the statistical investigative cycle (Wild & Pfannkuch, 1999). There are four components of the statistical investigative cycle: formulating questions, collecting data, analyzing results, and interpreting the results. Franklin et al. (2007) provides a summary of each phases of the statistical investigative cycle (Table 3).

Table 3. *Description of each phases of a statistical investigative cycle (Franklin et al., 2007)*

Phases of Statistical Investigation	Description
Formulate Questions	• Clarify the problem at hand • Formulate one (or more) questions that can be answered with data
Collect Data	• Design a plan to collect appropriate data • Employ the plan to collect the data
Analyze Data	• Select appropriate graphical and numerical methods • Use these methods to analyze the data
Interpret Results	• Interpret the analysis • Relate the interpretation to the original question

Therefore the GAISE framework consists of two dimensions: 1) four phases of the statistical investigative cycle and 2) the three developmental levels. Figure 5 shows how students progress through the GAISE levels as they continue to engage in the statistical investigative cycle and learn more sophisticated ways to engage in each part of the cycle. Thus PSMTs need statistical knowledge of the four phases of the statistical investigative cycle across the three GAISE levels to be prepared to teach high school students statistics.

Figure 5. Increased sophistication of statistical investigative cycle through GAISE levels.

Preparation of preservice teachers. Mathematics teacher education programs have the challenge to develop preservice teachers' MKT and SKT. This challenge becomes even more difficult since they must prepare preservice teachers to teach statistical content they likely did not experience in their own K-12 mathematics courses (Franklin et al., 2007). In response to the increased emphasis of statistics in CCSSM the Conference Board of the Mathematical Sciences (CBMS) published *The Mathematical Education of Teachers II* describing the courses and instructional outcomes of teacher education programs. One of the essential domains in preparing secondary mathematics teachers is on statistics and probability. CBMS recommends that PSMTs understand how to:

- Summarize, describe, and compare distributions of numerical data in terms of shape, center, and spread.
- Make interpretations of data based on the context.
- Calculate theoretical and experimental probabilities of simple and compound events, and understand why their values may differ.
- Describe statistical variability and its sources, and the role of randomness in statistical inference.
- Explore relationships between two variables by studying patterns in bivariate data.
- Use probability to make decisions (CBMS, 2012).

Therefore, in their coursework preservice teachers should "see real-world data sets, understand what makes a data set good or bad for answering the question at hand, appreciate the omnipresence of variability, and see the quantification and explanation of variability via statistical models that incorporate validity" (CBMS, 2012, p. 58). The preparation preservice teachers have received is often a formal course in the statistics department that may not be appropriate for this purpose. Instead the CBMS recommends "a modern version, given as one course or a two-course sequence, centers around statistical topics and real-world case studies, and makes use of technology in an active learning environment" (CBMS, 2012, p. 58).

To further expand on and clarify the recommendations in the MET II regarding statistics, the *Statistical Education of Teachers* (Franklin et al., 2015) report was commissioned by the ASA. The SET report emphasizes the differences in the mathematical preparation and the statistical preparation needed of preservice teachers and calls for mathematicians, statisticians, and mathematics teacher educators to collaborate in preparing

preservice teachers. According to Franklin et al. (2015), the three main goals for preparing PSMTs are:

1. Develop the necessary statistical reasoning skills along with the content knowledge in statistics beyond the typical introductory college course. Statistical topics should be developed through meaningful experiences with the statistical problem-solving process.
2. Develop an understanding of how statistical concepts develop throughout PreK–8 and how they connect to high-school statistics content, as well as develop an understanding of how statistical concepts are related, or not related, to mathematical topics.
3. Develop pedagogical content knowledge necessary for effective teaching of statistics. Pre-service and practicing teachers should be familiar with common student conceptions, content-specific teaching strategies, strategies for assessing statistical knowledge, and appropriate integration of technology for developing statistical concepts (Franklin et al., 2015, p. 43).

To meet recommendations one and two, the SET report suggests that PSMTs take a sequence of three statistics courses: 1) an introductory statistics course that emphasizes data analysis, simulation-based approaches to inference with technology, and an introduction to formal inference, 2) a statistical methods course building from the introductory course utilizing randomization and classical approaches to inference, and 3) a statistical modeling course based on multiple regression techniques. The introductory course would provide PSMTs with foundational statistical knowledge and experiences that they have not experienced in their own K-12 education and should be neither the traditional, procedurally-

focused courses nor calculus-based. Instead the introductory course should follow the recommendations of the *GAISE College Report* (Garfield et al., 2007) to develop statistical reasoning to:

- Emphasize statistical literacy and develop statistical thinking.
- Use real data.
- Stress conceptual understanding, rather than mere knowledge of procedures.
- Foster active learning in the classroom.
- Use technology for developing conceptual understanding and analyzing data.
- Use assessments to improve and evaluate student learning (p.2)

The third main goal of the SET report, which was not addressed in the MET II, is for PSMTs to develop pedagogical statistical knowledge. As a field we know that changing the content of the statistics courses is not the only answer; they need to take courses that will increase their statistical knowledge as well as their pedagogical statistical knowledge (Cobb & Moore, 1997). Thus the SET report authors suggest that there is a pedagogical component as a part of the first two courses in the sequence. PSMTs should learn about common student misconceptions, using dynamic statistical software to solve problems and as a teaching tool, and effective pedagogy for developing statistical reasoning.

Research on teachers' statistical knowledge. Over the last 25 years, there has been little research about the statistical knowledge of PSMTs or the misconceptions they develop (Batanero et al., 2011; Shaughnessy, 2007), even recently with the increased emphasis on statistics in high school mathematics with the adoption of CCSSM. The majority of research on preservice teachers' statistical knowledge has focused on elementary teachers (e.g., Browning et al., 2014; Groth & Bergner, 2006; Hu, 2015; Leavy, 2010; Leavy & O'Loughlin,

2006; Meletiou-Mavrotheris, Kleanthous, & Paparistodemou, 2014; Santos & da Ponte, 2014). The limited research conducted on PSMTs' statistical knowledge has been small-scale studies, from a small number of institutions on specific statistical content (e.g., Doerr & Jacob, 2011; Makar & Confrey, 2005). Several themes emerge from the research that has been conducted: 1) teachers' focus on procedures, computations, and algorithms, 2) their lack of reasoning skills, and 3) difficulty constructing and interpreting graphical representations.

The focus on procedures, computations and formulas has been demonstrated through several studies. In regards to the mean, teachers incorporate two approaches: balancing point and standard algorithm when reasoning about the mean but often rely on the formula to define and calculate the mean (Gfeller, Niess, & Lederman, 1999; Russell & Mokros, 1990). Research also shows that teachers are fluent with the procedures of descriptive statistics (Makar & Confrey, 2004) and in turn concentrate on computations and creating graphical displays and lose focus on the statistical investigation (Burgess, 2002; Heaton & Mickelson, 2002). The fact that this theme has emerged in the research is not surprising. Mathematics often has a correct answer and statistical reasoning requires drawing conclusions that are uncertain (Groth & Bergner, 2007; Rossman et al., 2006).

This concentration on computation may cause teachers to struggle with reasoning skills identified in the research. One area that has been studied by several researchers is teachers' abilities to interpret and analyze distributions. Researchers have found that teachers are confident in analyzing and interpreting symmetric distributions but struggle with skewed distributions (Doerr & Jacob, 2011; Rubin, Hammerman, Campbell, & Puttick, 2005). One example is the research Doerr and Jacob (2011) conducted with secondary mathematics teachers after taking a one-semester course that engaged teachers in statistical investigations.

On a post-test they found that 18% of the teachers did not understand that a distribution with the median larger than the mean is likely skewed left (Doerr & Jacob, 2011). Teachers also encounter difficulty distinguishing the sampling distribution from the population distribution. Researchers found teachers struggled differentiating between the variability of the distribution from the sampling distribution (Makar & Confrey, 2004) and describe the sampling distribution to be the same as the population distribution (Doerr & Jacob, 2011).

Lastly, research has showed that teachers struggled with the construction and interpretation of graphical representations (Meletiou-Mavrotheris & Lee, 2003). It is not surprising that they struggled with the interpretation of graphical representations since research has identified a lack of reasoning skills. But since teachers tend to focus on computations and procedures, it would seem that they would be able to construct graphical representations. Researchers found that teachers struggled constructing histograms and box plots, interpreting histograms and box plots, and confused a bar graph with a histogram (Meletiou-Mavrotheris & Lee, 2003).

To date, only one large-scale study, conducted by Lee et al. (2014) examined how 204 preservice mathematics teachers from eight universities used dynamic statistical tools to conduct a statistical investigation. They found that preservice teachers who pose a broad statistical question engaged in more graphical augmentations (e.g., adding shaded regions, reference lines, or statistical measures) using dynamic statistical software. These graphical augments allowed preservice teachers to dive deeper into the data analysis and make connections to the context to support claims.

Research about preservice or inservice middle or high school teachers' pedagogical statistical knowledge is even scarcer than research on their content knowledge. The little

research that has been conducted is around two themes: statistical questions posed and their impact, and the statistical language used by teachers.

Teachers must know how to write statistical questions so that students are engaging in activities that they find meaningful and allowing students to participate in the phases of the statistical investigative cycle. These activities should allow students to build off prior experiences to reason statistically (Cross, Hudson, Lee, & Vesperman, 2013; Franke, Kazemi, & Battey, 2007). By engaging in activities that are situated in contexts that students understand, the students develop a better understanding of the statistical concepts (Cross et al., 2013). Lee and Nickell (2014) found that after preservice teachers engaged with materials to increase their technological pedagogical statistical knowledge (Lee & Hollebrands, 2008), 72% were able to pose open-ended questions for students to engage in exploratory data analysis.

In terms of statistical language, one study by Makar and Confrey (2005) investigated the language used by teachers when discussing variation when comparing two distributions. They found that pre-service teachers use informal statistical language to compare distributions and throughout a course, teachers improve their use of informal language and transition to using formal statistical language. This is similar to research on students' development of statistical language (Bakker, 2004; Konold & Higgins, 2003). Thus teachers must provide students opportunities to use informal language and to transition their students to use formal statistical language. For the field to understand truly PSMTs' statistical knowledge and pedagogical statistical knowledge, more small and large-scale studies are needed.

Teachers' Beliefs

Thompson (1992) argued that researchers should not separate the study of teachers' beliefs from teachers' knowledge since they are intertwined. Recent studies in mathematics education have provided evidence to support Thompson's statement that teachers draw on their content and pedagogical knowledge (Ball et al., 2008) as well as their beliefs when teaching (Beswick, 2007; Bray, 2011; Gellert, 2000). Phillip (2007) defines beliefs as "psychologically held understandings, premises, or prepositions about the world that are thought to be true" (p.259).

Mathematics teachers hold beliefs about their teaching and the role of the teacher as well as beliefs about mathematics, students, and student learning (Calderhead, 1996). These beliefs teachers hold are important because they influence the actions that they take in the classroom (Cooney, Shealy, & Arvold, 1998; Phillip, 2007; Wilkins, 2008). Research has shown that not only does teachers' knowledge of mathematics and pedagogy positively influence student achievement (Baumert et al., 2010; Hill et al., 2005) but their beliefs about mathematics teaching and learning do as well (Love & Kruger, 2005).

Self-Efficacy. A major aspect of teachers' belief systems is their self-efficacy for specific tasks associated with teaching (McGee & Wang, 2014). Self-efficacy is defined as "people's beliefs in their capabilities to produce given attainments" (Bandura, 2006, p. 307). Self-efficacy has grown from Bandura's (1977) social cognitive theory which consists of two main constructs: efficacy expectations and outcome expectations. Efficacy expectations are the beliefs that an individual can successfully implement the behavior required to produce the outcomes. Outcome expectations are defined as an individual's expectation for performing a task that will lead to a certain outcome. Self-efficacy originates from the construct of efficacy

expectations. Judgments of one's own self-efficacy are task-specific (Bandura, 1977; Pajares, 1997) therefore there can be no global measure of self-efficacy (Bandura, 2006). Thus an individual teacher has two types of self-efficacy for a specific content: self-efficacy to do the content themselves and self-efficacy to teach the topic. The importance in studying self-efficacy is the ability to predict one's behavior because individuals choose to complete tasks that they feel more efficacious in and continue to persist at those tasks even when they face difficulties (Bandura, 1977, 1997; Stipek, 2002), and individuals with low self-efficacy tend to give up or avoid difficult tasks.

Teacher efficacy. Defining a teacher's self-efficacy for beliefs to teach is complex because teachers are "active, thinking decision-makers who make instructional choices by drawing on complex, practically-oriented, personalized, and context-specific networks of knowledge, thoughts, and beliefs" (Borg, 2003, p. 81). Thus defining the construct of teacher efficacy has been a debate over the last 40 years (Wyatt, 2014). Over the years teacher efficacy has been defined as a two-dimensional construct composed of personal teacher self-efficacy and teaching outcome expectancy (e.g., Dembo & Gibson, 1985; Enochs, Smith, & Huinker, 2000). In 2006, Bandura clearly made the distinction that outcome expectations are not a component of self-efficacy, thus the definition of teacher efficacy is not a two-dimensional construct. For this study, teacher efficacy is defined as "belief that they have the skills to bring about student learning" (Ashton, 1985, p. 142).

Bandura (1997) argued that there are four principle sources of efficacy information that can be utilized to enhance one's teaching efficacy: enactive mastery experiences, vicarious experiences, verbal persuasion, and physiological responses. Enactive mastery experiences are perceived successful experiences in completing a particular task (Bandura,

1997). In terms of teaching, mastery experiences take two forms: actual classroom teaching and cognitive mastery (Palmer, 2011). Cognitive mastery refers to teachers' perceived success in understanding content-specific topics or pedagogical concepts. Enactive mastery experiences can be taken to be the most important source of efficacy information because they provide authentic information – teachers can only assess their ability to teach only from actual teaching experiences (Bandura, 1997; Palmer, 2011; Tschannen-Moran, Hoy, & Hoy, 1998). Researchers have shown that enactive mastery experiences are a source that influences inservice and preservice teachers' teacher efficacy (Mulholland & Wallace, 2001; Pajares, 1997; Poulou, 2007).

Vicarious experiences are experiences where an individual watches another person model performing the behavior successfully (Bandura, 1997). This source is effective when the individual feels that he/she has similar abilities and experiences as the individual modeling the behavior. In terms of teachers, they experience vicarious experiences by watching videotaped lessons featuring other teachers, sharing successful lesson with professional learning communities, and through observing other teachers (Khourey-Bowers & Simonis, 2004; Posnanski, 2002; Ross & Bruce, 2007a). Research shows that this is not a common source identified by preservice teachers (Poulou, 2007).

Verbal persuasion occurs when an individual receives positive feedback and encouragement from others (Bandura, 1997). This can be effective for teachers when they receive verbal persuasion from peers, university instructors, school administrators, and parents of students (Tschannen-Moran & Hoy, 2007). Psychological responses refer to their ability to deal with difficult situations such as stress or fear (Bandura, 1997). Researchers have provided evidence that this is a source that influences inservice and preservice teachers

teacher efficacy and was found to be the least influential source for preservice teachers (Mulholland & Wallace, 2001; Poulou, 2007).

Mathematics teaching efficacy. As previously mentioned, mathematics teachers have two types of self-efficacy: mathematics self-efficacy and mathematics teaching efficacy. Mathematics self-efficacy is an individual's belief in their ability to do mathematics (Hackett & Betz, 1989). Mathematics teaching efficacy is a teacher's belief in his/her ability to teach mathematics to bring about student learning (Ashton, 1985; Enochs et al., 2000). Two instruments have been designed to measure mathematics teaching efficacy: Mathematics Teaching Efficacy Beliefs Instrument (MTEBI; Enochs et al., 2000) and Self-Efficacy for Teaching Mathematics Instrument (STEMI; McGee & Wang, 2014).

The MTEBI was the first mathematics specific validated instrument to measure mathematics teaching efficacy. It was created by Enochs, Smith, & Huinker by revising the Science Teaching Efficacy Beliefs Instrument (STEBI; Riggs & Enochs, 1990). MTEBI measures two constructs: personal mathematics teaching efficacy and mathematics teaching outcome efficacy. This instrument does not align with Bandura's (2006) construct of self-efficacy, which calls the validity of this instrument into question.

The STEMI was developed to provide a more content-specific mathematics teaching efficacy instrument that also aligns with Bandura's notions of self-efficacy. This instrument measures two constructs: efficacy for pedagogy in mathematics and efficacy for teaching mathematical content. The mathematical content that appears on the STEMI is content for elementary teachers such as: integers, rational numbers, irrational numbers, probability, size, quantity, and capacity. This instrument moves the field forward in researching mathematics teaching efficacy but only assesses it for a portion of all mathematics teachers.

Several studies conducted using these two mathematics teaching efficacy instruments have been focused on elementary teachers (Bursal & Paznokas, 2006; Greshman, 2008; McGee, Wang, & Polly, 2013; Moseley & Utley, 2006; Piel & Green, 1993; Polly et al., 2013; Swars, 2005; Swars, Daane, & Giesen, 2006; Swars, Hart, Smith, Smith, & Tolar, 2007). Elementary teachers often have a high level of anxiety towards mathematics (Greshman, 2008; Swars et al., 2006) and they have communicated their lack of confidence in doing mathematics and teaching mathematics (McGee et al., 2013; Piel & Green, 1993; Polly et al., 2013). There currently are no known validated instruments that measure mathematics teaching efficacy for middle or high school mathematics teachers or published research on mathematics teaching efficacy of middle or high school mathematics teachers.

Statistics teaching efficacy. Similar to mathematics, teachers have two types of self-efficacy: statistics self-efficacy and statistics teaching efficacy. Very little has been researched on teachers' self-efficacy and the majority of what has been done is on teachers' statistics self-efficacy. Two measures have been created to measure one's statistics self-efficacy: Current Statistics Self-Efficacy (CSSE) and Self-Efficacy to Learn Statistics (SELS; Finney & Schraw, 2003). The names of the instruments are also the names of the constructs that each measures. Current statistics self-efficacy refers to "confidence in one's abilities to solve specific tasks related to statistics" and self-efficacy to learn statistics refers to "confidence in one's abilities to learn the skills necessary to solve specific tasks related to statistics" (Finney & Schraw, 2003, p. 164). Neither the authors nor other researchers have examined the statistics self-efficacy of PSMTs. One of the only studies involving preservice elementary teachers found that their self-efficacy to learn statistics may affect their attitudes towards future professional development in statistics (Lancaster, 2008).

In terms of statistics teaching efficacy, only one instrument has been developed to measure statistics teaching efficacy. The Self-Efficacy for Teaching Statistics (SETS) instrument has two versions: one to measure the statistics teaching efficacy to teach middle school students (Harrell-Williams, Sorto, Pierce, Lesser, & Murphy, 2013) and the other to measure the statistics teaching efficacy to teach high school students (Harrell-Williams, Sorto, Pierce, Lesser, & Murphy, 2014). This instrument aligns with Bandura's (2006) construct of self-efficacy measuring efficacy expectations. It measures statistics teaching efficacy for specific statistical tasks and levels of those tasks aligned with CCSSM (National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010) and the GAISE framework (Franklin et al., 2007).

Due to the infancy of this instrument, limited studies have been conducted on statistics teaching efficacy of preservice teachers. There was only one study conducted by the authors of the SETS and it was to determine the relationship of preservice teachers' statistics teaching efficacy and statistics self-efficacy using the CSSE instrument (Finney & Schraw, 2003). Pearson correlation coefficient was used to examine the relationship of the total SETS score and CSSE to score, with a value of 0.819 (Harrell-Williams, personal communication). This provides evidence that an individual's self-efficacy to do statistics plays a crucial role in their statistics teaching efficacy. Two other research teams have qualitatively examined preservice teachers statistics teaching efficacy. Irakleous and Panaoura (2015) used interviews of two preservice elementary teachers in Cyprus for a case study. They found that the preservice teachers had overestimated their confidence to teach statistics. Fitzmaurice, Leavy, and Hannigan (2014) also examined the statistics teaching efficacy of preservice teachers through interviews and found that they were reluctant to teach statistics during their

student teaching placements. Preservice teachers who did teach statistics during their placement reported an increase in efficacy, confirming the role mastery experiences play in one's teaching efficacy.

Research on the Relationship between Teachers' Knowledge and Beliefs

Limited research has examined the relationship between teachers' mathematical knowledge and their mathematics teaching efficacy. The few published studies have been conducted with elementary teachers and have had inconsistent results. These studies have found that mathematics content knowledge and teacher efficacy beliefs are positively correlated (Swars, Smith, Smith, & Hart, 2009), weakly positively correlated (Bates, Latham, & Kim, 2011; McCoy, 2011), or uncorrelated (Swars et al., 2007). These inconsistent results are partially due to the instruments being used to measure mathematical content knowledge and mathematics teaching efficacy. For example, Bates et al. (2011) measured mathematics content knowledge using the Illinois Certification Testing System Basic Skills test, which is a test that is required for entrance into certified teacher education programs in Illinois. The Basic Skills test addresses more mathematical knowledge than the preservice elementary teachers are expected to teach. The mathematics teaching efficacy instrument used in McCoy (2011) and Swars et al. (2009), MTEBI, does not situate mathematics teaching efficacy beliefs in mathematical tasks; instead it refers to mathematics teaching in general. In a qualitative study, Austin (2013) provided more evidence of inconsistent results when the majority of teachers had high mathematics teaching efficacy and low mathematical knowledge for teaching.

At this point, research has not been published on the relationship of statistical knowledge and statistics teaching efficacy of teachers. However the relationship of statistics

content knowledge and other aspects of teachers' attitudes has been examined. Most of these studies have focused on elementary preservice teachers and have found a weak positive relationship between statistical knowledge and attitudes towards statistics (Nasser, 2004; Zientek, Carter, Taylor, & Capraro, 2011). These studies used classroom tests to measure statistical knowledge instead of a validated instrument. Hannigan, Gill, and Leavy (2013) investigated the relationship of statistical knowledge using a validated, widely-used instrument, Comprehensive Assessment of Outcomes in a First Statistics course (CAOS; delMas, Garfield, Ooms, & Chance, 2007). Hannigan and colleagues still found no strong positive correlation between statistical knowledge and attitudes toward statistics. Therefore having a positive attitude towards statistics is not enough and more emphasis needs to be placed on developing the statistical knowledge of teachers to prepare them to teach statistics.

Conceptual Framework

In order to frame this study, I drew on theoretical perspectives related to statistical knowledge, pedagogical statistical knowledge, statistics teaching efficacy, and factors that may influence those constructs.

Statistical knowledge. To be prepared to teach, statistical knowledge is foundational for developing the other aspects of statistical knowledge for teaching (Groth, 2013). For this study, statistical knowledge refers to knowledge of the statistics curriculum outlined in the GAISE framework (Franklin et al., 2007). PSMTs need statistical knowledge across all three GAISE levels and the four phases of the statistical investigative cycle.

Pedagogical statistical knowledge. Although acquiring content knowledge is foundational in preparing a teacher to teach statistics, it is not adequate to describe all the knowledge needed. Pedagogical statistical knowledge is another crucial component of a

teacher being prepared to teach statistics. This type of knowledge refers to a teacher's knowledge of potential student difficulties with statistics, developing strategies to support student's learning, teaching strategies to engage students in the statistical investigative cycle, and vertical and horizontal knowledge of the statistics and mathematics curricula (Ball et al., 2008; Groth, 2013). Groth's (2013) framework hypothesizes how this type of knowledge develops once a teacher has an understanding of the key developmental understandings of statistics.

Statistics teaching efficacy. Statistical knowledge directly influences one's statistical teaching efficacy (Palmer, 2011). Therefore to accurately measure a teacher's statistics teaching efficacy, researchers must target specific tasks at specific grade levels (Bandura, 1997). As a result it is necessary to examine statistics teaching efficacy across the three GAISE levels A, B, and C.

Statistics teaching efficacy is also related to pedagogical statistical knowledge. As teachers increase their pedagogical statistical knowledge through vicarious and mastery teaching experiences, their statistics teaching efficacy would also likely increase (Bandura, 1997). From the literature this relationship seems to also be symbiotic. As teachers increase their statistics teaching efficacy, they are more likely to investigate and incorporate more innovative teaching strategies (Czerniak & Chiarelott, 1990; Riggs & Enochs, 1990) and therefore increase their pedagogical statistical knowledge.

Potential influences on teachers' preparedness to teach statistics. In keeping with the purpose of this study, it is important to consider potential influences on teachers' content knowledge and statistics teaching efficacy that have been identified in the mathematics education literature (Begle, 1972; Bullock, 2011; Conner, Edenfield, Gleason, & Ersoz,

2011; Gallagher, Woloshyn, & Elliott, 2009; Maasz & Schlöglmann, 2009; Rubie-Davies, Flint, & McDonald, 2012; Smith & Levi-Ari, 2005; Stuart & Thurlow, 2000). Four categories that emerged from the literature and that influence preservice teachers' content knowledge and statistics teaching efficacy are: university experiences, K-12 experiences, teaching experience, and world experiences. University experiences refer to university-based statistics courses, mathematics methods courses, statistics methods courses. K-12 experiences refer to content knowledge gained in K-12 and knowledge of pedagogical strategies through vicarious experiences from one's K-12 teachers. For preservice teachers, teaching experience refers to tutoring experiences in statistics and field experiences were they observed or taught a statistics lesson. Lastly, world experiences refer to one's race, gender, and real-world experiences engaged in statistics.

Drawing upon these perspectives, I created the diagram in Figure 6 to frame this study. The diagram shows that statistical teaching efficacy is influenced by one's statistical knowledge and pedagogical statistical knowledge. Therefore there are three constructs that are intertwined that impact preservice teachers' preparedness to teach statistics. It also shows three types of experiences that can influence one's statistical knowledge, statistical teaching efficacy, and pedagogical statistical knowledge, which will therefore impact one's preparedness to teach statistics.

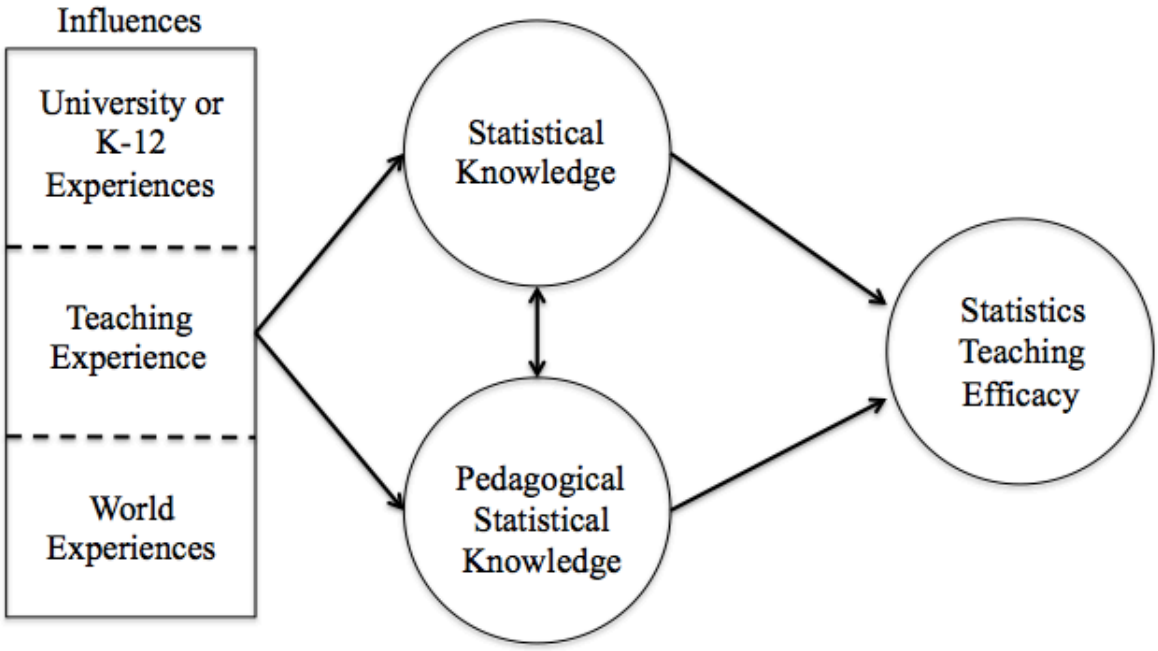

Figure 6. Conceptual framework for preservice teachers preparedness of to teach statistics.

Since this study focuses on preservice teachers, pedagogical statistical knowledge will not be measured. Preservice teachers develop minimal pedagogical knowledge until they enter the classroom (Stuart & Thurlow, 2000). Therefore this study focuses on the content knowledge component of statistical knowledge for teaching and statistics teaching efficacy in preparing teachers, rather than pedagogical statistical knowledge.

Chapter 3: Methodology

This study addressed preservice secondary mathematics teachers' (PSMTs) preparedness to teach high school statistics. The purpose of this two-phase, explanatory, sequential mixed methods study was to obtain descriptive quantitative results from a sample of PSMTs and then follow up with a portion of the individuals to explain those results in greater depth. In the first quantitative phase of the study, participants' statistical knowledge and statistics teaching efficacy was examined using validated instruments. The second, qualitative phase was conducted as a follow up to the quantitative results to help explain the quantitative results. In this follow-up, semi-structured interviews were used to explore factors that influence PSMTs' preparedness to teach high school statistics.

Therefore the research questions that were addressed are:

1. What is the preparedness of preservice secondary mathematics teachers [PSMT] to teach statistics as they enter student teaching?
    a. What is PSMTs' statistical knowledge of the high school content they are expected to teach using the phases of a statistical investigation?
    b. What is PSMTs' statistics teaching efficacy for high school? What particular topics or levels of statistics do they feel more or less prepared to teach?
    c. What self-reported factors (e.g., gender, number of statistics courses taken, whether or not they took AP Statistics in high school) influence PSMTs' statistical knowledge and statistics teaching efficacy?
    d. What is the relationship between PSMTs statistical knowledge and statistics teaching efficacy?

2. From the perspectives of PSMTs, what factors and experiences influence their preparedness to teach statistics?

In this chapter, I explain my choice of study design, describe the sampling frame and sample, data collected, and give an overview of data analysis processes. Validity and reliability issues are also addressed as well as ethical considerations.

Study Design

Why mixed methods? To answer the research questions, I used a mixed methods approach (Tashakkori & Teddlie, 2003), which integrated both quantitative and qualitative data in a single research study to draw interpretations from both sets of data (Creswell, 2015). When designing a mixed methods study, Creswell and Planco Clark (2011) identified four key principles that researchers should consider: using a fixed and/or emergent design, identifying a design approach to use, matching a design to the study's problem, purpose, and questions, and being explicit about the reasons for mixing methods.

The rationale for using a mixed-method approach is that neither qualitative nor quantitative data are sufficient to capture the complex issue of PSMTs' preparedness to teach high school statistics (Wyatt, 2014); however, used in combination, quantitative and qualitative data can complement each other and provide a more complete description of PSMTs' preparedness (Creswell & Planco Clark, 2011). To do so, a fixed explanatory sequential design will be used to align the research questions and purpose to capture the construct of preparedness to teach statistics.

Why explanatory design? Creswell and Planco Clark (2011) define an explanatory design as a study that occurs in two distinct phases. This design begins with the collection and analysis of quantitative data and is followed by the subsequent collection and analysis of

qualitative data. The first phase has the priority of addressing the research questions and the second phase is designed so that it follows from the results of the quantitative phase.

From this definition the quantitative data must be the priority to answer the research questions. In this study, the quantitative data answers research question one and aid in answering question two. The definition also suggests that the quantitative and qualitative data are interactive (Greene, 2007) and are mixed during data collection. This study began with quantitative data collection and analysis and then the results of the quantitative analysis informed the participants chosen, the qualitative data collected, and analysis (Figure 7).

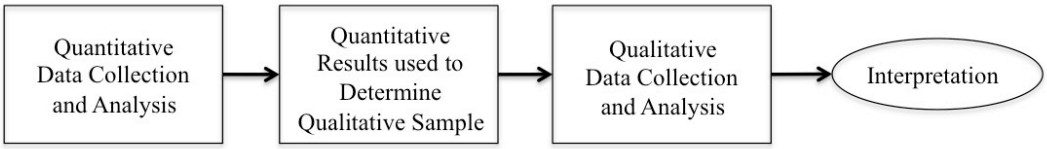

Figure 7. Explanatory sequential design of the study (Creswell & Planco Clark, 2011).

Sampling Frame

PSMTs are prepared to teach through a variety of teacher preparation programs. This study focused on PSMTs prepared through university-based teacher preparation programs in the United States. Over the past 15 years, the role of statistics in secondary mathematics education has increased; therefore several efforts have been made to increase awareness and preparation of mathematics teacher education faculty on ways they can prepare PSMTs for teaching statistics. Rather than using a random sample of all teacher preparation programs, this study began with a purposeful narrowing on PSMTs who currently attend institutions that have participated in the last 13 years in an NSF-funded or ASA-funded program to increase the emphasis of statistics education at that institution. Faculty from 57 institutions participated in the NSF-funded program, Preparing to Teach Mathematics with Technology

(PTMT), and/or the ASA-funded Math/Stat Teacher Education: Assessment, Methods, and Strategies (TEAMS) conference between 2002-2014. All 57 institutions were contacted through their undergraduate coordinator for mathematics education to inquire if the department/institution was interested in participating in the study (Appendix A). If the institution agreed to participate in the study, the coordinator was asked to identify the last mathematics teaching methods course PSMTs take before student teaching, the instructor of the course, and the semester(s) the course is offered. For participating in the study, each institution received an aggregate report for their institution's participants if 10 or more PSMTs participated in the study. 25 institutions agreed to participate in this study, producing a source of coverage error since all PSMTs at the 57 institutions did not have an opportunity to participate in the study since the undergraduate coordinator from 39 institutions did not agree for their PSMTs participate in the study (Groves et al., 2009).

From the 25 institutions who agreed to recruit participants, 18 institutions successfully recruited participants for the study. Of those 18 institutions all but one were public. Table 4 displays the Carnegie Classification™ (Carnegie Foundation for the Advancement of Teaching, 2011) for the institutions participating in the study. The majority of institutions (61.1%) had an enrollment profile of high undergraduate (between 10 and 25 percent of the students at the institution are graduate students) and the majority of participants attended institutions with a basic classification of research universities (very high), research universities (high), or Master's college and university with a larger program.

Table 4. *Carnegie Classification™ of Participating Institutions*

	Universities	Participants
	n (%)	*n* (%)
Enrollment Profile		
VHU: Very High Undergraduate	7 (38.9)	70 (29.7)
HU: High Undergraduate	11 (61.1)	166 (70.3)
Basic		
RU/VH: Research Universities (Very High)	4 (22.2)	70 (29.7)
RU/H: Research Universities (High)	6 (33.3)	62 (26.3)
D/RU: Doctoral/Research Universities	2 (11.1)	20 (8.5)
Master's L: Master's Colleges and Universities (Larger)	4 (22.2)	63 (26.7)
Master's S: Master's Colleges and Universities (Smaller)	1 (5.6)	8 (3.4)
Bac/Diverse: Baccalaureate College – Diverse Fields	1 (5.6)	13 (5.5)

Participants

Across institutions there were 236 PSMTs who participated in the study. The PSMTs were undergraduate juniors and seniors or graduate students earning initial licensure enrolled in their last mathematics education course prior to student teaching. The majority of the PSMTs were female (70.3%) and had taken one or two statistics courses at the time of the study. As data was collected from each participant, it was blinded and a code was used to indicate the university (U01, U,02…U18), semester of data collection (F14, S15) and student number (01, 02,…*n*).

Data Collection and Analysis

According to an explanatory sequential design, quantitative data and qualitative data are mixed during data collection (Creswell & Planco Clark, 2011). In phase 1, quantitative data was collected and analyzed to inform the participants chosen in phase 2, collection of qualitative data.

Phase 1. The data collected in Phase 1, quantitative phase, occurred in fall semester 2014 (n=154) and spring semester 2015 (n=81). Institutions who agreed to participate in the study, were asked to solicit student volunteers to complete the online statistics teaching efficacy survey, statistical knowledge assessment, and demographic questions.

SETS instrument. To examine PSMTs' statistics teaching efficacy, the Self-Efficacy to Teach Statistics (SETS; Harrell-Williams et al., 2014) instrument was administered. This instrument was chosen because it collects both qualitative and quantitative data about PSMTs' statistics teaching efficacy. Furthermore the SETS instrument is aligned with the GAISE framework, which reflects the content that PSMTs are expected to teach to high school students. Hence there is a close correspondence between the teaching efficacy instrument and the specific topics PSMTs need to know and to teach (Bandura, 1997; Finney & Schraw, 2003).

SETS instrument was administered towards the end of the participants' final mathematics methods course in fall semester 2014 and spring semester 2015 (Appendix C). The instrument contains 44 six-point Likert scale items. An earlier version of this instrument with 26 items aligned with levels A and B of GAISE was validated for use in measuring changes in elementary and middle grades preservice teachers' self-efficacy as a result of interventions, such as a course (Harrell-Williams et al., 2013). The high school version

contains the previous 26 items aligned to GAISE levels A and B and contains an additional 18 items validated and aligned to GAISE level C (Harrell-Williams & Pierce, 2015; Harrell-Williams et al., 2014)[1]. In addition to an overall score, the instrument provides sub-scale scores that correspond to Levels A, B and C in the GAISE framework. There are 11 Likert items for level A, 15 items for level B and 18 items for level C. For all Likert items, the stem of the question was

> "Rate your confidence in teaching high school students the skills necessary to complete successfully the task given by selecting your choice on the following scale: 1 = not at all confident, 2 = only a little confident, 3 = somewhat confident, 4 = confident, 5 = very confident, 6 = completely confident" (Harrell-Williams et al., 2014).

There is some concern about using self-reported data because PSMTs would have a tendency to present favorable images of their statistics teaching efficacy (Ross, 1989); however, research has shown that there is little motivation to misreport since the confidentiality was preserved (Baldwin, 2000). For each participant, the SETS instrument produces a confidence score to teach high school students statistics for GAISE Level A, Level B, Level C, and an overall score.

For the open-ended portion in each GAISE level category, PSMTs were asked to identify an item which they felt least confident to teach and an item which they felt most confident to teach to high school students and to explain their reasoning. There are two open-ended questions for each GAISE level A – C. One asks the respondent to "Choose one item that you indicated feeling LEAST confident about teaching high school students. Think about

[1] This 44-item version of SETS has been validated, with manuscript currently in preparation.

the reason(s) you feel this way. Use the space below to identify the item number and explain your reason(s)" and the other asks the respondent to "Choose one item that you indicated feeling MOST confident about teaching high school students. Think about the reason(s) you feel this way. Use the space below to identify the item number and explain your reason(s)" (Harrell-Williams et al., 2014). Therefore if a participant answered all open-ended questions, there will be three least confident responses and three most confident responses for each participant. While this qualitative data was collected in phase 1 due its tie to questions on the SETS instrument, it was used to answer Research Question 2 and analysis will occur in phase 2.

Based on preliminary analysis from data collected in fall semester 2014 (n=154), spring semester 2015 participants (n=81) were asked to answer an additional question comparing their preparedness across five areas of high school mathematics.

> "As a secondary mathematics teacher you have to be prepared to teach a wide variety of subjects: algebra, geometry, pre-calc/advanced algebra, calculus, and statistics. Given these five different topics, algebra, geometry, pre-calc/advanced algebra, calculus, and statistics, please rank these in order of how well you feel prepared to teach them from most to least."

This data was collected to explain PSMTs' statistics teaching efficacy in relationship to other areas of high school mathematics.

Statistics content assessment. To examine PSMTs' statistical knowledge, the Levels of Conceptual Understanding of Statistics (LOCUS) assessment (Jacobbe, Case, Whitaker, & Foti, 2014) was administered online (locus.statisticseducation.org). The LOCUS assessment is aligned with the CCSSM and assesses understanding across the three levels of

development in the GAISE framework (Franklin et al., 2007). A 23-item paper version of the LOCUS assessment has been validated as a measure to reliably assess current statistical understanding (Jacobbe, Case, et al., 2014).

Participants in this study took the Intermediate/Advanced Statistical Literacy version of the assessment, which was designed for students in grades 10 – 12. This test was chosen because this assessment represents the content that PSMTs are expected to teach to their students in the near future when they begin teaching. This assessment consists of 30 questions: 20 questions that align with levels B and C of the GAISE framework and 10 equator questions aligning with all three levels. These 30 questions contain the previously validated 23 items and an additional 7 items.[2] These 30 questions are also aligned with the four phases of an investigative cycle: Formulating Questions, Collecting Data, Analyzing Data, and Interpreting Results.

A sample LOCUS item for each phase of the statistical investigative cycle can be found in Appendix H. Items on the LOCUS are not procedural in nature like most large-scale assessments; instead the items are designed to measure one's conceptual understanding. Therefore, Jacobbe, Foti, Case, and Whitaker (2014) authors describe the items as

> Formulating Question areas were designed to "involve the planning process used in a statistical problem, including having the students determine if the problem is statistical in nature, decide what data is needed to answer the question, or consider what inferences can be drawn from the data" (p.3).
>
> Collecting Data items "involve executing or implementing sampling or experimental assignment of treatment techniques" (p.3).

[2] This 30-item version of LOCUS has been validated, with manuscript currently under review.

Analyze Data items "require students to analyze data through a statistical lens to show that they understand what the data is telling them" (p.3).

Interpret Results "have the students answer an initial question by drawing conclusions from the data" (p.4).

Therefore, for each participant, the LOCUS assessment produces a score for GAISE Level B, GAISE Level C, Forming Questions, Collecting Data, Analyzing Data, Interpreting Results, and an overall score. Table 5 displays the emphasis of these four phases on the Intermediate/Advanced version of the assessment.

Table 5. *Emphasis of four phases on LOCUS*

Four Phases of GAISE Framework	Emphasis on Intermediate/Advanced Assessment
Formulating Statistical Questions	15 – 20%
Collecting Data	20 – 25%
Analyzing Data	25 – 30%
Interpreting Results	30 – 35%

Demographic questions. A total of 12 different demographic questions were asked of respondents completing the LOCUS and SETS. Prior to completing the LOCUS assessment, participants were asked to answer four demographic questions regarding their gender, race, and if English is the most common language spoken in their home. At the conclusion of the SETS survey, participants were asked to answer eight demographic questions about themselves (Appendix D). These questions focused on if they took AP Statistics in high school, level of degree they would receive from their institution, grade levels they would be certified to teach, number of statistics course taken at their institution, comfort level with statistical technologies, and their overall preparedness to teach the high school CCSSM for

statistics. These 12 demographic questions were used to identify factors that influence PSMTs' statistical knowledge and/or statistics teaching efficacy.

Table 6 links the study's conceptual framework to the research questions and the data that was collected during phase 1. To investigate PSMTs' statistical knowledge, the LOCUS assessment will provide data for GAISE Levels B and C, each phase of the statistical investigative cycle, and a total score. To investigate PSMTs' statistics teaching efficacy, the SETS survey will provide data about their confidence on each items, each GAISE Level A, B, and C, a total confidence score, and PSMTs' ranking of five teaching topics. Lastly, to examine what factors influence PSMTs' statistical knowledge and statistics teaching efficacy the demographic questions from the SETS and LOCUS were used with the SETS and LOCUS scores.

Table 6. *Conceptual framework, research questions, and data collected in phase 1*

Conceptual Framework	Research Questions	Sources of Data
Statistical Knowledge	1a. What is the statistical knowledge of PSMTs in regards to the high school content and the phases of a statistical investigation? 1d. What is the relationship between PSTs statistical knowledge and statistics teaching efficacy?	LOCUS scores from: • Level B • Level C • Forming statistical questions • Collecting data • Analyzing data • Interpreting results • Total score
Statistics Teaching Efficacy	1b. What are PSMTs' statistics teaching efficacy for high school? What particular topics or levels of statistics that they feel more or less prepared to teach? 1d. What is the relationship between PSTs statistical knowledge and statistics teaching efficacy?	SETS confidence scores from: • Each item • Level A • Level B • Level C • Total confidence score • Rank order of 5 teaching topics
Influences:	1c. What factors influence PSMTs' statistical knowledge and statistics teaching efficacy?	• LOCUS scores • SETS scores • 8 demographic questions from SETS • 4 demographic questions from LOCUS • Rank order of 5 teaching topics

Analysis of phase 1. The analysis in phase 1 is designed as an exploratory analysis to examine the statistical knowledge of PSMTs, the statistics teaching efficacy they hold, factors that influence their content knowledge or statistics teaching efficacy, and the relationship between PSMTs' statistical knowledge and statistics teaching efficacy. The twelve demographic questions were used in analysis of the SETS and LOCUS scores to

identify factors that influence statistics teaching efficacy and statistical knowledge. In addition to examining aspects of research question one, the analysis of phase 1 provided the sample of participants for phase 2 as well as factors of interest that influence PSMTs' preparedness to examine in phase 2 analysis.

Through examination of the data, one factor was identified as unreliable to use for analysis. This was the type of course PSMTs had in their degree program that discussed instructional strategies for teaching statistics. PSMTs had four options to choose from: a course focused entirely on teaching methods for statistics, a course on teaching methods for mathematics that contained units or lessons focused on teaching statistics, a statistics course that had lessons or assignments on teaching methods for statistics, and other. Comparing PSMTs' responses to the program of study listed online for three institutions indicated that PSMTs did not answer this question reliably.

Statistical knowledge. To analyze the statistical knowledge of the PSMTs, analysis began with examining the time it took for participants to complete the LOCUS assessment. Four participants who took exceptionally less than to complete a content assessment than recommended by the authors of the assessment (less than ten minutes) were eliminated (Jacobbe, personal communication). Using SPSS, descriptive analysis of PSMTs' LOCUS overall scores and subscores was conducted, paired samples t-tests were used to test for significance of PSMTs' statistical knowledge between GAISE Levels B and C, and a repeated measures ANOVA was used to test for significant differences in PSMTs' statistical knowledge between the four phases of a statistical investigation. Paired samples t-tests were appropriate since the samples are independent and identically distributed. A repeated

measures ANOVA was used since the assumptions of normality and sphericity were not validated.

Following descriptive analysis, an item analysis was conducted to closely examine PSMTs' understandings and misconceptions. Then the LOCUS scores were explored to identify factors that increase PSMTs' statistical knowledge. For binary factors, two-sample t-tests were run to compare the difference in the means of the two groups to explore which factors have a significant difference on PSMTs' statistical knowledge. Two-sample t-tests are appropriate since the samples are approximately normal. For other factors such as basic Carnegie classification or number of statistics courses taken, a nonparametric Kruskal-Wallis test was conducted due to the small sample size in some groups. Table 7 displays the possible null hypothesis and alternative hypothesis that were tested for each factor. Table 7 only shows examples involving the total LOCUS scores, but all seven LOCUS scores were tested for each null hypothesis.

Table 7. *Possible null and alternative hypotheses for factors that influence statistical knowledge*

Factor	H_o	H_A
AP Statistics	The difference between the mean LOCUS scores of PSMTs who took AP statistics in high school is less than or equal to the mean LOCUS scores of PSMTs who did not take AP statistics.	The difference between the mean LOCUS scores of PSMTs who took AP statistics in high school is greater than the mean LOCUS scores of PSMTs who did not take AP statistics.
Gender	There is no difference between the mean LOCUS scores of male PSMTs and the mean LOCUS scores of female PSMTs.	There is a difference between the mean LOCUS scores of male PSMTs and the mean LOCUS scores of female PSMTs.
Degree program (undergraduate vs. graduate)	There is no difference between the mean LOCUS scores of undergraduate PSMTs and the mean LOCUS scores of graduate PSMTs.	There is a difference between the mean LOCUS scores of undergraduate PSMTs and the mean LOCUS scores of graduate PSMTs.

Statistics teaching efficacy. Analysis of PSMTs' statistics teaching efficacy began with accounting for missing data, since every PSMT did not complete every item on the SETS instrument. The percentage of missing values ranged from 0 to as high as 5.1% for some items, and only 91.1% of the 236 PSMTs in the sample would have been available for analysis under listwise deletion. Data was primarily missing due to nonresponse and this was addressed by using multiple imputation under the assumption that missing values are missing at random (Allison, 2002). Since approximately nine cases were missing, nine imputed data sets were created using SPSS (White, Royston, & Wood, 2011). Following imputation, an overall score was calculated for each PSMT as the sum of his/her Likert scores divided by the total number of items. Sub-scale scores for each GAISE level were also calculated for each PSMT using the sum of the Likert scores divided by the number of items in each level. This resulted in scores that corresponded to the six-point Likert scale for each of the nine imputed data sets.

Analyses run on each imputed data set were pooled to according to Rubin's (1987) rules and were examined to compare pooled values to the original data. The results were similar thus imputed results will be presented. A repeated measures ANOVA was used to test for significant differences in PSMTs' statistics teaching efficacy between the three GAISE levels. A repeated measures ANOVA was used since the assumptions of normality and sphericity were not validated. Following analysis of the overall and subscale scores, an item analysis was conducted to identify the specific topics that the PSMTs felt least and most confident in teaching. Then descriptive analysis of PSMTs' SETS total scores, level A scores, level B scores, level C scores and for each of the 44 items was conducted.

Following descriptive analysis, the four SETS scores were explored to identify factors from the 12 demographic questions that increase PSMTs' statistics teaching efficacy. For binary factors, two-sample t-tests were run to compare the difference in the means of the two groups to explore which factors have a significant difference on PSMTs' statistics teaching efficacy. Two-sample t-tests are appropriate since the samples are approximately normal. For other factors such as basic Carnegie classification or number of statistics courses taken, a nonparametric Kruskal-Wallis test was conducted due to the small sample size in some groups. Table 8 displays the possible null hypothesis and alternative hypothesis that were tested for each factor. Table 8 only shows examples involving the total SETS scores, but all four SETS scores were tested for each null hypothesis.

Table 8. *Possible null and alternative hypotheses for factors that influence statistics teaching efficacy*

Factor	H_o	H_A
AP Statistics	The difference between the mean SETS scores of PSMTs who took AP statistics in high school is less than or equal to the mean SETS scores of PSMTs who did not take AP statistics.	The difference between the mean SETS scores of PSMTs who took AP statistics in high school is greater than the mean SETS scores of PSMTs who did not take AP statistics.
Gender	There is no difference between the mean SETS scores of male PSMTs and the mean SETS scores of female PSMTs.	There is a difference between the mean SETS scores of male PSMTs and the mean SETS scores of female PSMTs.
Degree program (undergraduate vs. graduate)	There is no difference between the mean SETS scores of undergraduate PSMTs and the mean SETS scores of graduate PSMTs.	There is a difference between the mean SETS scores of undergraduate PSMTs and the mean LOCUS scores of graduate PSMTs.

Relationship between statistical knowledge and statistics teaching efficacy. To analyze the relationship between statistical knowledge and statistics teaching efficacy of

PSMTs, analysis began with examining their correlation. The correlation of the total LOCUS and SETS, Level B LOCUS and Level B SETS, and Level C LOCUS and Level C SETS were computed. Following correlation, multiple linear regression model was calculated to predict statistics teaching efficacy based on statistical knowledge, if AP Statistics was taken in high school, number of statistics courses, Carnegie classification, and gender.

Phase 2. The first part of the data for phase 2 was collected as part of the SETS survey (six open-ended responses) during Phase 1. This consists of responses to the six open-ended responses for 236 PSMTs. The additional data collected in phase 2, qualitative phase, occurred in late spring/early summer 2015. Of the participants who participated in the SETS and LOCUS in spring semester 2015 (n=81), 25 volunteers from the 81 Spring 2015 PSMTs participated in a semi-structured interview through video conferencing. These PSMTs represented half (n=9) of the universities in the sample.

Participants were invited through an email invitation (Appendix E) and the interviews took place using Google hangout or Skype and were recorded using a screen-capturing program. For each interview, the researcher prepared a PowerPoint containing the interview questions and all necessary information for each prompt in the interview. Each interview participant received $25 gift card for participating in this portion of the study. During the interview notes were taken and a brief summary and general impressions were written immediately following each interview.

Table 9. *Interview questions, purpose, and connection to the framework*

Interview Questions	General Purpose	Connection to the Framework
Tell me more about your confidence to teach item (least/most confident item)	This question started a conversation on different statistical topics in which PSMT feels least or most confident in.	K-12, university, teaching, and world experiences
From all the preservice teachers that took this survey, item _____ was commonly identified as a topic they are less confident in teaching to high school students. What do you think about that? Why do you think that is?	This question requires the PSMT to consider how and why PSMTs as a whole feel least or most confident about a topic.	K-12, university, teaching, and world experiences
Have you seen something like this before [referring to Figure 15]? *If yes*, tell me about it? If no, have you heard about the different phases that one might go through in the statistics investigative cycle? What experiences have you had with the statistical investigative cycle?	This questions targets the statistical knowledge of the PSMT in regards to the statistical investigative cycle and will lead into a conversation on the influence prior statistics courses.	K-12 and university experiences
From all the preservice teachers that took this test, this is the percent of items correct for each phase of the cycle. Why do you think that preservice teachers are weaker in _____ part of the cycle? How does this compare to your experience?	This question requires the PSMT to consider how and why PSMTs as a whole are weak in a topic. This will also lead into a conversation on the influence of prior coursework and teaching experiences.	K-12, university, and teaching experiences
To what extent did your teacher preparation courses prepare you to teach statistics?	This question specifically targets the PSMT's university methods courses.	University experiences
How do you think your understanding of the statistics content impacts your confidence to teach?	This question allows the PSMT to provide an overall description of his/her confidence and provide information that he/she has not mentioned in previous questions.	K-12, university, teaching, and world experiences
On the survey you were asked to rank five different topics: algebra, geometry, pre-calc/advanced algebra, calculus, and statistics. Here is how you rank them _____. Tell me more about how you ranked them.	This question provides the PSMT's perspective of how confident he/she is to teach statistics compared to other areas he/she is expected to teach.	K-12, university, teaching, and world experiences

The purpose of these interviews were to collect detailed information to support the quantitative results. PSMTs were asked questions to expand on their confidence to teach the items they identified in their open-ended responses, their experiences in their statistics courses and mathematics education methods courses, previous tutoring, field, and teaching experiences, and their comfort level with statistical technologies (Appendix F). Table 9 links the conceptual framework to the interview questions.

Table 10 links the study's conceptual framework to the research questions and the data that will be collected during phase 2. To investigate from the PSMTs' perspective the factors and experiences that influence their preparedness to teach, open-ended responses on the SETS survey were collected from all participants. To provide a more in-depth understanding of their perspectives, a sample of 25 PSMTs were interviewed about their preparedness to teach statistics. Transcripts of the interviews and field notes were used to write a narrative summary of interviewee's experiences relating to their confidence (Creswell, 2013; Merriam, 1998).

Table 10. *Conceptual framework, research questions, and data collected in phase 2*

Conceptual Framework	Research Questions	Sources of Data
Influences	2. From the perspectives of PSMTs, what factors and experiences influence their preparedness to teach statistics?	• SETS 6 open-ended responses ($n \approx 230$) • Interviews ($n \approx 20$)

Analysis phase 2. The analysis in phase 2 was designed to explore in-depth the quantitative results from phase 1. In phase 2 the analysis focused on describing factors and experiences that influence PSMTs' preparedness to teach statistics from their perspective through open-ended responses and interviews.

Open-ended responses. The six open-ended responses from the SETS survey were organized in ATLAS.ti to be analyzed. From there, qualitative analysis began with Decuir-Gunby, Marshall, and McCulloch's (2011) recommendations for the development of a codebook. I began by coding the open-ended responses using *a priori* codes from the three major components of the framework: experiences, statistical knowledge, and pedagogical statistical knowledge. Then I open-coded (Corbin & Strauss, 2008) within each component to identify themes. Iteratively, themes were collapsed into codes shown in Table 11.

Interviews. Transcripts of interviews and field notes were used to write a narrative summary of interviewee's experiences (Merriam, 1998). Only critical moments of the interviews were included in the summary for analysis of factors that influence PSMTs' preparedness. Critical moments are signified by the identification of factors and justifications of how those factors influenced PSMTs' preparedness to teach statistics. The interview summaries were coded in ATLAS.ti using the codebook developed for the open-ended responses to examine the factors that influence their preparedness to teach statistics. Then open coding was used to capture themes that appeared in interviews, but had not appeared in open-ended responses (Table 11).

Table 11. *Codes developed during analysis of open-ended responses*

Framework	Codes
Experiences	School Experiences K-12 Experience College Experience
	Teaching Experiences Lesson Plan Experience Tutoring Experience Field Experience
	World Experiences
	Recent Experiences Sufficient/Extended Experiences Prior Memories
Statistical Knowledge	Computational/Procedural Knowledge Conceptual Knowledge Nature of Statistics Recent Vocabulary Brush Perceived Difficulty of the Topic
Pedagogical Knowledge	Appropriate Teaching Strategies Knowledge of Students as Learners View Statistics Pedagogy the same as Mathematics Perceived Difficulty to Teach
Other	Technology Algebra

Once the open-ended responses and interviews were coded, the data was separated into two groups: responses describing confidence to teach statistics, and responses describing a lack of confidence to teach statistics. Codes from each group were examined to identify a list of factors that influence PSMTs' preparedness for each category (Table 12). From this list of factors, four categories were formed that influence PSMTs' preparedness from their perspective.

Table 12. *Factors that influence PSMTs' preparedness*

Reasons for not being confident	Reasons for being confident
Lack sufficient experiences Lack in-depth content knowledge Little or no experience with statistical technologies No/limited knowledge of student misconceptions No previous experience explaining statistics to others Nature and uncertainty of statistics	Extended Experiences with content Engagement with technology Tutoring/teaching experience View of statistics as a set of procedures

Validity/Reliability

As previously mentioned the instruments used in this study, SETS and LOCUS, have been previously validated by their respective research teams. In terms of reliability, it will be established through triangulation of sources. Denzin (as cited in Merriam, 2002) suggests that triangulation has four types: multiple investigators, multiple theories, multiple sources of data, or multiple methods. This study employs two of these strategies.

Even though there are not multiple investigators for this study, the study is being conducted in consultation with experts in the field. To maintain reliability and establish a reliable codebook, 15 PSMTs' responses were randomly chosen, and coded by another researcher, who is an expert in the field, for comparison. Little disagreements were found, which were then resolved through discussion.

This study also provided multiple sources of data for the participants. For each participant there were four sources of data: SETS Likert items, SETS open-ended responses, LOCUS assessment, and demographic questions. For a subgroup of participants there was also interview data to provide an in-depth understanding of their preparedness to teach statistics. The data was analyzed at different subunits to provide further support for conclusions. These subunits include looking at the individual and groups.

Ethical Considerations

This study received Internal Review Board (IRB) approval from North Carolina State to conduct this study (Appendix G). When an instructor provided an overview of the study, participants were told that participation or lack of participation in the study would not affect their grade in their course. In return for their cooperation, the undergraduate coordinator received aggregate results from both of the SETS surveys and LOCUS tests if 10 or more PSMTs participated in the study. The consent form and talking points provided to the instructors let participants know they could withdraw from the study at any time. There was minimal risk anticipated for participation in this study as all participants were assigned a unique numerical code and/or pseudonyms when audio or written work is presented. All data was stored on a password-protected external hard drive that was stored in a locked cabinet in the researcher's office. Back up files were stored on a secure, North Carolina State University server.

Chapter 4: Preservice Secondary Mathematics Teachers' Statistical Knowledge: A Snapshot of Strengths and Weaknesses

Journal

This article was written as a brief report in the *Journal for Research in Mathematics Education (JRME)*. Guidelines require that a brief report not exceed 3,600 words including tables, figures, and references. These quantitative results about PSMTs' statistics knowledge presented as a brief report would benefit the audience of JRME of researchers and mathematics teacher educators. For mathematics teacher educators, this manuscript provides timely evidence to support the need for increased emphasis on statistics in mathematics teacher preparation programs recommended in the *Mathematics Education of Teachers II* report and the recently released *Statistics Education of Teachers* report. For researchers, the result of the item analysis identifies areas of statistics in need of deeper research of preservice teachers' understandings and misunderstandings.

Abstract

Amid the implementation of new curriculum standard regarding statistics and new recommendations for preservice secondary mathematics teachers [PSMTs] to teach statistics, there is a need to examine the current state of PSMTs' knowledge of the statistical content they will be expected to teach. This study reports on the statistical knowledge of 217 PSMTs from a purposeful sample of 18 universities across the United States. The results show that PSMTs may not have strong statistical understandings that are needed to teach statistics to high school students. PSMTs' strengths include identifying appropriate measures of center, while weaknesses involve issues with variability, sampling distributions, p-values, and confidence intervals.

Introduction

Statistics has increasingly become a key component in many curricula in the U.S., especially those informed by recommendations from the National Council of Teachers of Mathematics (2000) and the Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010). In response, the *Mathematics Education of Teachers* reports I and II (Conference Board of the Mathematical Sciences, 2001, 2012) as well as the *Statistics Education of Teachers* report (Franklin et al., 2015) present recommendations for developing statistical knowledge and pedagogy needed by preservice mathematics teachers to teach statistics. While some smaller studies have suggested that preservice secondary teachers may struggle with statistics (e.g., Casey & Wasserman, 2015), there are no large-scale studies that describe the current state of new teachers' statistical knowledge. This study examines the statistical knowledge of a large cross-institutional sample of preservice secondary mathematics teachers [PSMTs] as they enter student teaching and aims to answer the following question: What knowledge do PSMTs demonstrate of the statistical content they will be expected to teach?

Theoretical Framework

To support inclusion of statistics in K-12 curriculum, the American Statistical Association published the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K-12 Curriculum Framework* (Franklin et al., 2007). The GAISE framework describes statistical content students need to develop in K-12 and partitions this content into three levels A, B, and C (Franklin et al., 2007). Although there are no explicit definitions given for each level, the levels increase in statistical sophistication and become

more abstract. The content in level A represents topics for early or novice learners of statistics (often introduced in elementary and middle school), level B represents slightly more advanced statistical content (often taught in middle school or early high school), and level C represents even more advanced content (typically taught in high school or introductory college courses) (Franklin et al., 2007). The GAISE report recommends that within each level, students should learn statistical topics through engaging in a statistical investigative cycle (Wild & Pfannkuch, 1999) consisting of: posing questions, collecting data, analyzing data, and interpreting results.

Figure 8 illustrates how, as students progress through three levels, they continue to engage in the statistical investigative cycle and learn more sophisticated ways to engage in statistical content.

Figure 8. Increased sophistication of statistical investigative cycle through three levels.

Methodology

The results reported here are part of a larger mixed methods study on preparedness of PSMTs to teach statistics. While the larger study included instruments to measure self-efficacy to teach statistics and interviews with PSMTs about their experiences in their teacher preparation programs, the focus here is on quantitative analysis of a measure of their statistical knowledge of the content they will be expected to teach. Results from the larger study will be reported elsewhere.

Participating institutions. This study focuses on PSMTs prepared through university-based teacher preparation programs in the US. Rather than using a random sample of all teacher preparation programs, this study began with a purposeful narrowing on PSMTs who currently attend institutions in which some faculty have participated in the last 13 years in particular National Science Foundation (NSF)-funded or American Statistical Association (ASA)-funded programs to increase the emphasis of statistics education at that institution. Faculty from 57 institutions participated in the NSF-funded program, Preparing to Teach Mathematics with Technology (PTMT, ptmt.fi.ncsu.edu), and/or the ASA-funded Math/Stat Teacher Education: Assessment, Methods, and Strategies (TEAMS, www.amstat.org/sections/educ/newsletter/v9n1/TEAMS.html) conference between 2002-2014. Both projects provided professional development to faculty members by focusing on explicit content and strategies to utilize in preparing PSMTs to teach statistics when implementing teacher preparation programs at their home institutions. All 57 institutions were contacted, and eighteen institutions agreed to participate.

Of the eighteen institutions in the study, all but one were public institutions. The majority of institutions (61.1%) had an Carnegie Classification™ (Carnegie Foundation for

the Advancement of Teaching, 2011) enrollment profile of high undergraduate. Approximately 84% of participants attended institutions with a basic classification of Research Universities/Very High, Research University/High or a Master's college and university with a larger program.

Participants. Across 18 institutions there were 221 PSMTs who took the assessment of their statistical understanding, described in the next section. Those who took exceptionally less time (10 minutes) to complete the content assessment than recommended by authors of the assessment were eliminated (Jacobbe, personal communication). This resulted in a sample size of 217 PSMTs. The PSMTs were undergraduate juniors and seniors, or graduate students earning initial licensure; all were enrolled in their last mathematics education course prior to student teaching. The majority of PSMTs were female (71%), and 88% were Caucasian. The majority (59%) of PSMTs reported they had taken one or two statistics courses at the time of the study.

Instrument. To examine PSMTs' statistical knowledge, the Levels of Conceptual Understanding of Statistics (LOCUS) assessment instrument (Jacobbe, Case, et al., 2014) was administered online (locus.statisticseducation.org). The LOCUS assessment is aligned with CCSSM and assesses understanding across the three levels of development in the GAISE framework. A 23-item paper version of the LOCUS assessment has been validated as a measure to reliably assess current statistical understanding (Jacobbe, Case, et al., 2014).

Participants took the Intermediate/Advanced Statistical Literacy version of the assessment, which was designed for students in grades 10 – 12. This assessment represents content that PSMTs are expected to teach to their students in the near future. The test consists of 30 questions: 20 questions that align with levels B and C of the GAISE framework and 10

equator questions aligning with all three levels. These 30 questions contain the previously validated 23 items and an additional 7 items.[3] Along with aligning with the GAISE levels, items align with the four phases of an investigative cycle: forming statistical questions, collecting data, analyzing data, and interpreting results.[4] While the actual items on the test cannot be released, sample items for the four categories at different levels are available on the LOCUS website (locus.statisticseducation.org/professional-development) and four sample items are in the Appendix. For each participant, the LOCUS assessment produces an overall score (percent correct) as well as sub-scores for Level B, Level C, Forming Questions, Collecting Data, Analyzing Data, and Interpreting Results.

Data analysis. To examine the statistical knowledge demonstrated by PSMTs, analysis began with examining the time it took for participants to complete the assessment. Descriptive statistics were computed for PSMTs' Overall scores, Level B scores, Level C scores and scores for each portion of the investigative cycle: Forming Questions, Collecting Data, Analyzing Data, and Interpreting Results. Paired samples t-tests were used to test for significance of PSMTs' statistical knowledge between GAISE Levels B and C and a repeated measures ANOVA was used to test for significant differences in PSMTs' statistical knowledge between the four phases of a statistical investigation. Following analysis of the overall and subscale scores, an item analysis was conducted to closely examine PSMTs' understandings and misconceptions.

[3] This 30-item version of LOCUS has been validated, with manuscript currently under review.
[4] For more information on the knowledge assessed on these items see Jacobbe, Foti, et al. (2014).

Results

The resulting scores on the LOCUS test can help in describing what PSMTs from these 18 universities currently understand about the statistics content they will soon be responsible for teaching. Table 11 reports summary statistics for PSMTs' scores on the assessment overall, and for Levels B and C. With a mean score of 69%, and a standard deviation of 14.06, these results suggest that PSMTs do not demonstrate a conceptual understanding of the statistical knowledge they will teach high school students. PSMTs scored, on average, significantly higher on the level B questions than on Level C questions ($t=5.772$, $p<0.001$), demonstrating that their statistical knowledge is weaker as items increase in sophistication. Figure 9 shows the distribution of PSMTs' scores. The boxplots show that for the overall scores and subscores, there are at least some PSMTs who scored 100% correct, indicating that they understand the statistical content they will soon be responsible to teach. Although, there is a concern since only one-quarter of PSMTs scored above 77%, and a quarter scored below 57% overall. The variation in scores seems somewhat similar for Level C scores. However higher variation in Level B scores is likely due to the increased quantity of low scoring individuals, indicated as outliers in *Figure 9*. (The star in *Figure 9* refers to an extreme outlier, which is a value more than three times the interquartile range.)

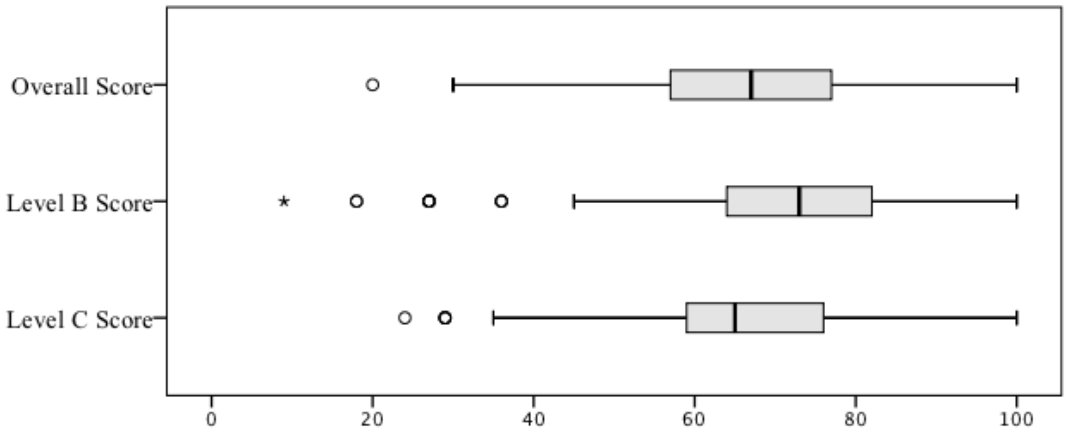

Figure 9. Distribution of PSMTs' LOCUS score.

Table 13. *PSMTs' Percent Correct on LOCUS instrument*

	Number of items	Mean	SD
Overall Score	30	68.61	14.06
GAISE Levels			
Level B Score	11	70.85	17.69
Level C Score	17	64.87	14.16
Phases of Statistical Investigative Cycle			
Formulating Questions	5	80.37	21.51
Collect Data	7	70.40	19.70
Analyze Data	7	63.34	22.22
Interpret Results	11	60.48	16.25

Note. Two items are classified as GAISE level A, but were not analyzed due to the small number of items.

Examining items by the phases in the statistical investigative cycle, Table 11 shows that PSMTs scored higher on average on Formulating Questions and lower as the cycle progresses, scoring the lowest on Interpreting Data items. A repeated measures ANOVA

determined that mean scores differed significantly between the four phases of the cycle [$F(3,648)=64.73$, $p<0.001$]. Post hoc tests using a Bonferroni correction revealed that PSMTs scored significantly lower on questions as the cycle progressed ($p<0.001$). However there was only a slight difference between mean scores for Analyze Data and Interpret Results ($p=0.32$). Figure 10 shows the distribution of scores across phases of the statistical investigative cycle. The boxplots show that for all four phases, there are some PSMTs who scored 100%, indicating that those PSMTs likely have the content knowledge that will be needed when teaching that phase of the investigative cycle. On Formulating Questions items, at least half of PSMTs scored 80% or higher, and a quarter of those scored 100%, indicating stronger understanding for these PSMTs about Formulating Questions. There are concerns regarding the other phases of the investigative cycle. Half of PSMTs scored below 71% on Collecting Data and Analyzing Data items, and half scored below 64% on Interpreting Results items. Even being conservative, this result is convincing that the majority of PSMTs do not have the statistical foundation needed to teach students key concepts related to Collecting Data, Analyzing Data, and Interpreting Results.

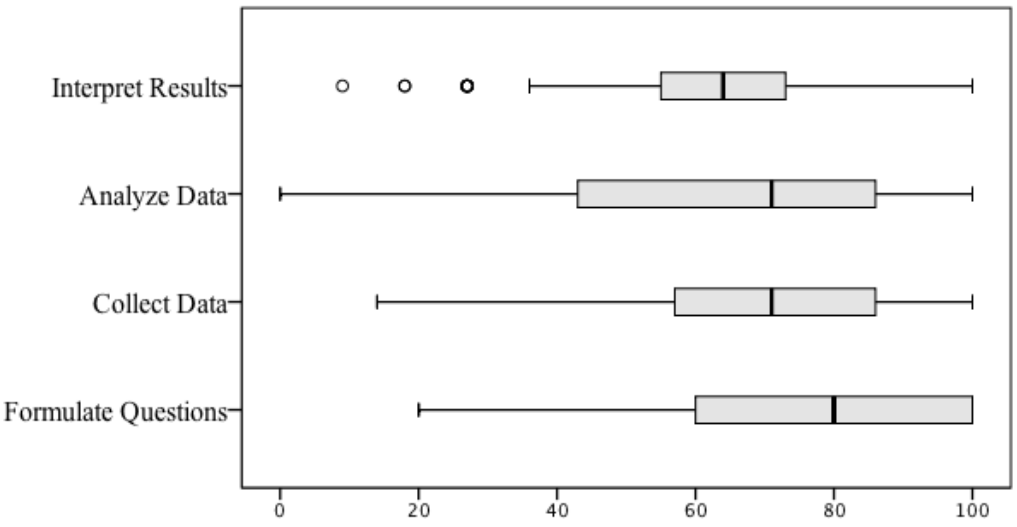

Figure 10. Distribution of PSMTs' LOCUS scores for phases of statistical investigative cycle.

Item analysis. Upon further analysis of individual items classified by the statistical investigative cycle, themes emerged concerning PSMTs' strengths and weakness. Four sample items, one from each phase of the investigative cycle, are in Appendix H. As previously mentioned, PSMTs scored the highest on average for Formulating Questions items. At least 87% of PSMTs were able to read a description of a study and measurements taken to identify the statistical question of interest. There was not a common misunderstanding demonstrated in the Formulating Questions items.

On average, PSMTs scored the next highest on Collecting Data items. Over 70% of PSMTs were able to identify: ways to improve a study design given a study and measurements; what was misleading about a graph given the raw data; which study design would be best based on a question of interest; and a data collection plan based on a study description. Even though PSMTs were able to develop a data collection plan, they struggled

more when asked to identify how to choose a sample to minimize bias. Only 64.5% were able to choose the correct sampling method; instead, 30% chose a convenience sample or a stratified sample that seemed complicated but not random.

Another common misunderstanding of PSMTs was the conclusion that could be drawn from a specific study design. Over 58% of PSMTs chose an answer that allowed a researcher to generalize results to an entire population based on a sample of volunteers. Of the 42% that knew the results from a volunteer sample could not be generalized to an entire population, only 31% identified a study design that would allow for comparison of a treatment and control group. These findings highlight PSMTs' need for a deeper understanding related to the ways in which study designs and data collection processes impact the conclusions that can be drawn.

PSMTs' average scores for Analyzing Data items were the second lowest among the phases and had the highest variability. 77% of PSMTs were able to identify which measure of center is appropriate for a given context and how measures of center and variation change when data values are changed. Approximately 70% of PSMTs demonstrated their ability to identify a justification of an association from a two-way table. PSMTs demonstrated more difficulty with Analyze Data items that involved the topics of variation of data represented as a histogram and distribution of sample means. Only 43% of PSMTs could identify a histogram containing data that varied the least from its mean. 30% of PSMTs chose a uniform distribution instead of a bell-shaped distribution and about 20% thought the variability from the mean was the same for all three distributions. PSMTs demonstrated another misunderstanding; they identified a distribution of sample means given the distribution of the population and population mean. 36% of PSMTs could not identify the

distribution of sample means, instead they chose distributions that resembled the population distribution. These results point to PSMTs' need for more knowledge in regards to variation, sample distributions, and distribution of sample statistics.

PSMTs scored the lowest on average on Interpret Results items. However on five of the 11 Interpret Results items, 84% or more of PSMTs answered the items correctly. Of these five items, four were Level B. PSMTs were able to: compare distributions in a context using the center and spread; demonstrate an understanding of the effect of sample size on sample mean; and interpret survey results with a given margin of error. These are important concepts often taught in middle and high school curricula. On the other six Interpret Results items, five items were from Level C. The percentage of PSMTs responding correctly to these items ranged from 21% to 48%, and their misunderstandings centered on ideas of formal inference. PSMTs struggled most on items regarding statistical significance, identifying and interpreting a p-value, and explaining confidence intervals. About half (48%) of PSMTs were able to interpret results given a large p-value and fail to reject the null hypothesis. However 40% of PSMTs chose a conclusion that a large p-value meant they could reject the null hypothesis. This result demonstrates PSMTs' lack of understanding of determining statistical significance given a p-value.

On another item regarding p-value, PSMTs were asked to reason if a p-value would be large or small for comparing means of two distributions given data on a dotplot. Only 35% of PSMTs were able to correctly identify that the p-value would be small due to the large gap between distributions. Almost 47% incorrectly answered that the p-value would be large due to a large gap between the distributions. These findings demonstrate that PSMTs on

average do not have an understanding of what it means to be statistically significant and what a p-value represents.

The item that PSMTs had the most difficulty with asked the test taker to explain the meaning of a 95% confidence interval for a mean. Approximately one-fifth chose the correct response that a 95% confidence interval represents that 95% of confidence intervals constructed from random samples, would capture the true mean. Almost half of PSMTs chose the response that there was a 95% probability that the mean was in between the lower and upper limits of the confidence interval. Approximately one-quarter of PSMTs chose a response that 95% of the data collected would fall between the lower and upper limits of the confidence interval. These misunderstandings highlight the need for PSMTs to have more experiences with interpreting and understanding confidence intervals.

Discussion and Conclusion

There are several findings of this study that are significant for mathematics teacher educators. The *Statistical Education of Teachers* report (Franklin et al., 2015) recommends that preservice teachers have opportunities in their statistics coursework to develop deep conceptual understandings of the statistical content they will be expected to teach. In addition, such conceptual understanding should be developed through many experiences engaging with all aspects of the statistical investigation cycle. These results provide empirical evidence that PSMTs in this study generally do not have strong conceptual understandings for teaching statistics to high school students. Additionally, PSMTs' statistical understanding decreases as the cycle progresses. Previous research has shown a similar trend with inservice teachers' and students' statistical knowledge measured by LOCUS (Jacobbe, 2015; Jacobbe, Foti, et al., 2014). Thus PSMTs need more experiences in

collecting data, analyzing data, and interpreting results to develop a deeper conceptual understanding of all aspects of the statistical investigative cycle.

As seen in the item analysis, PSMTs have developed similar strengths and weaknesses with concepts such as variation from a mean that high school and introductory college students develop (e.g., delMas & Liu, 2005). An important strength is that PSMTs are proficient at identifying an appropriate measure of center for a given context. This topic is heavily emphasized in school mathematics, yet research shows that students struggle understanding measures of center (e.g., Mokros, 1995). PSMTs' strength in understanding measures of center suggests they should be well equipped to assist their future students to develop stronger conceptions. PSMTs' weaknesses involve issues with variability, sampling distributions, p-values, and confidence intervals. Emphasis on these topics in high school has increased with the adoption of CCSSM. It appears that current experiences and opportunities for these PSMTs have not provided enough to foster a strong conceptual understanding.

In general, inservice teachers do not feel prepared to teach statistics due to a lack of content and pedagogical knowledge (Batanero et al., 2011). These findings, even though from a purposeful sample, suggest that the current cohort of PSMTs is likely no more prepared to teach statistics than most inservice teachers. There is a critical need for mathematics teacher education programs to reevaluate their programs to increase PSMTs' preparedness to teach statistics. Effort needs to be given to developing PSMTs' knowledge of and instructional approaches for concepts of variability, sampling distributions, and formal inference. One way to approach such program enhancements is for mathematics teacher educators to consider some of the boundary objects between mathematics and statistics, such as measurement and variability, as well as rich contextual problems, where fruitful lessons

could be developed (Groth, 2015). Lessons that incorporate real data and engage PSMTs in all phases of a statistical investigation could be structured to focus on comparing distributions to build from PSMTs' strengths in using measures of center to further develop their understanding of variability and formal inference.

Chapter 5: Preservice Secondary Mathematics Teachers Are Not Prepared to Teach Statistics: A Call for Reform in Mathematics Teacher Education Programs

Journal

This article was written as a report of original empirical research for the *Statistics Education Research Journal (SERJ)* whose audience consists of statisticians, statistics teacher educators, and mathematics teacher educators. This journal publishes research related to the teaching, learning, understanding, or assessment of statistics and probability. Guidelines require that a report not exceed 10,000 words in the body text. The analysis in this paper uses quantitative data about PSMTs' statistics content knowledge and their statistics teaching efficacy to highlight strengths and weaknesses and demographic factors that may or may not influence PSMTs' preparedness to teach statistics. The results would benefit the audience of SERJ of researchers, mathematics teacher educators, and statistics educators. For statistics and mathematics teacher educators, this manuscript provides timely evidence to support the need for increased emphasis on statistics in mathematics teacher preparation programs recommended in the *Mathematics Education of Teachers II* report and the recently released *Statistics Education of Teachers* report. For researchers, the results identify areas of statistics in need of deeper research of preservice teachers' statistical knowledge and statistics teaching efficacy.

Abstract

The purpose of this study is to provide researchers, mathematics educators, and statistics educators information about the current state in the U.S. of preservice secondary mathematics teachers' preparedness to teach statistics. To do so, statistical knowledge and

statistics teaching efficacy of 217 preservice secondary mathematics teachers (PSMTs) were examined. The results indicated that these preservice teachers do not have the statistical knowledge needed to teach high school students and are not confident to teach typical topics found in high school curricula. Factors that impact PSMTs' content knowledge and statistics teaching efficacy are reported as well as the relationship between statistical knowledge and statistics teaching efficacy. Implications and recommendations for mathematics teacher education programs are discussed.

Introduction

Statistics and data science are two of the fastest growing and most popular fields (Hardin et al., 2015). The pipeline to prepare the workforce for these disciplines begins in K-12, particularly in high school. Over the last decade there has been increased emphasis on statistics in standards documents meant to guide K-12 curriculum (Franklin et al., 2007; National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010; New Zealand Ministry of Education, 2006). However research has shown that inservice teachers are not prepared to teach statistics (e.g., Burrill & Biehler, 2011; Makar & Fielding-Wells, 2011) and struggle understanding how the statistical content should progress across grade levels (Jones & Tarr, 2010). Teachers often teach statistics procedurally, focusing on computations of statistical measures (Makar & Confrey, 2004) and creating graphical representations (Sorto, 2006). Therefore students who enroll in secondary mathematics teacher education programs in the U.S. have likely had minimal experience with statistics in their own K-12 educations. They have not had many opportunities to develop a conceptual understanding of the topics they are now expected to teach compared to other areas of mathematics; thus preservice teachers are likely no more prepared than inservice teachers to teach statistics (Franklin et al., 2007). To address the preparation of preservice secondary mathematics teachers (PSMTs), recent efforts in the U. S. such as the *Mathematics Education of Teachers II* report (Conference Board of the Mathematical Sciences, 2012) and the recent *Statistical Education of Teachers* (SET) report (Franklin et al., 2015) present recommendations for courses to develop statistical knowledge and pedagogy needed to teach statistics.

While those in the statistics education community do not need to be convinced that teachers are likely underprepared to teach statistics in secondary classrooms, the nuanced nature of their understandings, including strengths and weaknesses, and their beliefs and confidence in their abilities to teach statistics have not been documented on a large scale. Insight into PSMTs' understandings and confidence in teaching statistics can inform strategic changes to teacher education programs, which already include crowded curricula to prepare teachers for the myriad of responsibilities in teaching secondary mathematics. One of the fundamental recommendations from both the K-12 and College level Guidelines for Assessment and Instruction in Statistics Education (Franklin et al, 2007; Garfield et al, 2007) is that conceptual understanding of statistical ideas should be emphasized over procedures and computations. In order for teachers to enact this recommendation, they themselves must develop a conceptual understanding of the statistical content they are expected to teach (Franklin et al., 2015). If teachers do not have a conceptual understanding of statistics, they are not likely to have the knowledge they need to help students learn statistics content (Groth, 2013). Along with this statistical knowledge, teachers need pedagogical statistical knowledge "to assess students' level of understanding and plan next steps in the development of their statistical thinking" (Franklin et al. 2015, p. 3).

While examining knowledge needed to teach is important, researchers should also consider the non-cognitive aspects that teachers draw upon and how these are related to a teacher's preparedness to teach statistics (Ball et al., 2008; Fennema & Franke, 1992). Teachers' affect plays a crucial role in the pedagogical approaches they use, the time spent on a subject, and thus can impact students' learning (e.g., Love & Kruger, 2005; Pajares, 1992; Wilkins, 2008). Affect includes a teacher's beliefs, attitudes, and emotions towards

statistics. However there is a lack of research on secondary teachers' affect in regards to teaching statistics (Batanero et al., 2011). The limited research that has been conducted has been with elementary teachers. Researchers have found that a teacher's beliefs and attitudes towards statistics were related to their prior experiences with statistics, impact the choice of instructional tasks, and students' attitudes and beliefs towards statistics (Begg & Edwards, 1999; Eichler, 2008; Lancaster, 2008). A major aspect of teachers' belief systems is their self-efficacy for specific tasks associated with teaching, a construct known as teaching efficacy (McGee & Wang, 2014). Teachers with strong teaching efficacy for mathematics have been shown to use more innovative teaching strategies which in turn positively influence students' learning (Jarvis, Holford, & Giffin, 2003; Wilkins, 2008). Teachers who lack teaching efficacy for mathematics tend to use more procedure-based teaching strategies and tend not engage their students in high level tasks (Ross & Bruce, 2007b; Wilkins, 2008). Since a teacher's beliefs and attitudes play a large role, it is crucial when considering PSMTs' preparedness to teach statistics that PSMTs' affect as well as statistical knowledge is examined.

Background and Research Questions

Statistical knowledge. Building off of the work of Ball et al. (2008) on Mathematical Knowledge for Teaching, Groth (2007, 2013) developed a framework for Statistical Knowledge for Teaching. This framework consists of two domains: subject matter knowledge and pedagogical content knowledge. Within subject matter knowledge there are three types of content knowledge: common content knowledge, specialized content knowledge and horizon knowledge. Common content knowledge refers to knowledge gained through statistics courses and is considered common because it refers to knowledge used in

other professions that use statistics. This study examines the common content knowledge that PSMTs need to know to teach their high school students this same common content knowledge of statistics.

In 2007, the GAISE report laid out the statistical concepts that students need to develop in K-12 schooling and thus setting a minimum for the statistical knowledge PSMTs need to know to teach statistics. The GAISE framework consists of three levels A, B, and C (Franklin et al., 2007). Although there are not explicit definitions given for each level in the GAISE framework, the levels increase in statistical sophistication and become more abstract. Each level is aligned to specific statistical content. The content in level A represents topics for early or novice learners of statistics (no matter what grade level, but often introduced in elementary and middle school), level B represents slightly more advanced statistical content (often taught in middle school or early high school), and level C represents even more advanced content (typical taught in high school or introductory college courses) (Franklin et al., 2007).

The GAISE report recommends that within each of the three levels, students should learn statistical topics through engaging with the statistical investigative cycle (Wild & Pfannkuch, 1999). Though the investigative cycle is described slightly differently in different countries, in the US, and in the GAISE framework four components are emphasized: posing questions, collecting data, analyzing data, and interpreting results. Figure 11 shows how, as students' progress through the GAISE levels, they continue to engage in a statistical investigative cycle and learn more sophisticated ways to engage in each part of the cycle. To effectively prepare high school students to increase their sophistication across three levels

with all phases of a statistical investigation, PSMTs also need deep statistical knowledge across all three levels and all phases.

Figure 11. Increased sophistication of statistical investigative cycle through GAISE levels.

PSMTs' statistical knowledge. Over the last 25 years, there has been little research about the statistical knowledge of PSMTs or the misconceptions they develop (Batanero et al., 2011; Shaughnessy, 2007), even recently with the increased emphasis on statistics in high school mathematics with the adoption of CCSSM. The majority of research on preservice teachers' statistical knowledge has focused on elementary teachers (e.g., Browning et al., 2014; Groth & Bergner, 2006; Hu, 2015; Leavy, 2010; Leavy & O'Loughlin, 2006; Santos & da Ponte, 2014). The limited research conducted on PSMTs' statistical knowledge has been small-scale studies, from a small number of institutions on specific statistical content (e.g., Doerr & Jacob, 2011; Lesser et al., 2014; Makar & Confrey, 2005). For example, a recent

study by Casey and Wasserman (2015) examined 11 preservice teachers' statistical knowledge of informal lines of best fit from three universities. From these studies, research has shown that preservice secondary teachers: focus on procedures, computations, and algorithms; lack statistical reasoning skills; and have difficulty interpreting graphical representations. To date, only one large-scale study, conducted by Lee et al. (2014) examined how 204 preservice mathematics teachers from eight universities used dynamic statistical tools to conduct a statistical investigation. They found that preservice teachers who pose a broad statistical question engaged in more graphical augmentations (e.g., adding shaded regions, reference lines, or statistical measures) using dynamic statistical software. These graphical augments allowed preservice teachers to dive deeper into the data analysis and make connections to the context to support claims. For the field to truly understand PSMTs' statistical knowledge and pedagogical statistical knowledge, more small and large-scale studies are needed.

Statistics teaching efficacy. As mentioned before, one's preparedness to teach not only relies on cognitive aspects, but also affective constructs such as beliefs, attitudes, and self-efficacy. This focus of this study is on PSMTs' self-efficacy to do the job they for which they are preparing, which includes teaching statistics to high school students. Self-efficacy has grown from Bandura's (1977) social cognitive theory which consists of two main constructs: efficacy expectations and outcome expectations. Efficacy expectations are defined as the belief that an individual can successfully implement the behavior required to produce particular outcomes. Outcome expectations are defined as an individual's expectation of likely outcomes when performing a task. An individual's self-efficacy originates from the construct of efficacy expectations. Bandura (1986) defines self-efficacy

as “people’s judgments of their capabilities to organize and execute courses of action required to attain designated types of performance (p.391). Judgments of one’s own self-efficacy are task-specific and change over time (Bandura, 1977; Pajares, 1997). A teacher has two types of self-efficacy for each content area they teach: self-efficacy to know the content themselves and self-efficacy to teach the topic to students. In mathematics education, researchers have defined these two types of self-efficacy as: mathematics self-efficacy and mathematics teaching efficacy (Bates et al., 2011). Mathematics self-efficacy can be defined as a teacher’s belief in his/her ability to do mathematics (Hackett & Betz, 1989) and mathematics teaching efficacy can be defined as a teacher’s belief in his/her ability to teach mathematics to bring about student learning (Ashton, 1985; Enochs et al., 2000). Applying this to teaching statistics, teachers have two types of self-efficacy: statistics self-efficacy and statistics teaching efficacy. Statistics self-efficacy is a “teacher’s belief in his/her ability to do statistics” and statistics teaching efficacy as “a teacher’s belief in his/her ability to teach statistics to bring about student learning” (Lovett, Doerr, Thrasher, & Lee, under review, p. 4).

To measure statistics self-efficacy for college students, Finney and Schraw (2003) developed two instruments: Current Statistics Self-Efficacy (CSSE) and Self-Efficacy to Learn Statistics (SELS). CSSE measures one’s current statistics self-efficacy. Finney and Schraw define this as “confidence in one’s abilities to solve specific tasks related to statistics” (2003 p.164). SELS measures one’s self-efficacy to learn, which Finney and Schraw define this as “confidence in one’s abilities to learn the skills necessary to solve specific tasks related to statistics” (2003, p.164). These instruments, the first developed and validated to measure statistics self-efficacy, were designed to examine the statistics self-

efficacy of college students enrolled in a statistics course. Finney and Schraw found that the CSSE and SELS were positively related to each other and that there was a weak positive correlation between one's CSSE and one's statistical performance.

Only one instrument (known to the researchers) has been developed to measure statistics *teaching* efficacy: the Self-Efficacy for Teaching Statistics (SETS) instrument. This instrument has two versions: one to measure the statistics teaching efficacy to teach middle school students (Harrell-Williams et al., 2013) and the other to measure the statistics teaching efficacy to teach high school students (Harrell-Williams et al., 2014). Due to the recentness of this instrument, only a few studies have been conducted on statistics teaching efficacy on preservice teachers. One current study being conducted by the authors of SETS is to determine the relationship of preservice teachers' statistics teaching efficacy using the middle school SETS instrument and statistics self-efficacy using the CSSE instrument. Pearson correlation coefficient was computed to examine the relationship of the total SETS score and CSSE to score to be 0.819 (Harrell-Williams, personal communication). This provides evidence that an individual's self-efficacy to do statistics plays a crucial role in their statistics teaching efficacy. Fitzmaurice et al. (2014) have also examined the statistics teaching efficacy of preservice teachers through interviews and found that preservice teachers in their study were reluctant to teach statistics during their student teaching placements. Preservice teachers who did teach statistics during their placement reported an increase in efficacy following the placements.

Potential factors that influence PSMTs' statistical knowledge beliefs. Examining the literature on teacher education and mathematics education identified four factors that could be measured using participants' demographics that could potentially impact PSMTs'

statistical knowledge and/or statistics teaching efficacy: taking Advanced Placement (AP) Statistics, number of collegiate statistics courses, Carnegie Classification of the institution, and gender.

First, AP Statistics has grown rapidly in popularity. In 1999, the third year the course and exam were offered, over 25,000 students participated in the AP Exam (Roberts, Scheaffer, & Watkins, 1999). Now today over 180,000 students take the exam annually (Utts, 2015). The course is intended to cover the same material as a college introductory statistics course and is centered around four themes: exploratory data analysis, planning a study, probability, and statistical inference (Roberts et al., 1999). Thus, taking AP Statistics could potentially influence PSMTs' statistical knowledge and their statistics teaching efficacy.

Secondly, the influence of the number of mathematics courses taken at the university level on teachers' knowledge and beliefs and students' achievement has been a topic of research for over 45 years (Begle, 1972, 1979; Mewborn, 2001; Monk, 1994; Wilkins, 2008). Now with increased emphasis on preparing PSMTs to teach statistics and a recommendation in the SET report for PSMTs to take three statistic courses (Franklin et al., 2015), examining the influence of the number of statistics courses taken on PSMTs' statistical knowledge and beliefs is needed.

Research has also shown that universities with a doctoral and masters' programs are often better able to adopt policy and research recommendations for preparing preservice teachers (McCrory & Cannata, 2011). Therefore the Carnegie Classification™ may impact whether institutions have attended to suggestions for preparing teachers to teach statistics, and thus impact PSMTs' statistical knowledge and statistics teaching efficacy.

Finally, research on the relationship between gender and mathematics performance is abundant, with early studies indicated that males outperformed females in most areas of mathematics (e.g., Benbow & Stanley, 1982; Maccoby & Jacklin, 1974). Recent findings suggest the differences in mathematical performance have lessened (e.g., Devine, Fawcett, Szucs, & Dowker, 2012; Esienberg, Martin, & Fabes, 1996). However, in terms of mathematics self-efficacy there exists a gender difference called the "confidence gap" (Sadker & Sadker, 1994). Researchers have found that males often report higher levels of efficacy than females (Reis & Park, 2001; Seegers & Boekaerts, 1996), and gifted females are likely to underestimate their mathematics self-efficacy (Pajares, 1996; Reis & Park, 2001). Thus it is possible that these same findings could apply to statistical knowledge and statistics teaching efficacy.

Relationship between statistical knowledge and beliefs. Limited research has examined the relationship between teachers' mathematical knowledge and their mathematics teaching efficacy. The few published studies have been conducted with elementary teachers and have had inconsistent results. These studies have found that mathematics content knowledge and teacher efficacy beliefs are positively correlated (Swars et al., 2009), weakly positively correlated (Bates et al., 2011; McCoy, 2011), or uncorrelated (Swars et al., 2007). These inconsistent results are partially due to the instruments being used to measure mathematical content knowledge and mathematics teaching efficacy. For example, Bates et al. (2011) measured mathematics content knowledge using the Illinois Certification Testing System Basic Skills test, which is a test that is required for entrance into certified teacher education programs in Illinois. The skills on the Basic Skills test address more mathematical knowledge than the preservice elementary teachers are expected to teach. The mathematics

teaching efficacy instrument used in McCoy (2011) and Swars et al. (2009), MTEBI, does not situate mathematics teaching efficacy beliefs in mathematical tasks; instead it refers to mathematics teaching in general. In a qualitative study, Austin (2013) provided more evidence of inconsistent results when the majority of teachers had high mathematics teaching efficacy and low mathematical knowledge for teaching.

At this point, research on the relationship of statistical knowledge and statistics teaching efficacy of teachers is lacking. However, the relationship of statistical knowledge and other aspects of teachers' attitudes have been examined. Most of these studies have focused on elementary preservice teachers and have found a weak positive relationship between statistical knowledge and attitudes towards statistics (Nasser, 2004; Zientek et al., 2011). These studies used classroom tests to measure statistical knowledge instead of a validated instrument. Hannigan et al. (2013) investigated the relationship of statistical knowledge and attitudes towards statistics of preservice mathematics teachers using validated, widely-used instruments, the Comprehensive Assessment of Outcomes in a First Statistics course (CAOS; delMas et al., 2007) and Survey of Attitudes Towards Statistics (Schau, Stevens, Dauphine, & del Vecchio, 1995). Hannigan and colleagues still found no strong positive correlation between preservice teachers' statistical knowledge and attitudes toward statistics.

Purpose of the study. Given the context of statistics education in the US and increased demands on secondary teachers for teaching statistics, this study examines the preparedness of PSMTs to teach statistics as they enter student teaching. Therefore the research questions that will be addressed are:

1. What is PSMTs' statistical knowledge of the high school content they are expected to teach using the phases of a statistical investigation?
2. What is PSMTs' statistics teaching efficacy for teaching high school statistics?
3. What self-reported factors influence PSMTs' statistical knowledge and statistics teaching efficacy?
4. What is the relationship between PSMTs' statistical knowledge and statistics teaching efficacy?

Methodology

This paper is part of a larger mixed methods study on preparedness of PSMTs to teach statistics. The study utilizes an explanatory design, first quantitatively examining PSMTs' statistical knowledge and statistics teaching efficacy, and then qualitatively seeking factors and experiences that influence PSMTs' confidence through analysis of open-ended responses and interviews. This paper focuses on the results of the quantitative phase of the larger study. For more information on the qualitative results see Chapter 6.

Participating institutions. The population of interest for this study is PSMTs prepared through university-based teacher preparation programs in the United States. Over the past 15 years, several efforts have been made to increase the ways mathematics teacher education faculty prepare PSMTs for teaching statistics. Since a list of all universities in the US that prepare PSMTs is not readily accessible, a purposeful sampling was used rather than using a random sample. This study focused on PSMTs who currently attend institutions where at least one faculty member participated in either a National Science Foundation (NSF)-funded or American Statistical Association (ASA)-funded program to increase the emphasis of statistics education of teachers at that institution between 2002-2014. This

narrowed the possible sample to 57 institutions whose faculty had participated in the NSF-funded program, Preparing to Teach Mathematics with Technology (PTMT, ptmt.fi.ncsu.edu), and/or the ASA-funded Math/Stat Teacher Education: Assessment, Methods, and Strategies (TEAMS, http://www.amstat.org/sections/educ/newsletter/v9n1/TEAMS.html). All 57 institutions were contacted, and 18 institutions agreed to participate in this study during the 2014-2015 school year. Of the 18 institutions in the study all but one were public institutions.

Table 12 displays the Carnegie Classification™ (Carnegie Foundation for the Advancement of Teaching, 2011) for the institutions participating in the study. The majority of institutions (61.1%) had an enrollment profile of high undergraduate and the majority of participants attended institutions with a basic classification of research universities (very high), research universities (high), or Master's college and university with a larger program.

Table 14. *Carnegie Classification™ of participating institutions*

	Universities	Participants
	n (%)	*n* (%)
Enrollment Profile		
VHU: Very High Undergraduate	7 (38.9)	68 (31.3)
HU: High Undergraduate	11 (61.1)	149 (68.7)
Basic		
RU/VH: Research Universities (Very High)	4 (22.2)	67 (30.9)
RU/H: Research Universities (High)	6 (33.3)	54 (24.9)
D/RU: Doctoral/Research Universities	2 (11.1)	16 (7.4)
Master's L: Master's Colleges and Universities	4 (22.2)	61 (28.1)
(Larger)	1 (5.6)	6 (2.8)
Master's S: Master's Colleges and Universities		
(Smaller)	1 (5.6)	13 (5.9)
Bac/Diverse: Baccalaureate College – Diverse Fields		

Participants. Across 18 institutions there were 236 PSMTs who participated in some aspect of the study. Participants who did not complete all aspects of the study and those who took exceptionally less time to complete the content assessment than recommended by the authors of the assessment (less than ten minutes) were eliminated (Jacobbe, personal communication). This resulted in a sample size of 217 PSMTs consisting of undergraduate juniors and seniors, or graduate students earning initial licensure enrolled in their last mathematics education course prior to student teaching. The majority of PSMTs were female (71%) and 88% were Caucasian. Table 13 shows the demographics of the PSMTs in regards

to taking AP Statistics in high school, the self-reported number of college-level statistics courses taken, and the degree program in which they were enrolled. The majority of PSMTs (59%) had taken one or two statistics courses at the time of the study.

Table 15. *Demographics of the 217 participating PSMTs*

Characteristics of PSMTs	*n* (%)
High School Course	
AP Statistics	39 (18)
Number of University Statistics Courses	
0	10 (4.6)
1-2	128 (59)
3-5	71 (32.7)
6 or more	8 (3.7)
Degree Program	
Bachelor's	201 (92.6)
Master's	16 (7.4)

Instruments. Two instruments were used in this study to measure PSMTs' statistics teaching efficacy and their content understanding of statistics. The instruments were administered online in the final few weeks of the participants' last mathematics methods course before student teaching. Participants took the statistics teaching efficacy instrument first, and then shortly afterwards took the content assessment. Details about each instrument follow below.

Statistics teaching efficacy. To examine PSMTs' statistics teaching efficacy, the high school version of the Self Efficacy to Teach Statistics (SETS; Harrell-Williams et al., 2014)

instrument was administered. This instrument was chosen because it collects both qualitative and quantitative data about PSMTs' statistics teaching efficacy. Furthermore, the SETS instrument is aligned with the GAISE framework, which reflects the content that PSMTs are expected to teach to high school students. Hence there is a close correspondence between the teaching efficacy instrument and the specific topics PSMTs need to know and to teach. The closer this correspondence, the better prediction an instrument can make on the performance of the task, which in this case is teaching statistics to high school students (Finney & Schraw, 2003).

The instrument contains 44 six-point Likert scale items. An earlier version of this instrument with 26 items, aligned with levels A and B of GAISE, was validated for use in measuring changes in elementary and middle grades preservice teachers' self-efficacy as a result of interventions, such as a course (Harrell-Williams et al., 2013). The high school version contains the previous 26 items aligned to GAISE levels A and B and contains an additional 18 items validated and aligned to GAISE level C (Harrell-Williams, personal communication). In addition to an overall score, the instrument provides sub-scale scores that correspond to Levels A, B and C in the GAISE framework. There are 11 Likert items for level A, 15 items for level B and 18 items for level C. For all Likert items, the stem of the question was

> "Rate your confidence in teaching high school students the skills necessary to complete successfully the task given by selecting your choice on the following scale: 1 = not at all confident, 2 = only a little confident, 3 = somewhat confident, 4 = confident, 5 = very confident, 6 = completely confident" (Harrell-Williams et al., 2014).

Therefore, for each participant, the SETS instrument produces a confidence score to teach high school students statistics for GAISE level A, GAISE level B, GAISE level C, and an overall score. There may be concern that PSMTs would have a tendency to present favorable images of their statistics teaching efficacy since they are self-reporting (Ross, 1989); however, research has shown that there is little motivation to misreport since the confidentiality was preserved (Baldwin, 2000). At the conclusion of the SETS instrument, participants were asked to answer demographic questions about themselves and their statistical preparation.

Statistical knowledge. To examine PSMTs' statistical knowledge, the Levels of Conceptual Understanding of Statistics (LOCUS) assessment (Jacobbe, Case, et al., 2014) was administered online (locus.statisticseducation.org). The LOCUS assessment is aligned with the CCSSM and assesses understanding across the three levels of development in the GAISE framework (Franklin et al., 2007). A 23-item paper version of the LOCUS assessment has been validated as a measure to reliably assess current statistical understanding (Jacobbe, Case, et al., 2014). Participants in this study took the Intermediate/Advanced Statistical Literacy version of the assessment, which was designed for students in grades 10 – 12. This test was chosen because this assessment represents the content that PSMTs are expected to teach to their students in the near future when they begin teaching. This assessment consists of 30 questions: 20 questions that align with levels B and C of the GAISE framework and 10 equator questions aligning with all three levels. These 30 questions contain the previously validated 23 items and an additional 7 items.[5] These 30 questions are also aligned with the four phases of an investigative cycle: Forming Questions,

[5] This 30-item version of LOCUS has been validated, with manuscript currently under review (Jacobbe, personal communication).

Collecting Data, Analyzing Data, and Interpreting Results. For a description of each type of question and sample questions see Jacobbe, Foti, et al. (2014). Therefore for each participant, the LOCUS assessment produces a score for GAISE level B, GAISE level C, Forming Statistical Questions, Collecting Data, Analyzing Data, Interpreting Results, and an overall score.

Analysis of data. Analysis of the LOCUS scores began by generating descriptive statistics and distributions for PSMTs' LOCUS overall scores, Level B scores, Level C scores, and scores for each portion of the statistical investigative cycle. Paired sample t-tests were used to test for differences between PSMTs' statistical knowledge in GAISE Levels B and C, and a repeated measures ANOVA was used to test for significant differences in PSMTs' statistical knowledge between the four phases of a statistical investigation. Paired sample t-tests were appropriate since the samples are approximately normal. A repeated measures ANOVA was used since the assumptions of normality and sphericity were not validated.

For analysis of PSMTs' statistics teaching efficacy, it was necessary to account for missing data since every PSMT did not complete every SETS item. The percentage of missing values ranged from 0 to as high as 4.1% for some items, and only 91% of the 217 PSMTs in the sample would have been available for analysis under listwise deletion. Since data was primarily missing due to nonresponse on certain items, this was addressed by using multiple imputation (Allison, 2002). Nine imputed data sets were created using SPSS, since approximately nine cases were missing (White et al., 2011). Following imputation, an overall score was calculated for each PSMT by calculating the sum of his/her Likert scores for all of the items and then dividing by the total number of items. Sub-scale scores for each GAISE

level were also calculated for each PSMT by calculating the sum of the Likert scores and dividing by the number of items in each level. This resulted in four scores for each PSMT that corresponded to the six-point Likert scale for each of the nine imputed data sets. Using Rubin's (1987) rules, analyses run on each imputed data set were pooled and were examined to compare pooled values to the original data. The results of the pooled data were similar to the original data thus imputed results will be presented. To test for significant differences in PSMTs' statistics teaching efficacy between the three GAISE levels, repeated measures ANOVA tests were used. A repeated measures ANOVA was used since the assumptions of normality and sphericity were not validated.

Following descriptive analysis, the LOCUS and SETS scores were explored to identify demographic factors that may increase PSMTs' statistical knowledge and statistics teaching efficacy, respectively. For binary factors (e.g., took AP Statistics, did not take AP Statistics), two-sample *t*-tests were run to compare the difference in means of the two groups to examine for which factors there is a significant difference on PSMTs' statistical knowledge or statistics teaching efficacy. Two-sample *t*-tests are appropriate since the samples are independent and identically distributed. For other factors such as basic Carnegie classification or number of statistics courses taken, a nonparametric Kruskal-Wallis test was conducted due to the small sample size in some groups. Finally, to analyze the relationship between statistical knowledge and statistics teaching efficacy of PSMTs, analysis included examining the correlation of the total LOCUS and SETS, level B LOCUS and level B SETS, and level C LOCUS and level C SETS.

Results

First, the statistical knowledge and statistics teaching efficacy of PSMTs measured by the LOCUS and SETS, respectively is described. Next, the factors identified in the demographic questions and whether they appear to influence PSMTs' scores are presented. Finally, the relationship between PSMTs' statistical knowledge and statistics teaching efficacy is discussed.

Statistical knowledge. These results describe what PSMTs from 18 universities currently understand about the statistics content they will soon be responsible for teaching. Table 14 reports summary statistics for PSMTs' scores on the LOCUS assessment overall, for GAISE Levels B and C. With an overall mean score of 69% and a standard deviation of 14.06, these results suggest that these PSMTs do not demonstrate a strong conceptual understanding of the statistics content they will soon teach high school students. Figure 12 shows the distribution of PSMTs' LOCUS scores. As seen in the boxplots for the overall scores, there are at least some PSMTs who scored in the 90-100%, indicating a strong statistical knowledge of topics they will soon be responsible to teach. However only one fourth of PSMTs scored above 77%, and one fourth scored below 57% overall.

Table 16. *PSMTs' statistics content scores*

	Number of items	Mean	SD
Overall Score	30	68.61	14.06
GAISE Levels			
Level B Score	11	70.85	17.69
Level C Score	17	64.87	14.16
Phases of Statistical Investigative Cycle			
Formulating Questions	5	80.37	21.51
Collect Data	7	70.40	19.70
Analyze Data	7	63.34	22.22
Interpret Results	11	60.48	16.25

Note. Two items are classified as GAISE level A, but were not analyzed due to the small number.

In terms of the GAISE level subscores, PSMTs scored, on average, significantly higher on the Level B items than on Level C items ($t=5.772$, $p<0.001$), demonstrating that their statistical knowledge is weaker as items increase in sophistication. The boxplots show that for the subscores, there are at least some PSMTs who demonstrated a strong grasp of the statistical concepts they will soon be responsible to teach for both GAISE levels; however, there is a concern for the majority of PSMTs since more than 75 percent of them scored below 80% overall. The interquartile range for the overall score and Levels B and C are similar, though more outliers exist for Level B (the star in Figure 12 refers to an extreme outlier, which is a value more than three times the interquartile range less than either Q1 or Q3),

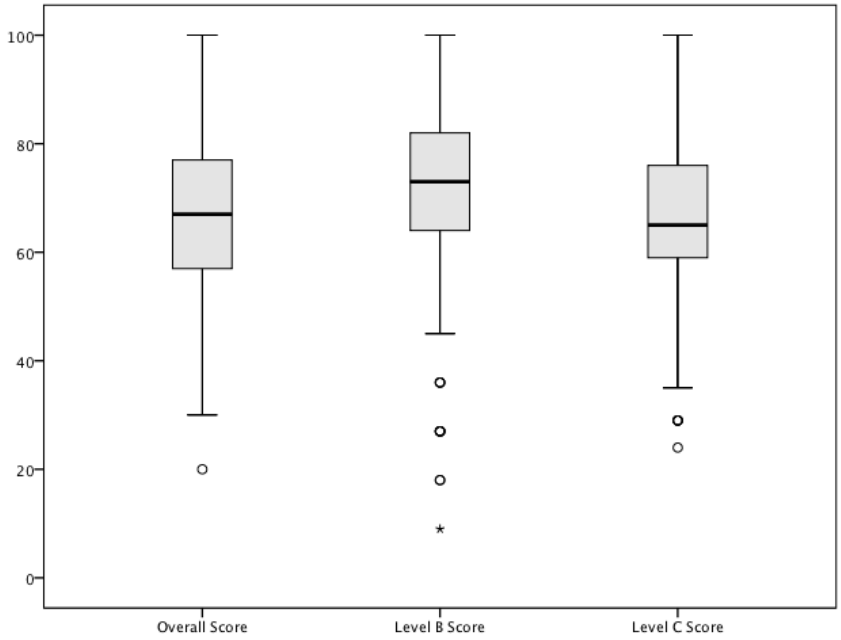

Figure 12. Distribution of PSMTs' LOCUS scores.

Examining scores by the phases in the statistical investigative cycle, Table 14 and Figure 13 shows that PSMTs scored highest on Formulating Questions and the lowest on Interpreting Results items. A repeated measures ANOVA determined that mean scores differed significantly between the four phases of the cycle [$F(3,648)=64.73$, $p<0.001$]. Post hoc tests using a Bonferroni correction revealed that PSMTs scored significantly lower on questions as the cycle progressed ($p<0.001$). There was no real difference between mean scores for Analyze Data and Interpret Results ($p=0.32$). The boxplots in Figure 13 show that for all four phases, there are some PSMTs who scored very well, indicating that those PSMTs likely have the content knowledge needed for teaching that phase of an investigative cycle. On Formulating Questions items, at least half of PSMTs scored 80% or higher, and a quarter of those scored 100%, indicating stronger understanding for these PSMTs about

Formulating Questions. There are concerns regarding the other phases of the investigative cycle. Half of PSMTs scored below 71% on Collecting Data and Analyzing Data items, and half scored below 64% on Interpreting Results items. Even being conservative, this result is convincing that the majority of PSMTs do not have the statistical foundation needed to teach students key concepts related to Collecting Data, Analyzing Data, and Interpreting Results.

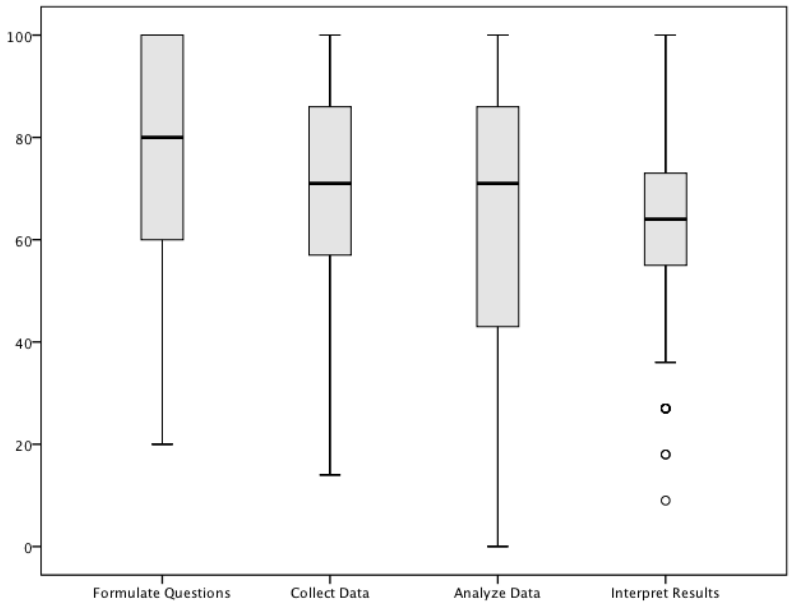

Figure 13. Distribution of PSMTs' scores for phases of statistical investigative cycle.

An examination of individual items identified several strengths and weakness of PSMTs' statistical knowledge. As previously mentioned, PSMTs scored the highest on average for Formulating Questions items, in these items PSMTs' demonstrated a strength in their ability to read a description of a study and measurements taken to identify the statistical question of interest. For Collecting Data items, one strength of PSMTs' was their ability to identify a data collection plan based on a study description. In terms of Analyzing Data and Interpreting Results, PSMTs are proficient at identifying an appropriate measure of center for

a given context and comparing distributions in a context using the center and spread, topics that are heavily emphasized in school mathematics. This strength in understanding measures of center suggests that PSMTs should be well equipped to assist their future students develop stronger conceptions of measures of center beyond the standard algorithms.

Weaknesses demonstrated by the largest number of PSMTs were seen in the Analyze Data and Interpret Results items. This is not surprising since PSMTs scored on average the lowest in these two phases. These weaknesses involve issues in understanding variability, sampling distributions, *p*-values, and confidence intervals. Even though emphasis on these topics in high school has increased with the adoption of CCSSM, PSMTs' statistics and mathematics education courses have not developed a deep understanding of these topics. For a deeper description of the item analysis, see Chapter 4.

Statistics teaching efficacy. PSMTs completed the SETS instrument by rating their confidence to teach statistics from 1 to 6 so that 1 = not at all confident, 2 = only a little confident, 3 = somewhat confident, 4 = confident, 5 = very confident, 6 = completely confident" (Harrell-Williams et al., 2014). Table 15 reports summary statistics for PSMTs' confidence scores on the SETS instrument overall and for GAISE Levels A, B and C. These results show that on average PSMTs are *confident* to teach high school students statistics and are most confident in GAISE Level A items. On average, teachers only felt between *somewhat confident* and *confident* in their ability to teach level C topics. PSMTs' confidence decreased as the statistical sophistication of the items increased. A repeated measures ANOVA determined that mean scores differed significantly between the three levels [$F(2,432)=60.04$, $p<0.001$]. Post hoc tests using a Bonferroni correction indicate that PSMTs scored significantly lower as the statistical sophistication of items increased ($p<0.001$).

Table 17. *PSMTs' confidence scores for teaching statistics*

	Number of items	Mean	SD
Overall Score	44	4.10	0.78
GAISE Levels			
Level A Score	11	4.54	0.79
Level B Score	15	4.12	0.82
Level C Score	18	3.80	0.89

Factors influence statistical knowledge or statistics teaching efficacy. On the SETS and the LOCUS assessment, PSMTs answered several demographic questions that were used to determine whether any factors appear to influence statistical knowledge and/or statistics teaching efficacy. Potential factors that could influence at least one construct were if the PSMT took AP Statistics in high school, number of statistics course taken at their university, the Carnegie Classification of their university, and PSMTs' gender. These factors were tested for their influence on statistical knowledge and statistics teaching efficacy.

***AP Statistics*.** Of the 217 PSMTs who participated in the study, 39 reported (18 %) they took AP Statistics in high school. Taking AP Statistics was a factor shown to be statistically significant for influencing both statistical knowledge and statistics teaching efficacy. To examine the influence of taking AP Statistics on statistical knowledge, independent two samples t-tests were conducted to compare overall scores and GAISE Levels B, and C, respectively, on the LOCUS instrument for PSMTs who took AP Statistics in high school and those who did not. Table 16 shows the relationship between the statistical knowledge of PSMTs and if they took AP Statistics. PSMTs who took AP Statistics scored higher on average on all LOCUS items except for items regarding collecting data. PSMTs

who took AP Statistics scored statistically significantly higher on GAISE Level B, Analyzing Data, and Interpreting Results items than those who did not.

Table 18. *Statistics content scores by whether a PSMT took AP Statistics*

	AP Statistics		*t*	*df*
	Yes N=39	No N=178		
Overall Score	69.82 (13.61)	65.90 (14.10)	-1.58	215
GAISE Levels				
Level B	75.26 (15.95)	69.89 (17.95)	-1.72*	215
Level C	66.80 (15.25)	64.44 (13.92)	-0.94	215
Phases of Statistical Investigative Cycle				
Formulating Questions	82.56 (21.12)	79.89 (21.63)	-0.70	215
Collect Data	67.39 (21.74)	71.06 (19.23)	1.05	215
Analyze Data	71.36 (18.83)	61.58 (22.56)	-2.51**	215
Interpret Results	64.74 (15.28)	59.54 (16.35)	-1.82*	215

Note. * = $p \leq 0.05$, ** = $p \leq 0.01$. Standard deviations appear in parenthesis below means.

To examine the influence of taking AP Statistics on statistics teaching efficacy, independent two samples t-tests were conducted to compare overall confidence and GAISE Levels A, B, and C, respectively, on confidence scores for PSMTs who took AP Statistics in high school and those who did not (see Table 17). PSMTs who took AP Statistics reported on

average higher statistics teaching efficacy on all SETS items; however, those 39 participants who took AP Statistics only had statistically significant higher confidence for Level C items than those who did not take AP Statistics.

Table 19. *Confidence scores according to whether a PSMT took AP Statistics*

	AP Statistics		*t*	*df*
	Yes N=39	No N=178		
Overall Score	4.25 (0.63)	4.06 (0.81)	-1.33	215
GAISE Levels				
Level A	4.60 (0.67)	4.53 (0.82)	-0.50	215
Level B	4.25 (0.66)	4.09 (0.85)	-1.05	215
Level C	4.03 (0.69)	3.75 (0.93)	-1.78*	215

Note. * = $p \leq 0.05$. Standard deviations appear in parenthesis below means.

Number of statistics courses. To analyze the impact of the number of statistics courses taken at a university on the statistical knowledge of PSMTs a Kruskal-Wallis test was conducted to evaluate differences among the four categories of statistical courses taken (0, 1 – 2, 3 – 5, 6 or more) on median scores from the LOCUS assessment. The test, which corrects for tied ranks, was not statistically significant for the overall score or any subscore categories [overall (χ^2 *df (3, 217) = 5.784, p=0.12),* Level B (χ^2 *df (3, 217) = 5.118, p=0.16),* Level C (χ^2 *df (3, 217) = 6.018, p=0.11)*]. The LOCUS assessment represents statistical knowledge that PSMTs will be teaching, thus the number of statistics college

courses does not appear to statistically increase PSMTs' understanding of the school statistics they will need to teach.

Even though the number of statistics courses taken at an institution did not statistically influence PSMTs' scores on a test of their statistical knowledge, it did impact the statistics teaching efficacy. A Kruskal-Wallis test was conducted to evaluate differences among the four categories of the number of statistics courses present in this study (0, 1-2, 3-5, 6 or more) on median scores on SETS instrument. The test, which correct for tied ranks, was significant for overall scores (χ^2 *df (3, 217) = 7.65, p=0.05),* Level A scores (χ^2 *df (3, 217) = 8.99, p=0.03)*, and Level B scores (χ^2 *df (3, 217) = 9.70, p=0.02)*. Follow-up tests were conducted to evaluate pairwise differences among the four groups, controlling for Type 1 error across tests by using the Bonferroni approach. The results of these tests indicated significant differences between PSMTs who did not take statistics, who took one to five courses, and who took six or more. The typical confidence score on the SETS instrument increased with the number of statistics courses taken. However, there was not a significant difference between PSMTs who took 1 – 2 courses and those who took 3 – 5 courses.

Carnegie Classification. To analyze the influence of Carnegie Classification on statistical knowledge and statistics teaching efficacy, non-parametric tests were run due to the low number of PSMTs for some classifications. The basic Carnegie Classification for institutions was statistically significant for both statistical knowledge and statistics teaching efficacy. The type of institution attended by a PSMT showed to have an impact on their statistical knowledge. A Kruskal-Wallis test was conducted to evaluate differences in median content scores among the six basic Carnegie Classifications present in this study (RU/VH, RU/H, D/RU, Master's L, Master's S, and Bac/Diverse). The test was significant for overall

scores (χ^2 *df (5, 217) = 20.86, p=0.001),* Level B scores (χ^2 *df (5, 217) = 26.61, p=0.000),* Level C scores (χ^2 *df (5, 217) = 12.77, p=0.026),* Collect Data scores (χ^2 *df (5, 217) = 16.87, p=0.005)*, Analyze Data scores (χ^2 *df (5, 217) = 11.66, p=0.04)*, and Interpret Results scores (χ^2 *df (5, 217) = 18.751, p=0.002).* Follow-up tests were conducted to evaluate pairwise differences among the six groups, controlling for Type 1 error across tests by using the Bonferroni approach. The results of these tests indicated significant differences within institutions with three research classifications (RU/VH, RU/H, D/RU) and differences within institutions with the two master's classifications (ML, MS). For universities with a research classification, PSMTs' overall, Level B, Level C, Collect Data, Analyze Data, and Interpret Results scores increased as the level of research in the university increased. For universities with a Masters' classification, PSMTs' overall, Level B, Level C, Collect Data, Analyze Data, and Interpret Results scores increased with the size of the Masters' program.

To examine the influence of the Carnegie Classification on PSMTs' statistics teaching efficacy, a Kruskal-Wallis test was conducted to evaluate differences among the six basic Carnegie Classifications present in this study (RU/VH, RU/H, D/RU, Master's L, Master's S, and Bac/Diverse) on median scores on the SETS instrument. The test was significant for overall scores (χ^2 *df (5, 217) = 21.32, p=0.04)* and Level C scores (χ^2 *df (5, 217) = 11.49, p=0.04).* Follow-up tests to evaluate pairwise differences among the six groups, controlling for Type 1 error across tests by using the Bonferroni approach, revealed no trend in the research level of the institution and PSMTs' statistics teaching efficacy.

Gender. To examine whether gender differences exist for PSMTs' statistical knowledge and statistics teaching efficacy, independent t-tests were conducted to compare content scores and statistics teaching efficacy scores for males (n=63) and females (n=154).

While there was not a significant difference based on gender for either type of score, for statistics teaching efficacy scores, there was a trend that male PSMTs had slightly higher, but non-significant, statistics teaching efficacy than female PSMTs overall, and in each GAISE level A, B, and C.

Relationship between statistical knowledge and statistics teaching efficacy. To analyze the relationship between statistical knowledge and statistics teaching efficacy of PSMTs, Pearson product-moment coefficients were computed for the overall scores and subscores, respectively. The overall content score was weakly positively correlated with the overall teaching efficacy score (r =0.22, p = 0.0001). PSMTs with more statistical knowledge, measured by the LOCUS, were more likely to feel more confident to teach statistics, as measured by SETS. Examining the Level B scores on both instruments shows that PSMTs' level B content scores were also weakly positively correlated with their teaching efficacy scores (r =0.247, $p < 0.0001$). PSMTs' Level C content scores were also weakly positively correlated with the Level C teaching efficacy scores (r =0.14, p = 0.046). Level C scores showed the weakest relationship between statistical knowledge and statistics teaching efficacy.

To predict the relationship of statistics teaching efficacy based on statistical knowledge, a simple linear regression was calculated since all assumptions were met. A significant regression equation was found ($F(1, 215) = 10.974$, $p=0.001$), with an R^2 of 0.044. PSMTs' predicted statistics teaching efficacy is 3.28 + 0.012 (statistical knowledge) measured by LOCUS when statistics teaching efficacy is measured by SETS. Since this model is only accounting for 4.4 percent of the total variation, a multiple linear regression model was calculated to predict statistics teaching efficacy based on statistical knowledge, if

AP Statistics was taken in high school, number of statistics courses, the university's Carnegie Classification, and gender. A significant regression equation was found ($F(5, 211) = 4.114$, $p=0.001$), with an R^2 of 0.067, with the LOCUS score being statistically significant ($t=3.145$, $p=0.002$), but other factors were not significant in the model, specifically, taking AP Statistics ($t=0.759$, $p=0.448$), number of statistics courses ($t=1.658$, $p=0.097$), Carnegie Classification ($t=1.303$, 0.193), or gender ($t=1.838$, $p=0.66$). Only 6.7% of the variation in statistics teaching efficacy is explained by the LOCUS score, taking AP Statistics, the number of statistics courses, Carnegie Classification of the institution, and gender. Therefore the fitted regression line from this model containing 217 PSMTs is the simple linear regression model 3.28 + 0.012 (statistical knowledge) measured by LOCUS when statistics teaching efficacy is measured by SETS. Since this model has a large amount of unexplained variance there is likely many other variables contributing to statistics teaching efficacy beyond what was measured in this study and used in the model.

Discussion

There are several findings of this study that are significant for statistics educators and mathematics teacher educators working with PSMTs. First, because of the increased emphasis of statistics in the U.S. high school curriculum, it is important that PSMTs are prepared to teach statistics. The results of this study provide insight on the current landscape of PSMTs' statistical knowledge and statistics teaching efficacy. PSMTs in this study were chosen from a purposeful sample of universities that had faculty members who have participated in programs to increase statistics education. Thus I expected that these might be receiving more emphasis on statistics in their mathematics teacher education programs than the average PSMT. However, these PSMTs still generally do not have strong conceptual

understandings of the statistics content they will be expected to teach and over half do not feel confident to teach statistics. Additionally, PSMTs' statistical understandings decrease as the investigative cycle progresses, that is, they are far less knowledgeable about analyzing data and interpreting results than they are about formulating questions and collecting data. Previous research has shown a similar trend with inservice teachers' and students' statistical knowledge measured by LOCUS (Jacobbe, 2015; Jacobbe, Foti, et al., 2014). Similar to statistical knowledge, PSMTs' statistics teaching efficacy decreased as topics get more sophisticated, being, on average, only *somewhat confident* to *confident* to teach high school level statistics content. Thus the experiences of PSMTs during the mathematics teacher education programs are not adequate in preparing PSMTs for all aspects of the statistical investigative cycle and especially with content at a GAISE Level C in sophistication.

Secondly, the fact that the number of statistics courses, Carnegie Classification, and taking AP Statistics were identified as factors that had an impact on PSMTs' statistical knowledge and/or statistics teaching efficacy provides evidence to support prior research and recommendations in the *SET* Report (Franklin et al, 2015). These results about the impact of the number of statistics courses taken in college supports previous research on preservice teachers' attitudes towards statistics; specifically, the number of advanced mathematics (statistics) courses do not increase preservice teachers' understanding of school mathematics (statistics), but do increase their attitudes towards mathematics (Ball, 1990; Wilkins, 2008). In the results reported here, I do not have details about the nature and content of the statistics courses taken by these PSMTs[6]. This study only used their self-reported data on courses taken, without asking for names or content of those courses. Some may have taken a typical

[6] A manuscript is under review that reports more on PSMTs' perspectives on the nature of the statistics courses taken by the participants in this study.

introductory statistics course, and others may have taken calculus-based statistics and/or probability theory courses that they reported as statistics courses. In the SET report, Franklin and colleagues (2015) recommend that preservice secondary mathematics teachers take three statistics courses with the following emphases: 1) an introductory statistics course that emphasizes data analysis, simulation-based approaches to inference with technology, and an introduction to formal inference, 2) a statistical methods course building from the introductory course utilizing randomization and classical approaches to inference, and 3) a statistical modeling course based on multiple regression techniques. The introductory course would provide PSMTs with foundational statistical knowledge and experiences that they have not experienced in their own K-12 education. The results of this study support this recommendation since the small number of PSMTs who took AP Statistics in high school, experiencing data analysis and introduction to formal inference, demonstrated statistically significantly higher statistics teaching efficacy and statistical knowledge than those that did not.

Finally as seen in the analysis, results indicate that statistics teaching efficacy is weakly positively correlated with statistical knowledge. This finding supports previous research of preservice elementary teachers that their mathematics teaching efficacy is weakly positively correlated to mathematical knowledge (Bates et al., 2011). This study was the first to compare statistics or mathematics teaching efficacy to content knowledge using a teaching efficacy instrument that is context specific, asking participants to consider teaching specific topics. Since efficacy beliefs are task specific constructs (Bandura, 1997), I anticipated a closer relationship between statistics teaching efficacy and statistical knowledge as measured by the SETS and LOCUS. Even with the SETS instrument measuring specific tasks across

three GAISE levels, and thus having a closer alignment with the LOCUS assessment, there is not a strong relationship between PSMTs' statistics teaching efficacy and statistical knowledge in our sample. PSMTs in this study took the SETS instrument prior to taking the LOCUS assessment. Therefore, PSMTs may have over- or under-estimated their statistics teaching efficacy because they were not aware of the content they were expected to teach. Further research is needed to determine if the order in which PSMTs take the LOCUS and SETS influences the relationship between statistics teaching efficacy and statistical knowledge.

Also, since researchers continue to find a weak positive relationship between content knowledge and self-efficacy (e.g., Bates et al., 2011; McCoy, 2011), this suggests that content knowledge alone is not a strong predictor or maybe not even a predictor of one's self-efficacy. As seen in the model, taking AP Statistics, the number of statistics courses, the university's Carnegie Classification, or gender leave a large amount of unexplained variance. If a relationship exists between statistics teaching efficacy and statistical knowledge, there must be other factors to explain the relationship.

These results also lead to several recommendations for mathematics teacher education programs. First, mathematics teacher education programs cannot rely on PSMTs to learn all of their statistical knowledge through statistics courses. Even though requiring PSMTs to take an introductory course will possibly increase PSMTs' statistical knowledge and confidence to teach statistics, that alone is not enough to prepare PSMTs to teach statistics. Research has shown, that in addition to content, teachers need to develop pedagogical content knowledge (Hill, Ball, & Schilling, 2008). Thus mathematics teacher education programs need to attend to developing PSMTs' statistical knowledge and pedagogical

statistical knowledge in mathematics methods courses. Pedagogical *statistical* knowledge includes a teacher's knowledge of potential student difficulties with statistics, strategies to support student's learning, strategies to engage students in a statistical investigative cycle, and knowledge of statistics curricula (e.g., Groth, 2013). I suggest a substantial focus on three areas. First, mathematics teacher educators should work with statistics instructors to identify high school topics that PSMTs are not getting enough experience with such as variability, sampling distributions, and inference as identified in this study. These topics should be included so that PSMTs can develop statistical knowledge of these topics that are difficult to understand and teach. Secondly, mathematics teacher educators should allow PSMTs to develop a deeper understanding of statistical topics by engaging them in tasks that include all four phases of the statistical investigative cycle. The tasks will model the way PSMTs should engage their future students in exploring statistical topics and deepening their knowledge simultaneously. As shown by Lee et al (2014), statistical investigations can be greatly enhanced when PSMTs use dynamic statistical software to investigate broad statistical questions. Thus the final recommendation is that mathematics teacher education programs include experiences for PSMTs to: engage with dynamic statistical software to during meaningful statistical investigations; develop concepts such as sampling distribution and simulation approaches to inference; and assist gain pedagogical strategies for developing students' conceptions.

This study had a number of limitations, which should be taken into consideration when interpreting the findings. First, this study was a purposeful sample on institutions across the U.S. and was not a random sample of all PSMTs. While most studies in teacher education that have explored PSMTs' statistical knowledge and/or teaching efficacy have

been conducted at one institution or a small number of institutions, this study was conducted across 18 institutions varying in program size and location in the U.S., which may expand the potential usefulness of the results. However, of importance is that this purposeful sample was from those institutions with at least one faculty member that had participated in a project aimed at increasing preparation of mathematics teachers to teach statistics. While I have no evidence of how the various teacher education programs actually attend to the preparation of secondary mathematics teachers to teach statistics, these results suggest that these efforts may not be having a strong impact on the current cohort of PSMTs represented in this study. This suggests that efforts are needed on a larger and more sustainable scale that can truly transform mathematics teacher education programs in ways that provide the needed preparation for the increased demands on high school teachers for teaching statistical content.

In a separate study, the SETS instrument has been found to have some limitations when measuring statistics teaching efficacy (Lovett et al., under review). The SETS instrument is aligned with recommendations from the GAISE framework, but places a large focus on analyzing data. Since all four phases of the statistical investigative cycle do not receive the same emphasis as in the LOCUS instrument, this may also explain the amount of unexplained variance in the model. There may not be a close enough correspondence in the instruments to detect the true relationship between statistical knowledge and statistics teaching efficacy.

Conclusion

It is time for secondary mathematics teacher education programs to attend to the needs of their preservice teachers in preparing to teach the statistics and data science concepts that high schoolers need to learn to be college and workforce ready. This study

provides strong evidence and recommendations for changes. These results also highlight that projects that aim at professional development of individual faculty are likely not enough to create changes at the program level to increase the preparedness of PSMTs to teach statistics. PSMTs graduating from these mathematics teacher education programs do not possess the statistical knowledge or the confidence to teach statistics. While continued efforts towards professional development of statistics faculty and mathematics teacher education faculty in terms statistics education can be helpful, they may not be enough to make the dramatic changes needed to better prepare PSMTs to teach statistics. The statistics education community needs to develop sustainable and large-scale models for infusing statistics and statistics teaching as a core component of all secondary mathematics teachers' preparation. The universities in this study with Carnegie Classification of Research/Very High and Master's (Large Program) classifications are preparing their PSMTs significantly more in statistics content understanding and self-efficacy for teaching statistics than other universities classified as research or masters universities respectively. There is a need to further examine *how* these specific mathematics teacher education programs are preparing their PSMTs to teach statistics. Such case studies could provide recommendations to make large-scale changes for mathematics teacher education programs across the country. Such large-scale changes that increase the emphasis on statistics in secondary mathematics teacher preparation are what is needed to stop the cycle of new mathematics teachers being unprepared to teach the statistical standards in today's curriculum.

Chapter 6: New Standards Require Teaching More Statistics: Are Preservice Secondary Mathematics Teachers Ready?

Journal

This article was written as a research manuscript for the *Journal of Teacher Education* in response to the call for a Special Issue on Teaching to Changing Standards. Specifically, the manuscript addresses aspects related to the first two suggested topics in the call for papers:

- What approaches in teaching methods courses assist teachers in adopting instructional practices effective in teaching to new standards?
- What aspects of teachers' content knowledge must be strengthened to meet the demands of new standards?

Guidelines require that a manuscript not exceed 10,000 words including table, figures, and references. These results would benefit the audience of JTE of educational researchers, teacher educators, and mathematics teacher educators. While this manuscript provides some of the quantitative results about statistics content and statistics teaching efficacy, it focuses on the qualitative data to unpack the nuances of the influences on PSMTs' preparedness to teach statistics, from their perspective. These findings provide a clear focus for mathematics teacher educators on areas that should be emphasized in mathematics methods courses that could strengthen PSMTs' preparedness to teach statistics. For mathematics teacher education programs, these results provide evidence to make strategic changes to increase the focus on statistics.

Abstract

New standards for high school mathematics in the United States include a strong emphasis in statistics. This paper reports results from a mixed methods cross-institutional study examining the preparedness of preservice secondary mathematics teachers to teach statistics and identifying factors and experiences that influence their preparedness. The results suggest that the cohort of teachers entering secondary mathematics classrooms in 2015 were not well prepared to teach statistics. Specific suggestions are given for how teacher education programs must rise to the challenge of preparing their graduates to teach these new and challenging standards.

Introduction

As of 2015, most states have either adopted the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practice & Council of Chief State School Officers, 2010) or modified their previously adopted standards to align with CCSSM (Academic Benchmarks, 2015). While the National Council of Teachers of Mathematics has long advocated for the inclusion of statistics and probability in middle and high school mathematics curricula (1989, 2000), the CCSSM has increased the emphasis on statistics in these grade levels. Mathematics teacher education programs are faced with the challenge of preparing preservice secondary (grades 6-12) mathematics teachers (PSMTs) to teach statistics.

In 2007, Franklin and colleagues authored the Guidelines for Assessment and Instruction in Statistics Education (GAISE) for K-12 students, endorsed and published by the American Statistical Association (ASA). Unfortunately, even with ASA's efforts to influence K-12 curricula and teaching practices, many PSMTs have had minimal experience with statistics in their own K-12 education. To prepare PSMTs to succeed in teaching statistics standards in CCSSM, the *Mathematics Education of Teachers II* report (Conference Board of the Mathematical Sciences, 2012) and the recent *Statistical Education of Teachers* (SET) report (Franklin et al., 2015) presented recommendations for courses that should be included in secondary mathematics teacher education programs to develop statistical knowledge and pedagogy needed to teach statistics. In addition, the last decade has included recommendations to assist college faculty to reform statistics courses, particularly at the introductory statistics level (e.g., Cobb, 2015; Garfield et al., 2007). Given these changes in the emphasis of statistics, the purpose of this study is to examine and describe experiences in

PSMTs' preparedness to teach statistics that can assist teacher education programs in making strategic changes.

Statistical Knowledge for Teaching

In typical secondary mathematics teacher preparation programs, PSMTs should have opportunities to develop statistical knowledge for teaching as learners in statistics courses as well as courses focused on pedagogy, often referred to as "methods" courses. Regardless of where these opportunities arise, it is critical that PSMTs engage in experiences that can develop their statistical knowledge for teaching. For this study, statistical knowledge refers to teachers' knowledge of topics outlined in the K-12 GAISE report for which teachers must assist students in learning. These topics range in sophistication across three developmental levels labeled A, B and C (typically taught in elementary, middle, and high school) and emphasize four phases of a statistical investigation: pose questions, collect data, analyze data, and interpret results.

Although developing statistical knowledge is foundational for teaching statistics, it is not adequate to describe all knowledge needed (Groth, 2013). Along with statistical knowledge, PSMTs need opportunities to develop pedagogical statistical knowledge (Franklin et al., 2015). Hill et al. (2008) have shown that in addition to content knowledge, pedagogical content knowledge: is needed for responding to students' difficulties (Cengiz, Kline, & Grant, 2011); affects instructional strategies used (Baumert et al., 2010); and is linked to student achievement (Hill et al., 2005). Pedagogical *statistical* knowledge includes a teacher's knowledge of potential student difficulties with statistics, strategies to support student's learning, strategies to engage students in a statistical investigative cycle, and knowledge of statistics curricula (e.g., Groth, 2013).

Statistics has changed dramatically because of affordances of technological tools. Not surprisingly, Lee and Hollebrands (2011) and others have noted that knowledge for teaching statistics must also include understanding how to use technological tools to solve statistical problems and to teach statistical topics through investigations. In the *SET* report, Franklin et al. (2015) recommend that PSMTs need to be "well versed in dynamic statistical software to solve and understand problems" and "feel comfortable teaching a statistical concept using technology as tool" (p.43).

Teaching Efficacy

A teacher's preparedness not only relies on cognitive aspects, but also affective constructs such as beliefs, attitudes, and self-efficacy (e.g., Ball et al., 2008). Teachers' beliefs play a crucial role in the pedagogical approaches they use, the time spent on a subject, and impact students' learning (e.g., Wilkins, 2008). However there is a lack of research on secondary teachers' beliefs in regards to statistics and teaching statistics (Batanero et al., 2011). This study focuses on PSMTs' self-efficacy for teaching high school students the statistical standards in CCSSM. Bandura (1986) defines self-efficacy as "people's judgments of their capabilities to organize and execute courses of action required to attain designated types of performance" (p.391). Judgments of one's own self-efficacy are task-specific and change over time (Bandura, 1997; Pajares, 1997). A teacher has two types of self-efficacy for each content area they teach: self-efficacy to know the content themselves, and self-efficacy to teach the topic to students, known as teaching efficacy.

There are two sources Bandura (1997) suggests can impact a teacher's efficacy that are focused on in this study: mastery and vicarious experiences. Bandura describes mastery experiences as prior experiences in performing a task that are perceived to be successful.

Palmer (2011) further elaborated this into two forms of mastery experiences: classroom teaching experiences and cognitive mastery. Palmer describes cognitive mastery as a teacher's perceived success in understanding the content and pedagogy necessary to teach a specific topic and vicarious experiences as those experiences where an individual watches another person perform the behavior successfully.

Most research on teaching efficacy has been done with inservice teachers and points to the positive impact of mastery experiences, as well as developing teachers' cognitive mastery of content and pedagogy (Lovett et al., under review; Palmer, 2011). While limited research of the impact of sources of preservice teachers' efficacy has been conducted, one study of preservice elementary teachers found that mastery experiences, specifically cognitive mastery, had the greatest impact on preservice teachers' teaching efficacy for mathematics and science (Newton, Leonard, Evans, & Eastburn, 2012).

Other research suggests three kinds of experiences that potentially influence PSMTs' teaching efficacy: university experiences, teaching experience, and world experiences (e.g., Conner et al., 2011; Rubie-Davies et al., 2012). In the context of this study, university experiences refer to cognitive mastery gained through university-based statistics courses and mathematics methods courses; teaching experience refers to prior experience teaching (or tutoring) mathematics or statistics at the K-16 level; and world experiences refer to workplace or everyday experiences with statistics.

The PSMTs preparing to teach high school mathematics have both statistics self-efficacy and statistics teaching efficacy. Statistics self-efficacy is a "teacher's belief in his/her ability to do statistics" and statistics teaching efficacy as "a teacher's belief in his/her ability to teach statistics to bring about student learning" (Lovett et al., under review). In this

study, I focus on the latter but, as the results will show, statistics self-efficacy and knowledge will come into play.

Only a few studies have been conducted on statistics teaching efficacy of preservice or inservice teachers. One exception is Fitzmaurice et al. (2014) examined the statistics teaching efficacy of preservice teachers through interviews and found that they were reluctant to teach statistics during their student teaching placements. Preservice teachers who did teach statistics during their placement reported an increase in efficacy.

Methodology

This paper is part of a larger mixed methods study on preparedness of PSMTs to teach statistics. The study utilizes an explanatory design, first quantitatively examining PSMTs' statistical knowledge and statistics teaching efficacy, and then qualitatively seeking factors and experiences that influence PSMTs' confidence through analysis of open-ended responses and interviews. Thus three research questions are investigated:

1. What is PSMTs' understanding of statistics content and statistics teaching efficacy for content they are expected to teach?
2. How does PSMTs' confidence to teach statistics compare to other areas of high school mathematics?
3. From the perspectives of PSMTs, what factors and experiences influence their preparedness to teach statistics?

Participants. This study focuses on PSMTs prepared through university-based teacher preparation programs in the United States. Over the past 15 years, several efforts have been made to increase preparation of mathematics teacher education faculty on ways to prepare PSMTs for teaching statistics. This study began with a purposeful narrowing on

PSMTs who currently attend institutions whose faculty have participated in the last 13 years in professional development efforts through either the National Science Foundation (NSF)-funded *Preparing to Teach Mathematics with Technology* project (ptmt.fi.ncsu.edu), or the ASA-funded Math/Stat Teacher Education: Assessment, Methods, and Strategies program to increase the emphasis of statistics education at that institution. Faculty from 57 institutions participated in either (or both) programs. All 57 institutions were contacted, and 18 agreed to participate in this study, with all but one being public institutions. The majority of institutions (61.1%) had an enrollment profile of high undergraduate, and the majority of participants attended institutions with a basic *Carnegie Classification*^{TM} of research universities (very high), research universities (high), or Master's college and university with a larger program.

There were 236 PSMTs who participated in the study. The PSMTs were undergraduate juniors and seniors or graduate students earning initial licensure, all in their last few weeks of their final mathematics education methods course prior to student teaching in Fall 2014 and Spring 2015. The majority of PSMTs were female (70.3%), and had taken one or two statistics courses. All data was blinded so that participants and universities remained anonymous.

Data collection.

Statistical Content Assessment. To examine PSMTs' statistical knowledge, the Levels of Conceptual Understanding of Statistics (LOCUS) assessment instrument (Jacobbe, Case, et al., 2014) was administered online (locus.statisticseducation.org). Participants took the Intermediate/Advanced Statistical Literacy version of the assessment, which was designed to assess statistics content taught in grades 10 – 12. It is aligned with CCSSM and assesses understanding across the three levels of development in the GAISE framework and

four phases of an investigative cycle. This assessment represents content that PSMTs are expected to teach to their students in the near future. A 23-item paper version of the LOCUS assessment has been validated as a measure to reliably assess current statistical understanding (Jacobbe, Case, et al., 2014). The version used in this study consists of 30 questions and contains the previously validated 23 items and an additional 7 items.[7] For each participant, the LOCUS assessment produces an overall score (percent correct), as well as sub-scores for Level B, Level C, Forming Questions, Collecting Data, Analyzing Data, and Interpreting Results.

Statistics Teaching Efficacy Instrument. To examine PSMTs' statistics teaching efficacy, the Self-Efficacy to Teach Statistics (SETS; Harrell-Williams et al., 2014) instrument was administered. This instrument collects both qualitative and quantitative data about PSMTs' statistics teaching efficacy. Furthermore, the SETS instrument is aligned with the GAISE framework, which reflects statistics content PSMTs are expected to teach. Hence there is a close correspondence between the teaching efficacy instrument and the topics PSMTs need to know and teach.

The instrument contains 44 six-point Likert scale items. An earlier version of this instrument with 26 items aligned with levels A and B of GAISE was validated for use with elementary and middle grades preservice teachers' self-efficacy (Harrell-Williams et al., 2013). The high school version contains the previous 26 items aligned to GAISE levels A and B, and contains an additional 18 items validated and aligned to GAISE level C (Harrell-Williams & Pierce, 2015; Harrell-Williams et al., 2014)[8]. In addition to an overall score, the

[7] This 30-item version of LOCUS has been validated, with manuscript currently under review.
[8] This 44-item version of SETS has been validated, with manuscript currently in preparation.

instrument provides sub-scale scores that correspond to Levels A (11 items), B (15 items) and C (18 items). For all Likert items, the stem of the question was

> "Rate your confidence in teaching high school students the skills necessary to complete successfully the task given by selecting your choice on the following scale: 1 = not at all confident, 2 = only a little confident, 3 = somewhat confident, 4 = confident, 5 = very confident, 6 = completely confident" (Harrell-Williams et al., 2014).

Therefore, for each participant, the SETS instrument produces a confidence score to teach high school students statistics for GAISE level A, GAISE level B, GAISE level C, and overall confidence.

For the open-ended portion, in each GAISE level, PSMTs were asked to identify an item which they felt *least* confident to teach and an item which they felt *most* confident to teach and to explain their reasoning. Thus there were up to six open-ended responses possible for each PSMT. There is some concern about using self-reported data due to a tendency of participants to present favorable images of themselves (Ross, 1989); however, there is little motivation to misreport when confidentiality is preserved (Baldwin, 2000).

Based on preliminary analysis from data in Fall 2014 (n=154), Spring 2015 participants (n=81) were asked an additional question in the demographic section of the SETS survey. They were asked to compare their preparedness across five areas of high school mathematics (algebra, geometry, pre-calculus/advanced algebra, calculus, and statistics) and to rank these in order of how well they feel prepared to teach them from most to least. This ranking was collected to help situate PSMTs' statistics teaching efficacy in relationship to other areas of high school mathematics.

Interviews. To further understand PSMTs' preparedness to teach statistics and the experiences and factors that impact this, 25 volunteers from the 81 Spring 2015 PSMTs participated in a semi-structured interview through video conferencing. These PSMTs represented half (n=9) of the sample universities. PSMTs were asked to: expand on their confidence as expressed in their open-ended responses; their experiences in statistics courses and mathematics education methods courses; previous tutoring, field, and teaching experiences; and their comfort level with statistical technologies. Transcripts of interviews and field notes were used to write a narrative summary of each interviewee's experiences (Merriam, 1998).

Data analysis. Analysis occurred in two phases. Phase one was an analysis of the quantitative data from the LOCUS and SETS instruments. To examine the statistical knowledge demonstrated by PSMTs, analysis began with examining the time it took for participants to complete the assessment; those taking less than 10 minutes were eliminated. Descriptive statistics were computed for PSMTs' Overall scores, Level B scores, Level C scores and scores for each portion of the investigative cycle: Forming Questions, Collecting Data, Analyzing Data, and Interpreting Results. Paired samples t-tests were used to test for significance of PSMTs' statistical knowledge between GAISE Levels B and C.

For the SETS survey, analysis began with accounting for missing data, since every PSMT did not respond to all 44 Likert items. The percentage of missing values ranged from 0 to as high as 5.1% for some items, and only 91.1% of the 236 PSMTs in the sample would have been available for analysis under listwise deletion. Data was primarily missing due to nonresponse, and this was addressed by using multiple imputation under the assumption that missing values are missing at random (Allison, 2002). Since approximately nine cases were

missing, nine imputed data sets were created using SPSS (White et al., 2011). After using imputation to replace missing values, an overall score and 3 subscales were calculated for each PSMT as the sum of his/her Likert scores on the items divided by the total number of items, resulting in scores that corresponded to the six-point Likert scale. Analyses on each imputed data set were pooled according to Rubin's (1987) rules and the pooled values were compared to the original data. The results were similar, thus imputed results will be presented. Since assumptions of normality and sphericity were not validated, a repeated measures ANOVA was used to test for significant differences in PSMTs' statistics teaching efficacy between the three GAISE levels.

Qualitative analysis began with Decuir-Gunby, Marshall, and McCulloch's (2011) recommendations for the development of a codebook. I began by coding the open-ended responses using *a priori* codes from the three major components of the framework: experiences, statistical knowledge, and pedagogical statistical knowledge. Then I open-coded (Corbin & Strauss, 2008) within each component to identify themes. Iteratively, themes were collapsed into codes. All open-ended responses were coded and 15 PSMTs' responses were randomly chosen for comparison with another researcher and little disagreements were found. Any disagreements were then resolved through discussion. Analysis was then focused on coding 25 interviewee summaries using the resulting codebook from the analysis of open responses. Then open coding was used to capture themes that appeared in interviews, but had not appeared in open-ended responses.

Once all data was coded, the data was separated into two categories: responses describing confidence to teach statistics, and responses describing a lack of confidence to

teach statistics. Codes from each category were examined to identify a list of factors that influence PSMTs' confidence for each category.

Results

Statistical knowledge. Due to the limited scope of this manuscript, I only briefly present the most important findings about PSMTs' statistical knowledge to help situate results in their confidence to teach. A more detailed analysis and discussion is available in Chapter 4. The PSMTs had a mean statistical content score of 69%, with a standard deviation of 14.06. These results suggest that overall, PSMTs did not demonstrate a strong conceptual understanding of the statistical knowledge they will soon teach high school students. PSMTs scored, on average, significantly higher on the GAISE Level B questions than on GAISE Level C questions ($t=5.772$, $p<0.001$), indicating their statistical knowledge is weaker as topics increase in statistical sophistication. Examining items by phases in the statistical investigative cycle, PSMTs scored highest on Formulating Questions and lower as the cycle progresses, scoring the lowest on Interpreting Results items. A repeated measures ANOVA determined that mean scores differed significantly between the four phases of the cycle [$F(3,648)=64.73$, $p<0.001$]. Post hoc tests using a Bonferroni correction revealed that PSMTs scored significantly lower on questions as the cycle progressed ($p<0.001$). There was only a slight difference between mean scores for Analyze Data and Interpret Results ($p=0.32$).

An examination of items identified several strengths and weakness of PSMTs' statistical knowledge. Two important strengths are that PSMTs are proficient at identifying an appropriate measure of center for a given context, a topic that is heavily emphasized in school mathematics. This strength in understanding measures of center suggests that PSMTs'

should be well equipped to assist their future students develop stronger conceptions of measures of center beyond the standard algorithms. PSMTs' weaknesses involve issues with variability, sampling distributions, *p*-values, and confidence intervals. Emphasis on these topics in high school has increased with the adoption of CCSSM.

Statistics teaching efficacy. Recall that the rating scale for their confidence to teach statistics is from 1 (not at all confident) to 6 (completely confident). Table 18 reports summary statistics for PSMTs' confidence scores overall and for GAISE Levels A, B and C. These results show that PSMTs are typically between *somewhat confident* (4) and *confident* (5) to teach high school students statistics, with the highest confidence in Level A items. A repeated measures ANOVA determined that mean confidence scores differed significantly between the three levels [$F(2,470)=66.54$, $p<0.001$]. Post hoc tests using a Bonferroni correction revealed that PSMTs' confidence was significantly lower as the statistical sophistication of items increased ($p<0.001$).

Table 20. *PSMTs' confidence on SETS instrument*

	Number of items	Mean	SD
Overall Score	44	4.06	0.80
GAISE Levels			
Level A Score	11	4.51	0.80
Level B Score	15	4.08	0.85
Level C Score	18	3.76	0.91

Examining scores by item identified specific topics that PSMTs felt least confident and most confident to teach. Within Level A, PSMTs were most confident to teach creating graphical representations and recognizing that statistical results may differ from group to group. They were least confident to teach selecting appropriate graphical displays and generalizing results from a small to large group. For Level B, PSMTs were most confident to teach creating histograms and computing the interquartile range, and least confident to teach interpreting measures of association. For Level C, PSMTs were most confident to teach characteristics of a normal distribution and identifying slope and y-intercept in a regression equation. PSMTs are least confident to teach finding conditional and marginal frequencies using two-way tables, and formal ideas of inference using simulations, such as margin of error and testing for statistical significance.

Confidence compared to other areas of high school mathematics. To put the results of PSMTs' statistics teaching efficacy in context, I examined the ranked order of 81 PSMTs' confidence (1 is highest confidence) for five areas of high school mathematics they will be expected to teach: algebra, geometry, algebra 2/pre-calc, statistics, and calculus. Figure 14 shows that 74% percent of PSMTs were most confident to teach algebra, while

63% of PSMTs were least confident to teach statistics. These results show that even though on average PSMTs are *confident* to teach statistics as measured by SETS, their confidence in teaching statistics is lower than for other high school topics.

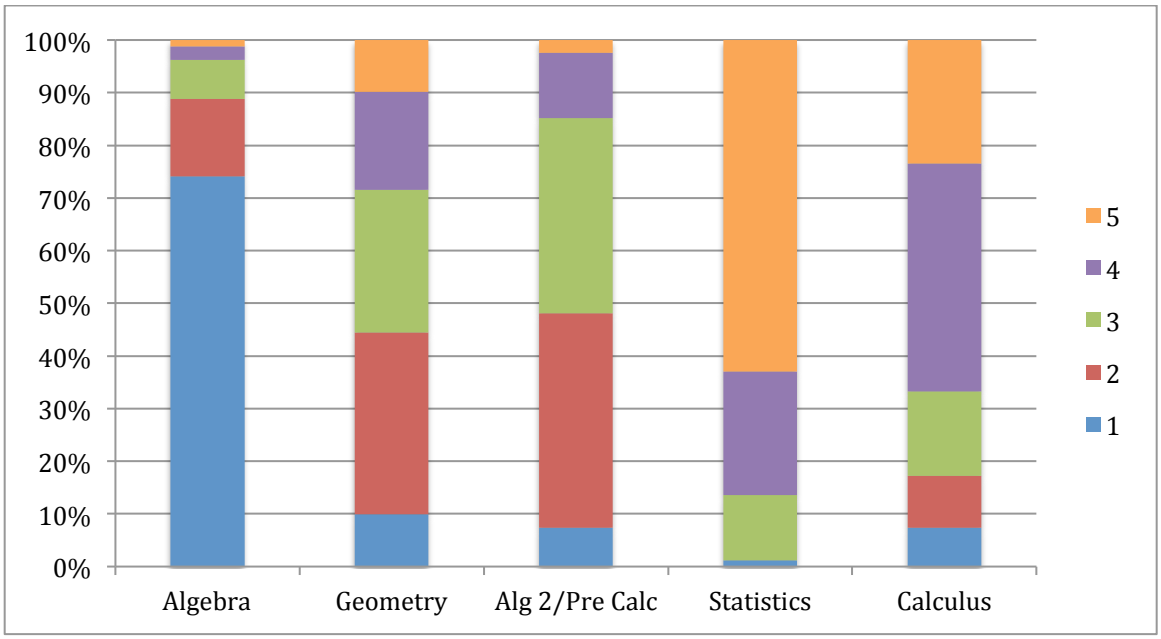

Figure 14. PSMTs' ranking of confidence to teach high school topics from most (1) to least (5).

Influences on preparedness. While the quantitative results identified the statistical knowledge and statistics teaching efficacy, the analysis of the open-ended SETS responses and interview summaries sought to provide insight as to why PSMTs' confidence to teach statistics is not as high as other areas of mathematics, and what experiences and factors may be contributing to their perceived preparedness. Because every PSMT did not answer each open-ended response, there were 657 responses for items that PSMTs felt least confident to teach, and 659 responses for items they felt most confident to teach. Through coding, four major categories emerged: role of statistics knowledge, role of pedagogical knowledge,

impact of using technology, and view of statistics. In this section, I report insights gained into PSMTs' confidence or lack of confidence for each of these categories in order of prevalence.

Role of statistical knowledge. Of the 657 responses from PSMTs about feeling the least confident to teach, 585 (89%) referred to their lack of statistics knowledge. In more than half of these responses (55.2%) PSMTs made short statements and did not provide much information on experiences that related to their statistical knowledge (e.g., "I'm not sure that I know what a two-way table is" U16F1410).

In approximately one-third of responses, PSMTs discussed previous experiences related to their statistical knowledge that impacted their confidence. One theme that emerged was PSMTs' need to review or "brush up" since they had not recently had experiences with the topic. Creating and interpreting graphical displays was a common topic that PSMTs felt least confident to teach because they had not reviewed the material in their college statistics courses. This theme often occurred when PSMTs were discussing graphical representations, measures of association, two-way tables, and items involving formal inference. For example, this theme emerged in PSMTs' responses regarding ideas of formal inference, such as developing a margin of error for an estimated population mean, that they learned in their college statistics courses. A response that typified this theme was from U6S1508,

> "I feel least confident here [margin of error] because I have only touched upon this in one of my early classes post secondary, and it was not in the context of teaching it, but learning it. I could do research and learn it again so I could teach it, but right now I would not feel confident in teaching this."

Like many PSMTs, she identified that she had learned this material in her previous college courses, and although she didn't feel like she remembered the content, she could review and teach it to herself *and then* feel confident in teaching it.

Since the interviews were focused on PSMTs' experiences with statistics, all interviewees discussed their college statistics courses during the interview. Approximately one-third of the 25 PSMTs interviewed discussed that their courses were procedurally focused and they did not develop a deep understanding of the statistics because of the nature of the course. When Interviewee 24 was describing her overall confidence to teach statistics she said,

> "Even the statistics content I do feel comfortable with was more of a memorize and regurgitate the process and repeat it kind of approach instead of understanding it. I don't feel like I learned a whole bunch of statistics, I memorized a whole bunch of statistics."

While many PSMTs acknowledged they only had a procedural understanding of statistics and felt that this had developed due to the nature of their statistics courses. Other PSMTs reflected on their professors being "old-school" or "set in their ways" because their courses did not utilize technology to learn and do statistics. All 25 interviewees were also asked about their experiences using simulations to develop a sampling distribution and test for statistical significance, yet only one recalled having such previous experience with these topics in the CCSSM standards they will be expected to teach (HSS.IC.B.4, HSS.IC.B.5). Interviewee 15 felt that the reason she and other PSMTs didn't have experience with simulations was because "A lot of teachers we had are, I don't want to say old fashion, but

old fashion in a way that they don't use a lot of technology and they don't get the creativeness in using simulations."

When discussing items PSMTs were *most confident* to teach in the open-ended responses there were 588 (89%) instances where they referred to their statistical knowledge. Some PSMTs continued to make short statements and did not provide much information about their previous experiences. For example, in regards to the normal distribution, a PSMT wrote, "I remember how to look at shape and symmetry of a graph and what it looks like" (U12F1404). However in more than half of the responses, PSMTs discussed their previous experiences to support their confidence. For example, "I feel very confident with histograms because that is mostly what my statistics class covered" (U18F1408).

To understand more about PSMTs' experiences, all interviewees were asked about their high school and college statistics courses and their mathematics methods courses. Almost all PSMTs (n=21) felt most confident to teach topics with which they have had extended experiences. For instance, in discussing her confidence to teach creating histograms, Interviewee 15 said, "Histograms have been incorporated into the regular math classes. Pretty much any math class I have ever been in since middle school has included a stat unit and histograms have been a part of it." It is not surprising that PSMTs felt confident in areas such as creating graphical representations and computing measures of center because of extended experiences with them in middle and high school. Another commonly taught topic in high school is identifying slope and y-intercept of a linear model and interpreting these in the context of data. Interviewee 2 discussed her confidence in this topic, "Even though I didn't have any statistics classes in high school, that was something that I did in middle school and high school. So it is something I've done for a long time."

Role of pedagogical knowledge. There were 127 (19.3%) responses where PSMTs' justifications were coded as using pedagogical knowledge. When a response concerned why they were least confident to teach a topic, I considered their response an indication of a lack of pedagogical statistical knowledge. The majority of time (67.7%) these responses still included PSMTs reflecting on their statistical knowledge, as well as their pedagogical statistical knowledge. From these responses, two themes emerged regarding PSMTs' lack of knowledge of appropriate teaching strategies, and lack of knowledge of students' approaches and difficulties.

PSMTs frequently discussed that they did not know how to teach or explain a topic when justifying their lack of confidence. As previously mentioned, often this stemmed from their content knowledge, but it was expressed in terms of the pedagogical statistical knowledge as well. PSMTs lacked confidence to teach topics because they were lacking teaching strategies and resources. For example, a PSMT attributed her lack of confidence to teach variability to her lack of an appropriate teaching strategy; "I would be least confident in this because I wouldn't know how to go about showing this to students explicitly without just saying it" (U4S1503). Similarly, many PSMTs discussed they would need to learn more about the best approach to teaching a topic beforehand. This idea came up as well during interviews when PSMTs were asked what other knowledge or experiences they need to feel prepared to teach statistics. Several PSMTs responded similar to Interviewee 11 as she discussed needing more experiences "actually discussing more of how to do it [teach statistics] with students." PSMTs feel they need to know appropriate teaching strategies and resources to feel more prepared to teach statistics.

PSMTs also often expressed lack of confidence because they did not know different ways students could approach a topic. One PSMT discussed her lack of confidence in teaching variability because she felt "it's difficult to prepare for a solution path that could occur if you are unable to think in the same manner as that particular student" (U8F1410). Another PSMT discussed what he thinks he knows about students' difficulties with association that are impacting his confidence to teach it, "I think students would have a difficult time understanding how association and cause and effect are not exactly the same thing" (U8F1413). Both of these types of justifications pointed to PSMTs recognizing that pedagogical knowledge for teaching statistics specifically was important, but that they were not well-equipped with knowledge of students' understandings and misconceptions regarding statistical topics.

To investigate this further, all interviewees were asked about learning about students' understandings and misconceptions in their teacher education program. All 25 remembered that students' understandings and misconceptions were discussed in their mathematics methods courses, but only half recalled understandings and misconceptions being discussed during their unit on teaching statistics. For example, Interviewee 9 discussed that "we [PSMTs at her university] studied so much about how students think about geometry … but statistics, I don't know how they think about it. I think that would be really helpful." Interviewee 7 expressed similar feelings that "it would be helpful to know more about how kids think about statistics. Because I don't know anything about the misconceptions students would have."

Over half of the PSMTs interviewed explicitly stated that their mathematics methods course(s) did not prepare them to teach statistics. The other PSMTs felt that their

mathematics methods courses were a "nice start" or "it was just like scratching the surface." PSMTs reflected that one reason they feel their mathematics education course(s) didn't prepare them to teach statistics was that their courses were largely focused on preparing them to teach algebra. For example, Interviewee 16 noted:

> "Week after week we were working with factoring, graphing, it was such a big focus on algebra 1. When we are secondary teachers and statistics is going to be such a big focus now. I feel like more time should definitely be spent on it. It shouldn't be something you stick in for one week."

Interviewee 5 (different university) also felt that "there was a lot of emphasis on how to teach algebra" and that she felt like she could "tell you more about how abstract algebra relates to teaching algebra than I can about some basic statistics topics." Not only across different universities did PSMTs express an emphasis on algebra, they also expressed that the statistics unit (often one week) in their mathematics methods courses mostly occurred near the end of the semester and "was thrown in at the end and cut off early" (Interviewee 25).

PSMTs that were interviewed did feel like their mathematics methods courses prepared them well to teach other areas of secondary mathematics. For example, Interviewee 5 discussed how her mathematics methods course changed her view of teaching mathematics, "It [mathematics methods course] challenged me to see all the different fields within math in a more big picture way I guess. Not needing a specific procedure to feel comfortable teaching a lesson." Methods courses for secondary mathematics are perceived by PSMTs as preparing them holistically for teaching mathematics and giving them strategies and understanding of students' conceptions in topics like algebra and geometry, but lacking in statistics.

When discussing items PSMTs were *most confident* to teach, there were 230 (34.9%) instances that were categorized into pedagogical statistical knowledge. In 67 of the 230 (29.1%), PSMTs discussed their teaching experience where they had an opportunity to teach others a statistical topic. The PSMTs that were interviewed were asked about field experiences and tutoring experiences regarding statistics. Of the 25 PSMTs that were interviewed, five said that they had tutoring or teaching experience with statistical topics. These PSMTs felt more confident in statistical topics they had experience teaching and described the value of such experience. For example, Interviewee 6 commented she had learned about graphing a line of best fit but her experience teaching it was what gave her confidence, and Interviewee 12 felt like she "really began to understand it more because I had to not only explain it [when tutoring] in terms she would understand it, but also explain from my math perspective." Perhaps one of the most intriguing quotes came from Interviewee 24:

> "I wish I would have [had tutoring or teaching experience in statistics] now because I am going to teach statistics in a month and I have no experience what-so-ever. It could have been a valuable learning opportunity for me to get comfortable with it in a one-on-one setting before I have 30 teenagers staring at me."

In general, PSMTs seem acutely aware that developing pedagogical knowledge specific to statistics is crucial in building their confidence, and that practical teaching experiences would be immensely helpful.

***Impact of using technology*.** The use of graphing calculators and dynamic statistical software such as *TinkerPlots* (Konold & Miller, 2011) and *Fathom* (Finzer, 2007) emerged in PSMTs' least and most confident responses. There were only 26 (4.0%) instances where

PSMTs justified their *lack* of confidence because of their previous experiences with technology. These responses focused on not having the knowledge or experience using technology, even the graphing calculator, to fit linear models, plotting residuals, or calculating the correlation coefficient. One example is from a PSMT who expressed her lack of confidence to teach fitting linear models as, "I have little knowledge on this content material or understanding for how to apply it to various types of technology; even the graphing calculator" (U1F1435). Thus a few PSMTs seem to recognize that they should have more agile abilities and knowledge of how to use technology for these concepts.

During interviews, seven of the 25 PSMTs elaborated on their lack of experiences using technology and acknowledged they needed to know more about technology for learning statistics and how to use in their lessons. These seven PSMTs had no previous experiences with dynamic statistical software; they had only used a graphing calculator or statistical computing software (e.g., SAS). Interviewee 17 commented that she needed a program "like GeoGebra for statistics" to be able to teach it because "Technology is most important in statistics because you can't do it without any technology." These seven PSMTs recognized the power of technology in teaching, especially statistics, and were aware that they are lacking experiences with technology that would make them more prepared to teach statistics.

PSMTs also reflected on their experiences with technology to justify a topic they were *most* confident to teach. In these 58 (8.8%) instances in the open-ended responses, two themes emerged. PSMTs viewed using technology as a computational tool that they were confident in using, such as computational statistical software. One PSMT noted "It's [calculating correlation coefficient] easy. The software should just pop this value out"

(U3F1408) and a few mentioned that they can use dynamic statistical software for computations "I know how to click the right button on Fathom or TinkerPlots that will give me the fit for a model. I am confident that I can show students these same buttons" (U4S1506). The majority of PSMTs discussed that fitting an appropriate model using their graphing calculator was a simple procedure, illustrated by the following:

> "because it [fitting an appropriate model] is easy enough to take your data points and enter them within a calculator and then calculate a model that best fits your data set by using different calculations with the calculator and graphing the models" (U12F1407).

Thus some PSMTs referenced work with technology to support their confidence because they were comfortable quickly generating computations, but made no reference of using dynamic statistical tools for addressing conceptual issues.

Some PSMTs discussed experiences they had with technology that helped them develop a deeper understanding of statistical topics. This theme emerged only in responses when PSMTs discussed their confidence to teaching graphical representations, typified by a PSMT's confidence in teaching comparing distributions, "I have had great training in TinkerPlots and Fathom, this is a great way to teach students about the spread of data" (U8F1414). Some PSMTs went more in depth in describing the knowledge they gained through working with technology:

> "We not only learned multiple technologies that can assist us in our lessons, but we had to work through the material ourselves. I feel that this has really helped us to develop a better understanding of the material and which aspects may be difficult for

our students. Thus, it helps us see how to form a lesson that can help our students understand the material better as well" (U8F1410).

To further understand the impact of dynamic technology on PSMTs' confidence, interviewees were asked to reflect on their experiences with technologies, what topics they examined using those technologies, and to discuss how useful they view those technologies for their future teaching. Sixteen of the 25 PSMTs interviewed reported previous experiences using dynamic statistical software in a mathematics methods course. These interviewees were asked about topics they had experience with, and if and how they would incorporate it into their future classrooms. PSMTs only recalled experiences creating graphical representations, examining measures of center, and fitting linear models. Several interviewees described how experiences using TinkerPlots and Fathom provided experiences to deeper explore data, therefore increasing their confidence in graphical representations.

All 16 PSMTs with experiences using dynamic statistical software expressed that their experiences using those technologies deepened their understanding and they would use those technologies to provide their future students with similar opportunities. For example, Interviewee 6 discussed that TinkerPlots is "a good visual for them [students]" and would engage her future students in "exploratory lessons like we did" in her methods courses.

***View of statistics*.** The PSMTs' view of statistics emerged as another category that influenced why they were least or most confident to teach a statistical topic. In PSMTs' responses regarding topics they were least confident to teach, there were 26 (4.0%) instances where they referred to some topics in statistics as being a "gray area", being "uncertain" in the correct answer, or no "algorithm" to do it. This occurred the most frequently in PSMTs' Level A responses regarding variability, sampling, and informal inference. The following

response shows how a PSMT acknowledged her view of statistics, "Most of the other tasks were more objective - making plots, comparing plots. They were more technical in nature. Recognizing limitations, is more obscure, … it is an art" (U3F1408). Here she attributes her lack of confidence to teach recognizing the limitations in making inference as an area of uncertainty, instead of a procedure like creating a graphical representation. One PSMT's response typifies the uneasiness with uncertainty in statistics, but also shows a perception of how her future students will deal with uncertainty involved in making statistical claims.

> "When I have done this (informal inference), answers are always subjective and based on whether you can justify your answer or not, especially when it comes to describing whether there is a meaningful difference from looking at the overlap. There is no specific way to describe distributions so I know students will struggle with the uncertainty of their answers [sic]" (U4S1507).

This theme of their view of statistics also emerged during interviews. Eight of the 25 interviewees discussed that the statistical topic they felt least confident to teach was different from mathematics because it did not have a procedure or a right answer. Interviewee 7 discussed in her previous experiences with statistics she was "always looking for a right answer" and lacked confidence in herself because she "could never come up with the right answer." When Interviewee 5 was asked to reflect on topics that PSMTs in general did not have confidence to teach, her response highlighted the nature of statistics, "I would assume so many people are uncomfortable with statistics because so much of it is centered around thinking on your own rather than following a set procedure." Thus, PSMTs seem very uncomfortable in areas of statistics that require statistical thinking that are inherently built on uncertainty.

In PSMTs' responses regarding topics they were *most* confident to teach, two themes emerged: viewing statistical topics as procedures, and viewing statistics as algebra. Viewing statistical topics as procedures emerged in 99 (15.1%) responses regarding PSMTs' confidence in teaching graphical representations and computing measures. An example response that typifies confidence in teaching graphical representations is, "it [creating histograms] is a simple idea and is somewhat procedural. It does not require a lot of cognitive demand to create a histogram of data that you're given" (U15S1502). Many PSMTs discussed their confidence in teaching how to compute interquartile range because "This is more procedural and easy computations, which is why I feel the most confident to teach this" (U17F1405). When PSMTs responded in this way, they were reflecting on topics being solely procedural and didn't consider other non-procedural aspects such as determining bin width for intervals of a histogram based on problem context.

This theme also occurred when 20 of 25 PSMT interviewees discussed they felt most confident to teach topics that they described as "simple", "procedural" or "computational." Interviewee 9's response was typical because "either you know interquartile range or you don't. There is not that much understanding to computing it. I feel like I can tell someone how to do a procedure and they can do it." Interviewee 9 focused not just on the computational nature, but that teaching a computation was perceived as easy to do.

In the 29 (4.4%) responses where PSMTs were viewing statistics as algebra, PSMTs were comparing statistical topics to algebraic ones or referencing they had learned the topic in high school algebra courses. This most frequently occurred when PSMTs justified their confidence to teach identifying the slope and y-intercept in a linear model and interpreting them in the context of data, because "I feel most confident about teaching this topic since it

directly relates to algebra which I am very familiar with" (U4S1508). Often PSMTs focused on identifying slope and *y*-intercept, and did not comment on interpreting these in the context of data when justifying their confidence. This theme emerged as well when eight of 25 interviewees mentioned that this particular item "sounds like algebra, it is easy to do. It is just math not statistics." Thus, overall PSMTs are confident when they view an aspect of statistics as procedural or "algebra-like", but less confident when it involves uncertainty.

Occurrence of factors. The four major categories give insight to the most dominant factors that PSMTs attribute to influencing their confidence to teach statistics, but the prevalence of the categories differed greatly (see Table 19). PSMTs' perception of their statistical knowledge seemed to have the same large impact in both their least and most confident to teach responses. However, there is a large difference in the other categories. When PSMTs justified statistical topics they were most confident in teaching, they more frequently described aspects of pedagogical statistical knowledge, the impact of using technology, and their view of statistics. It is not surprising that PSMTs' felt more confident in items for which they had teaching experience. Also, since PSMTs in this study feel most confident to teach algebra (Figure 14), it makes sense that PSMTs were confident in teaching items they viewed as teaching a procedure or teaching algebra.

Table 21. *Occurrence of factors in open-ended responses*

	Least Confident to Teach *n* (%)	Most Confident to Teach *n* (%)
Role of Statistical Knowledge	585 (89.0)	588 (89.0)
Role of Pedagogical Knowledge	127 (19.3)	230 (34.9)
Impact of using Technology	26 (4.0)	58 (8.8)
View of Statistics	26 (4.0)	99 (15.1)

Summary and Conclusion

With increased emphasis on statistics in high school mathematics, this cross-institutional study investigated how prepared PSMTs were in statistics knowledge and confident they felt to teach statistics as they completed their teacher education program. Thus, this study aimed to provide evidence to support recommendations for how teacher education programs should attend to the preparation of PSMTs to teach statistics.

PSMTs' statistical knowledge. This study provides insight into the current landscape of how well PSMTs demonstrate the statistical knowledge needed to teach statistical topics in CCSSM. As we have shown, PSMTs' statistical understandings diminish as the sophistication of topics increase from GAISE Level B to C, and as the cycle of a statistical investigation progresses. Previous research has shown a similar trend with inservice teachers' and students' statistical knowledge measured by LOCUS (Jacobbe, 2015; Jacobbe, Foti, et al., 2014). Quite simply, either PSMTs are not getting enough opportunities to learn the statistics they are going to teach in high school, or they are not developing their understandings in ways that are useful for teaching.

PSMTs' statistics teaching efficacy. As I have shown, PSMTs' confidence diminishes as the difficulty of topics increases from GAISE level A, to B, to C, and thus, they are least confident in teaching typical high school content in the CCSSM. PSMTs felt much more confident to teach algebra and geometry than statistics, and accordingly, felt more confident to teach aspects of statistics they perceived as procedural (e.g., creating graphical representations, computing measures, describing normal distributions, identifying slope and y-intercept). Areas that PSMTs felt least confident to teach include selecting appropriate graphical representations, measures of association, two-way tables, and ideas of formal inference using simulation. PSMTs' confidence to teach creating graphical representations and computing measures, and lack of confidence to teach interpreting measures of association is consistent with previous research (Harrell-Williams, Sorto, Pierce, Lesser, & Murphy, 2015). These findings provide a clear focus on areas that should be emphasized in courses that could strengthen PSMTs' preparedness to teach statistics.

University statistics courses. The results show that PSMTs' confidence or lack of confidence to teach statistics stems largely from their experiences and content knowledge gained or not gained in their high school and university statistics courses. PSMTs report that their college courses were typically traditional lectures that did not incorporate active learning, were procedurally focused, and did not use simulation approaches to inference. Taught in this way, these courses are not deepening PSMTs statistical knowledge or engaging them in statistical reasoning, and do not seem to be implementing the 2007 GAISE College level standards (Garfield et al., 2007). Widespread efforts like the Consortium for the Advancement of Undergraduate Statistics Education (www.causeweb.org) have been advocating for changes to statistics courses for over a decade, but these results indicate that

they may not be implemented on a large scale. Mathematics teacher education and statistics faculty should work together to reexamine the statistics courses PSMTs are required to take to align with updated guidelines for statistics courses from the GAISE College report and SET recommendations (Franklin et al. 2015). To break this cycle of underprepared high school teachers, these recommendations need to be implemented nationally to increase the preparedness of PSMTs to teach the statistical standards in CCSSM.

Mathematics methods courses. The need for mathematics methods courses to place greater emphasis on statistics for PSMTs to feel prepared to teach it is apparent throughout the results. Mathematics teacher education programs should be providing PSMTs opportunities to deepen their statistical knowledge for teaching (Groth, 2013). These courses should include topics that PSMTs feel comfortable with, such as creating graphical representations, and build from this knowledge to develop a deeper understanding through engaging in the statistical investigative cycle. Also, mathematics methods courses should provide PSMTs with experiences in other areas of statistics, such as selecting appropriate graphs, measures of association, two-way tables, and simulation-based inference since PSMTs do not feel confident to teach these areas.

PSMTs in this study discussed that they understand the importance of learning about teaching strategies and student misconceptions to effectively teach mathematics. Therefore PSMTs expressed that they feel less prepared to teach statistics than algebra and geometry because they have not had opportunities to learn about pedagogical issues in statistics like they have had for algebra and geometry. This result points to the need for secondary mathematics teacher programs to incorporate substantial units into methods courses that

focus on developing pedagogical knowledge for topics difficult to understand and teach such as variability, sampling distributions, and simulation-based inference

As seen in the results, PSMTs do not recall being provided opportunities to experience teaching statistics in their fieldwork or through writing lesson plans. Mathematics teacher education programs should build opportunities for PSMTs to experience teaching a statistical topic. This could begin with requiring PSMTs to develop a lesson plan on a statistical topic. Another approach could be arranging for PSMTs to serve as tutors or teaching assistants for the introductory statistics classes, providing PSMTs opportunities to explain statistics to others. Finally, programs could ensure that PSMTs are observing a variety of topics during their field experiences by working with inservice teachers. These experiences will add to the pedagogical statistical knowledge PSMTs develop and aid in the feeling of preparedness to teach statistics.

Use of dynamic statistical software. The results highlighted that PSMTs' feel that experiences with dynamic statistical software increase their confidence to teach statistics. Dynamic statistical software can be used two ways in a classroom: 1) as an amplifier to create graphical representations and perform computations quickly, and 2) as a reorganizer to deepen students' statistical understanding through using the dynamic components of the software (Garfield & Ben-Zvi, 2004). PSMTs in this study seemed to experience using dynamic statistical software both as an amplifier and a reorganizer, but their only reported experiences of it being used as a reorganizer was with examining center, spread, and trends in graphical representations. PSMTs attributed their confidence to teaching graphical representations to their deep understanding they developed using dynamic statistical software. For other statistical topics, the software seemed to only be used as an amplifier.

There needs to be a shift to providing opportunities for PSMTs to engage with dynamic statistical software as a reorganizer to build conceptual understandings of concepts such as correlation, variability, and simulation-based approaches to inference.

Limitations and conclusion. There were a number of limitations, which should be taken into consideration when interpreting the findings. First, this study was a purposeful sample of U. S. institutions and was not a random sample of all PSMTs. While most studies in teacher education have been conducted at one institution or a small number of institutions with small sample sizes, this study includes 18 institutions varying in program size and location, which may expand the potential usefulness of results. Another limitation is the SETS instrument focuses largely on procedural aspects of statistics; therefore, many of PSMTs' responses reflected on the procedures as well. To capture PSMTs' true statistics teaching efficacy for how statistics should be taught, changes should be made to the instrument to better align with recommendations from the GAISE framework. Teachers need to engage their students in all phases of posing questions, collecting data, analyzing data, and interpreting results; but the instrument focuses heavily on their confidence to teach students to analyze data. Secondly, more items are needed involving use of technology as a learning tool and confidence in using it as a teaching tool.

We know that inservice teachers do not feel prepared to teach statistics (Batanero et al., 2011). The findings suggest that the current cohort of PSMTs is likely no more prepared to teach statistics than most inservice teachers. There is a critical need for mathematics teacher education programs to reevaluate their programs in strategic ways that can increase PSMTs' preparedness to teach statistics, while also maintaining their strong attention to other content areas taught in secondary mathematics to meet new standards. The results provide

fruitful directions for programs to consider for how they can increase the emphasis on statistics and statistics teaching, ideally in collaboration with faculty who teach the statistics content courses for teachers. If secondary mathematics teacher education programs continue *not* to emphasize the importance of statistics in high school curricula and lack opportunities that provide PSMTs the content and pedagogical approaches they need, new teachers entering the field will continue to be underprepared and perhaps devalue statistics in their curriculum and instruction. It is time to rise to meet this challenge.

Chapter 7: Summary and Conclusion

The final chapter presents the central findings of this study to determine the current state of preparedness of PSMTs to teach statistics. The explanatory mixed methods design resulted in two research questions. This chapter begins with reviewing those research questions and uses quantitative and qualitative data to answer those questions. Explicit connections are made between the findings and existing literature. The following section describes the limitations of the study. The chapter concludes with implications for mathematics teacher education programs and recommendations for future research.

Summary of Research Question 1 and Findings

What is the preparedness of preservice secondary mathematics teachers [PSMT] to teach statistics as they enter student teaching?

Statistical knowledge. Due to the increased emphasis of statistics in the U.S. high school curriculum, it is important that PSMTs are prepared to teach statistics. The results of this study provide insight on the current landscape of PSMTs' statistical knowledge. PSMTs in this study were chosen from a purposeful sample of universities that had faculty members who have participated in programs to increase statistics education. However, these PSMTs still generally do not have strong conceptual understandings of the statistics content they will be expected to teach. Additionally, PSMTs' statistical understandings decrease as the investigative cycle progresses, that is, they are far less knowledgeable about analyzing data and interpreting results than they are about formulating questions and collecting data. Previous research has shown a similar trend with inservice teachers' and high school students' statistical knowledge measured by LOCUS (Jacobbe, 2015; Jacobbe, Foti, et al., 2014). Thus PSMTs need more experiences in collecting data, analyzing data, and

interpreting results to develop a deeper conceptual understanding of all aspects of the statistical investigative cycle.

As seen in the item analysis, PSMTs have developed similar strengths and weaknesses as high school and introductory college students develop with concepts such as variation from a mean (e.g., delMas & Liu, 2005; Pfannkuch, Wild, & Parsonage, 2012). In terms of Formulating Questions items, a strength PSMTs demonstrated is their ability to read a description of a study and measurements taken to identify the statistical question of interest. For Collecting Data items, PSMTs demonstrated their ability to identify a data collection plan based on a study description. These strengths suggest that PSMTs are prepared to engage their students in crucial aspects of designing a study and collecting data to investigate a statistical question of interest. In terms of Analyzing Data and Interpreting Results, PSMTs are proficient at identifying an appropriate measure of center for a given context and comparing distributions in a context using the center and spread, topics that are heavily emphasized in school mathematics. Identifying appropriate measures of center for a given context is heavily emphasized in school mathematics, yet research shows that students struggle to understand measures of center (e.g., Mokros, 1995; Watson, 2007). PSMTs' strength in understanding measures of center suggests that these future teachers should be well equipped to assist their students develop stronger conceptions of measures of center beyond the standard algorithms.

Weaknesses demonstrated by the largest number of PSMTs were seen in the Analyze Data and Interpret Results items. This is not surprising since PSMTs scored on average the lowest in these two phases. These weaknesses involve issues in understanding variability, sampling distributions, *p*-values, and confidence intervals. Even though emphasis on these

topics in high school has increased with the adoption of CCSSM, PSMTs' statistics and mathematics education courses do not appear to be assisting them in developing a deep understanding of these topics.

Statistics teaching efficacy. Preparedness to teach also includes non-cognitive aspects that teachers draw upon (Ball et al., 2008; Beswick, 2007; Bray, 2011; Gellert, 2000; Thompson, 1992). It is critical that mathematics teacher education programs consider PSMTs' affect when preparing them to teach (Barlow & Reddish, 2006). The results of this study provide insight on the current landscape of PSMTs' statistics teaching efficacy. PSMTs' confidence diminishes as the difficulty of topics increases from GAISE level A, to B, to C, and thus, they are least confident in teaching typical high school content in the CCSSM. PSMTs felt much more confident to teach algebra and geometry than statistics, and accordingly, felt more confident to teach aspects of statistics they perceived as procedural (e.g., creating graphical representations, computing measures, describing normal distributions, identifying slope and y-intercept). Areas that PSMTs felt least confident to teach include selecting appropriate graphical representations, measures of association, two-way tables, and ideas of formal inference using simulation. PSMTs' confidence to teach creating graphical representations, computing measures, and identifying slope and y-intercept, and lack of confidence to teach interpreting measures of association and formal inference using simulations are consistent with previous research on preservice and inservice teachers' statistics teaching efficacy (Harrell-Williams et al., 2015; Lovett et al., under review). These findings provide a clear focus on areas that should be emphasized in courses that could strengthen PSMTs' preparedness to teach statistics.

Self-reported factors influence statistical knowledge and statistics teaching efficacy. The number of statistics courses, a university's Carnegie Classification, and taking AP Statistics were identified as factors that had an impact on PSMTs' statistical knowledge and/or statistics teaching efficacy, while gender had no impact. These results provide evidence to support prior research and recommendations in the *SET* Report (Franklin et al, 2015). The results about the impact of the number of statistics courses taken in college supports previous research on preservice teachers' attitudes towards statistics; specifically that the number of advanced mathematics (statistics) courses do not increase preservice teachers' understanding of school mathematics (statistics), but do increase their attitudes towards mathematics (Ball, 1990; Wilkins, 2008). This study only used their self-reported data on courses taken, without asking for names or content of those courses. Some may have taken a typical introductory statistics course, and others may have taken calculus-based statistics and/or probability theory courses that they reported as statistics courses.

Also, as seen in the results, PSMTs who took AP Statistics in high school, experiencing data analysis and introduction to formal inference, demonstrated statistically significant higher statistics teaching efficacy and statistical knowledge than those that did not. Since AP Statistics is not taken by all PSMTs while in high school, an introductory course should provide PSMTs with foundational statistical knowledge and experiences that they would have experienced in AP Statistics, as long as the course aligns with recommendations in the GAISE College report (Garfield et al., 2007).

The universities in this study with Carnegie Classification of Research/Very High and Master's (Large Program) classifications are preparing their PSMTs significantly more in statistics content understanding than other universities classified as research or masters

universities respectively. These findings demonstrate that the research emphasis and size of the university play a role in implementing recommendations for mathematics teacher education programs and that professional development projects aimed at individual faculty are likely not enough to create changes at the program level to increase the preparedness of PSMTs to teach statistics.

Relationship between statistical knowledge and statistics teaching efficacy. Many researchers have examined the relationship between knowledge and efficacy (Appleton, 1995; Bates et al., 2011; McCoy, 2011; Newton et al., 2012; Swackhamer, Koellner, Basile, & Kimbrough, 2009; Swars et al., 2007). The results of this study indicate that statistics teaching efficacy is weakly positively correlated with statistical knowledge. This finding supports previous research of preservice elementary teachers that their mathematics teaching efficacy is weakly positively correlated to mathematical knowledge (Bates et al., 2011). However the study was the first to compare statistics or mathematics teaching efficacy to content knowledge using a teaching efficacy instrument that is context specific, asking participants to consider teaching specific topics. Since efficacy beliefs are task-specific constructs (Bandura, 1997; Pajares, 1997), I anticipated a closer relationship between statistics teaching efficacy and statistical knowledge as measured by the SETS and LOCUS. PSMTs in this study took the SETS instrument prior to taking the LOCUS assessment, that is, they self assessed their confidence before taking an assessment that may have given them insight into their statistics knowledge. Even with the SETS instrument measuring specific tasks across three GAISE levels, and thus having a closer alignment with the LOCUS instrument, there is not a strong relationship between PSMTs' statistics teaching efficacy and statistical knowledge in the sample. Since researchers continue to find a weak positive

relationship between content knowledge and self-efficacy (e.g., Bates et al., 2011; McCoy, 2011), this suggests that content knowledge alone is not a strong predictor of one's self-efficacy.

If there is a relationship between statistical knowledge and statistics teaching efficacy the factors that were tested or the design of the study may have hindered detecting it. As seen in the model presented in Chapter 6, taking AP Statistics, the number of statistics courses, the university's Carnegie Classification, or gender leave a large amount of unexplained variance. Therefore, if a relationship exists between statistics teaching efficacy and statistical knowledge, there must be other factors to explain the relationship. Also, PSMTs may have over- or under-estimated their statistics teaching efficacy because they were not aware of the content they were expected to teach. Further research is needed to determine if the order in which PSMTs take the LOCUS and SETS influences the relationship between statistics teaching efficacy and statistical knowledge.

Summary of Research Question 2 and Findings

From the perspectives of PSMTs, what factors and experiences influence their preparedness to teach statistics?

The results of this study showed that there were three main factors that influence PSMTs' perspectives on their preparedness to teach statistics: university statistics courses, mathematics methods courses, and the use of dynamic statistical software.

University statistics courses. PSMTs' confidence or lack of confidence to teach statistics stems largely from their experiences and content knowledge gained or not gained in their high school and university statistics courses. PSMTs report that their college courses were typically traditional lecture that did not incorporate active learning, were procedurally

focused, and did not use simulation approaches to inference. Taught in this way, these courses are not deepening PSMTs' statistical knowledge or engaging them in statistical reasoning, and do not seem to be implementing the 2007 GAISE College level standards (Garfield et al., 2007).

Widespread efforts like the Consortium for the Advancement of Undergraduate Statistics Education (www.causeweb.org) have been advocating for changes to statistics courses for over a decade, but the results from this study indicate that they may not be implemented on a large scale. Mathematics teacher education and statistics faculty should work together to reexamine the statistics courses PSMTs are required to take to align with updated guidelines for statistics courses from the GAISE College report and SET recommendations (Franklin et al. 2015). To break this cycle of underprepared high school teachers, these recommendations need to be implemented nationally to increase the preparedness of PSMTs to teach the statistical standards in CCSSM.

Mathematics methods courses. In mathematics methods courses, there should be a greater emphasis on strategies and issues in teaching statistics for PSMTs to feel prepared to teach it. Mathematics teacher education programs should be providing PSMTs opportunities to deepen their statistical knowledge for teaching (Groth, 2013). These courses should include topics with which PSMTs feel comfortable, such as creating graphical representations, and build from this knowledge to develop a deeper understanding through engaging in the statistical investigative cycle. Also, mathematics methods courses should provide PSMTs with experiences in other areas of statistics, such as selecting appropriate graphs, measures of association, two-way tables, and simulation-based inference since PSMTs do not feel confident to teach these areas.

PSMTs in this study discussed that they understand and value the importance of learning about teaching strategies and student misconceptions to effectively teach mathematics. Therefore, PSMTs feel less prepared to teach statistics than algebra and geometry because they have not had opportunities to learn about pedagogical issues in statistics like they have had for algebra and geometry. This result points to the need for secondary mathematics teacher programs to incorporate substantial units into methods courses that focus on developing pedagogical knowledge for topics difficult to understand and teach such as variability, sampling distributions, and simulation-based inference.

As seen in the results, PSMTs do not recall being provided opportunities to experience teaching statistics in their fieldwork nor through writing lesson plans. Mathematics teacher education programs should build opportunities for PSMTs to experience teaching a statistical topic. This could begin with requiring PSMTs to develop a lesson plan on a statistical topic. Another approach could be arranging for PSMTs to serve as tutors or teaching assistants for the introductory statistics classes, providing PSMTs opportunities to explain statistics to others. Finally, programs could ensure that PSMTs are observing a variety of topics during their field experiences by working with inservice teachers. These experiences will add to the pedagogical statistical knowledge PSMTs develop and aid in the feeling of preparedness to teach statistics.

Use of dynamic statistical software. The results highlighted that PSMTs feel that experiences with dynamic statistical software increase their confidence to teach statistics. Dynamic statistical software can be used two ways in a classroom: 1) as an *amplifier* to create graphical representations and perform computations quickly, and 2) as a *reorganizer* to deepen students' statistical understanding through using the dynamic components of the

software (Garfield & Ben-Zvi, 2004; Pea, 1985). PSMTs in this study seemed to experience using dynamic statistical software both as an amplifier and a reorganizer, but their only reported experiences of it being used as a reorganizer was with examining center, spread, and trends in graphical representations. PSMTs attributed their confidence to teaching graphical representations to their deep understanding they developed using dynamic statistical software. However for other statistical topics, the software seemed to only be used as an amplifier. There needs to be a shift to providing opportunities for PSMTs to engage with dynamic statistical software as a reorganizer to build conceptual understandings of concepts such as correlation, variability, sampling distributions, and simulation-based approaches to inference.

Limitations

There were a number of limitations that should be taken into consideration when interpreting the findings. First, this study was a purposeful sample of U. S. institutions and was not a random sample of all PSMTs. While most studies in teacher education have been conducted at one institution or a small number of institutions with small sample sizes, this study includes 18 institutions varying in program size and location, which may expand the potential usefulness of results. However of importance is that this purposeful sample was from those institutions with at least one faculty member that had participated in a project aimed at increasing preparation of mathematics teachers to teach statistics.

Another limitation of data collection was that only the 81 PSMTs who participated in spring 2015 were eligible to be interviewed. The 25 interviewees were volunteers representing half (n=9) of the institutions in the study. Even though this is a limitation of the study, there is no reason to believe that participants from phase 2 had different factors or

experiences that influenced their preparedness than phase 1 participants. The in-depth interviews provided rich descriptions of these factors and influences that lead to generalizability of other PSMTs within the sampling frame. There were also recording issues with 12 interviewees. Those 12 interviewees' summaries did not contain quotes from the interview and were written using only the notes taken by the researcher, limiting analysis that could be conducted.

Another limitation is the SETS instrument focuses largely on procedural aspects of statistics, therefore many of PSMTs' responses reflected on the procedures as well (Lovett et al., under review). The SETS instrument is aligned with recommendations from the GAISE framework, but places a large focus on analyzing data. Since all four phases of the statistical investigative cycle do not receive the same emphasis as in the LOCUS instrument, there may not be a close enough correspondence in the instruments to detect a relationship between statistical knowledge and statistics teaching efficacy. To capture PSMTs' true statistics teaching efficacy for how statistics should be taught, changes should be made to the instrument to better align with recommendations from the GAISE framework. Secondly, more items are needed involving use of technology as a learning tool and confidence in using it as a teaching tool.

Implications

These findings suggest that the current cohort of PSMTs is likely no more prepared to teach statistics than most inservice teachers. There is a critical need for mathematics teacher education programs to reevaluate their programs in strategic ways that can increase PSMTs' preparedness to teach statistics, while also maintaining their strong attention to other content areas taught in secondary mathematics. The results provide fruitful directions for programs to

consider for how they can increase the emphasis on statistics and statistics teaching, ideally in collaboration with faculty who teach the statistics content courses for teachers.

One implication is that mathematics teacher education programs cannot rely on PSMTs to learn all of their statistical knowledge through statistics courses. In the SET report, Franklin and colleagues (2015) recommend that preservice secondary mathematics teachers take three statistics courses with the following emphases: 1) an introductory statistics course that emphasizes data analysis, simulation-based approaches to inference with technology, and an introduction to formal inference, 2) a statistical methods course building from the introductory course utilizing randomization and classical approaches to inference, and 3) a statistical modeling course based on multiple regression techniques. The introductory course would provide PSMTs with foundational statistical knowledge and experiences that they have not experienced in their own K-12 education. The results of this study support this recommendation since the small number of PSMTs who took AP Statistics in high school, experiencing data analysis and introduction to formal inference, demonstrated statistically significant higher statistics teaching efficacy and statistical knowledge than those that did not.

Even though requiring PSMTs to take an introductory course will possibly increase PSMTs' statistical knowledge and confidence to teach statistics, which alone is not enough to prepare PSMTs to teach statistics. Research has shown, that in addition to content teachers need to develop pedagogical content knowledge (Hill et al., 2008). Thus mathematics teacher education programs need to attend to developing PSMTs' statistical knowledge and pedagogical statistical knowledge in mathematics methods courses. Pedagogical *statistical* knowledge includes a teacher's knowledge of potential student difficulties with statistics,

strategies to support student's learning, strategies to engage students in a statistical investigative cycle, and knowledge of statistics curricula (e.g., Groth, 2013). I suggest a substantial focus on three areas. First, mathematics teacher educators should work with statistics instructors to identify high school topics that PSMTs are not getting enough experience with such as variability, sampling distributions, and inference as identified in this study. These topics should be included so that PSMTs can develop statistical knowledge of these topics that are difficult to understand and teach. One way to approach such program enhancements is for mathematics teacher educators to consider some of the boundary objects between mathematics and statistics, such as measurement and variability, as well as rich contextual problems, where fruitful lessons could be developed (Groth, 2015). Lessons that incorporate real data and engage PSMTs in all phases of a statistical investigation could be structured to focus on comparing distributions to build from PSMTs' strengths in using measures of center to further develop their understanding of variability and formal inference.

Secondly, mathematics teacher educators should allow PSMTs to develop a deeper understanding of statistical topics by engaging them in tasks that include all four phases of the statistical investigative cycle. The tasks will model the way PSMTs should engage their future students in exploring statistical topics and deepening their knowledge simultaneously. As shown by Lee et al (2014), statistical investigations can be greatly enhanced when PSMTs use dynamic statistical software to investigate broad statistical questions. Thus the final recommendation is that mathematics teacher education programs should include experiences for PSMTs to engage with dynamic statistical software to engage in meaningful statistical investigations, develop concepts such as sampling distribution and simulation approaches to

inference, and assist PSMTs in developing appropriate pedagogical strategies for developing students' conceptions.

Implementing these recommendations pose a great challenge for many mathematics teacher educators since typically they do not have expertise in statistics education. To develop statistical knowledge and pedagogical statistical knowledge, mathematics teacher educators need expertise with dynamic statistical technology, access to curricular resources, and knowledge in implementing recommendations and resources. To assist in these recommendations, wide-scale professional development of mathematics teacher education faculty and strong curriculum materials are needed. This study showed that NSF-funded and ASA-funded professional development projects that targeted individual faculty were not enough to make significant progress in preparing PSMTs to teach statistics.

Also, current widespread curricular materials developed for teacher education are not robust enough to develop statistical knowledge for teaching. Some materials focus on developing statistical knowledge of "big ideas" and not developing deep conceptual understanding of statistics through engaging in the statistical investigative cycle (e.g., Peck, Gould, & Miller, 2013) or do not focus on developing pedagogical statistical knowledge (e.g., Bargagliotti et al., 2014). There are mathematics teacher educators, who have expertise in statistics education, developing curricular materials that are focused on important hard-to-teach content and pedagogy in statistics (e.g., Casey, Ross, Groth, & Zejullahi, 2015), but those materials have predominately been shared through conference presentations, to a small number of mathematics teacher educators. Recently, efforts have been made to disseminate statistics teacher education resources to the mathematics teacher education community through open education resources such as MOOC-Eds (http://friday.institute/tsdi.) and

PTMT's new interactive educational portal (http://friday.institute/ptmt). To help mathematics teacher education *programs* implement these recommendations and ultimately increase the preparation of PSMTs' to teach statistics, these open resources are not enough.

Recommendations for Future Research

The results highlight three areas for future research that I plan to pursue: PSMTs' statistical knowledge for teaching, PSMTs' statistics teaching efficacy, and mathematics teacher education programs. The research base on PSMTs' statistical knowledge for teaching and students' statistical knowledge is inadequate for mathematics teacher education faculty to effectively prepare PSMTs. More research is needed on secondary students' statistical knowledge, specifically on GAISE level C topics. Since topics like simulation-based inference is new to the high school curriculum, most of the research on these topics is at the tertiary level. As more research is conducted on students' statistical knowledge, that research can be used to increase the pedagogical statistical knowledge of PSMTs. Until there is more research on students' understanding of these topics, materials development and research on PSMTs' pedagogical statistical knowledge will lag behind. Thus I plan to start investigating students' knowledge of these topics to create curriculum materials for PSMTs and conduct research on PSMTs' statistical and pedagogical statistical knowledge of these topics using the curriculum materials.

As previously mentioned, the SETS instrument may not be capturing the true nature of the construct of statistics teaching efficacy. Future work is needed in this field to develop another instrument and investigate PSMTs' statistics teaching efficacy to get a true understanding of their confidence to teach statistics. I plan to form a team with a statistician

and a learning scientist to develop an statistics teaching efficacy instrument that focuses on all four phases of the statistical investigative cycle.

Recall that there were PSMTs from groups of universities that outperformed others in similar classified institutions. Therefore is a need to further examine *how* these specific mathematics teacher education programs are preparing their PSMTs to teach statistics. Such case studies could provide recommendations to make large-scale changes for mathematics teacher education programs across the country. Such large-scale changes and large scale models that increase the emphasis on statistics in secondary mathematics teacher preparation are what is needed to stop the cycle of new mathematics teachers being unprepared to teach the statistical standards in today's curriculum. Therefore the final way I plan to extend the research from my dissertation is to conduct a case study of high and low performing institutions that participated in this study to try to determine the university factors that are influencing PSMTs' preparedness to teach statistics.

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Appendices

Appendix A: Letter of Invitation

NCSU Box 7801
502 Poe Hall
Raleigh, NC 27606
(513) 544-6261

Undergraduate Coordinator of Mathematics Education or Administrative Equivalent
Department of STEM Education
Box 8205 North Carolina State University
Raleigh, NC 27695

To Whom It May Concern:

We are writing to invite you and some of your mathematics education students to participate in a research study. The purpose of the study is to examine how prepared advanced preservice teachers are to teach statistics. Given the long-standing recommendations of NCTM and the American Statistics Association (ASA), as well as the recent emphasis in the 6-12 Common Core State Standards for Mathematics, it is important to the field of mathematics teacher education to better understand how well prepared novice secondary mathematics teachers are to teach statistics. This research aims to examine how two constructs, self-efficacy and statistics content knowledge, contribute to a teachers' preparation for teaching statistics. The results of this research can benefit your university, as well as inform other mathematics education programs.

We would like to ask for your help recruiting a professor to administer two instruments: 1) an online survey to assess their self-efficacy to teach statistics (SETS), and 2) an online test of the statistical content found in the Common Core standards (LOCUS). The SETS should take about 20-30 minutes, while the content test would likely take between 45-60 minutes. The target level of the preservice teachers is near the end of their last "methods for teaching mathematics" course, *before* they begin full-time student teaching. Individual professors can choose whether they wish to offer course credit (e.g., as an assignment) for the completion of one or both instruments. If you choose to invite preservice secondary (9-12 or 6-12 licensure) teachers in their last methods course to participate, then you will receive a report that describes the aggregate results from participants in your program.

If you are willing to be a participant in this research study or would like further information before making a decision, please do not hesitate to contact us. I look forward to hearing from you.

Thank you,

Jennifer Nickell
Ph.D. Student in Mathematics Education
North Carolina State University
jnickel@ncsu.edu

Hollylynne Lee
Professor of Mathematics Education
North Carolina State University
hollylynne@ncsu.edu

Appendix B: Undergraduate Student Participant Consent Form

North Carolina State University
INFORMED CONSENT FORM for RESEARCH
Undergraduate Participants
Title of the Study: Preparedness of Preservice Teachers to Teach Statistics
Principal Investigator: Jennifer Nickell Faculty Sponsor: Hollylynne Lee

What are some general things you should know about research studies?
You are being asked to take part in a research study. Your participation in this study is voluntary. You have the right to be a part of this study, to choose not to participate or to stop participating at any time without penalty. The purpose of research studies is to gain a better understanding of a certain topic or issue. You are not guaranteed any personal benefits from being in a study. Research studies also may pose risks to those that participate. In this consent form you will find specific details about the research in which you are being asked to participate. If you do not understand something in this form it is your right to ask the researcher for clarification or more information. A copy of this consent form will be provided to you. If at any time you have questions about your participation, do not hesitate to contact the researcher named above.

What is the purpose of this study?
The purpose of this study is to examine the preparedness of preservice teachers to teach statistical content in middle or high school.

What will happen if you take part in the study?
If you agree to be a part of the study you will contribute to this research by completing an online survey about how prepared you feel to teach certain statistics topics (about 20-30 minutes), and an online statistics concept inventory (45-60 minutes). After completing the survey and test the Undergraduate coordinator for your university will receive students' results in the aggregate. Some participants may be asked to participate in a recorded online interview to further explore the your self-efficacy and content knowledge. You may decline to participate in an interview.

Risks/Benefits
There are no physical or emotional risks associated with participation in this study. The preservice teachers that participate in this study will potentially gain a better understanding of their preparedness to teach statistics.

Confidentiality
The information in the study records will be kept confidential to the full extent allowed by law. Data will be de-identified and given a code to match the participants' responses on both instruments. The principle investigator will store data securely. Pseudonyms will be used in oral or written reports to avoid linking you to the study. Identifiable and De-identified data may be shared with the developers of the two instruments. When identifiable data is shared, the developer will de-identify the data according to the procedures of their study. These results are being shared for further item of the instruments and test validation studies.

Compensation
There is no compensation for completing the content test or self-efficacy survey. If agree and complete an interview you will receive $25.

What if you have questions about this study?
If you have questions at any time about the study or the procedures, you may contact Jennifer Nickell at Poe 502 Campus Box 7801 NCSU Raleigh, NC 27695 or jnickel@ncsu.edu. If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Deb Paxton, Regulatory Compliance Administrator, Box 7514, NCSU Campus (919/515-4514).

Participation
Participation in this study is not a course requirement and your participation or lack thereof, will not affect your class standing or grades at your university. Your participation in this study is voluntary; you may decline to participate without penalty. If you decide to participate, you may withdraw from the study at any time without penalty and without loss of benefits to which you are otherwise entitled. If you withdraw from the study before data collection is completed your data will be returned to you or destroyed at your request.

**********CONSENT**********
I have read and understood the above information. I have received a copy of this form.
☐ I agree to participate in this study.
☐ I do not agree to participate in this study.

Appendix C: SETS Survey

Self-Efficacy to Teach Statistics (SETS) in High School
Survey

INSTRUCTIONS

- Rate your confidence in ***teaching high school students*** the skills necessary to complete successfully the task given by circling your choice, for example:

	Not at all Confident					Completely confident
1. Collect data to answer a posed statistical question in contexts of interest to high school students.	1	2	3	(4)	5	6

- For the open-ended questions, please include as much detail as you feel comfortable sharing.

- Your responses are voluntary and confidential. You may simply skip any question you are unable or unwilling to answer, but we hope that you will answer as many questions as possible.

Using a scale of (1, 2, 3, 4, 5, 6) where 1 = not at all confident, 2 = only a little confident, 3 = somewhat confident, 4 = confident, 5 = very confident, 6 = completely confident, please rate your confidence in ***teaching high school students*** the skills necessary to complete the following tasks successfully:

	Not at all confident					Completely confident
1. Collect data to answer a posed statistical question in contexts of interest to high school students.	1	2	3	4	5	6
2. Recognize that there will be natural variability between observations for individuals.	1	2	3	4	5	6
3. Select appropriate graphical displays and numerical summaries to compare individuals to each other and an individual to a group.	1	2	3	4	5	6
4. Create dotplot, stem and leaf plot, and tables (using counts) for *summarizing* distributions.	1	2	3	4	5	6
5. Use dotplot, stem and leaf plot, and tables (using counts) for *describing* distributions.	1	2	3	4	5	6
6. Create boxplots for *summarizing* distributions.	1	2	3	4	5	6
7. Use boxplots, median, and range for *describing* distributions.	1	2	3	4	5	6
8. Identify the association between two variables from scatterplots.	1	2	3	4	5	6
9. Generalize a statistical result from a small group to a larger group such as the whole class.	1	2	3	4	5	6
10. Recognize that statistical results may be different in another class or group.	1	2	3	4	5	6
11. Recognize the limitation of making inference (i.e. generalization) from a classroom dataset to any population beyond the classroom.	1	2	3	4	5	6

Open-Ended Question A:

Please review your responses to **items 1 – 11 (on the previous page only).**

a) Choose one item from **items 1 – 11 (on the previous page only)** that you indicated feeling **LEAST** confident about teaching high school students. Think about the reason(s) you feel this way. Use the space below (and the back of this paper, if necessary) to identify the item number and explain your reason(s).

b) Choose one item from **items 1 – 11 (on the previous page only)** that you indicated feeling **MOST** confident about teaching high school students. Think about the reason(s) you feel this way. Use the space below (and the back of this paper, if necessary) to identify the item number and explain your reason(s).

Using a scale of (1, 2, 3, 4, 5, 6) where 1 = not at all confident, 2 = only a little confident, 3 = somewhat confident, 4 = confident, 5 = very confident, 6 = completely confident, please rate your confidence in ***teaching high school students*** the skills necessary to complete the following tasks successfully:

	Not at all confident					Completely confident
12. Distinguish between a question based on data that vary and a question based on a deterministic model (for example, specific values of rate and time determines a particular value for distance in the model $d = r \times t$).	1	2	3	4	5	6
13. Identify what variables to measure and how to measure them in order to address the question posed.	1	2	3	4	5	6
14. Describe numerically the variability between individuals within the same group.	1	2	3	4	5	6
15. Create histograms for summarizing distributions.	1	2	3	4	5	6
16. Use histograms for comparing distributions.	1	2	3	4	5	6
17. Compute interquartile range and five-number summaries for summarizing distributions.	1	2	3	4	5	6
18. Use interquartile range, five-number summaries, and boxplots for comparing distributions.	1	2	3	4	5	6
19. Recognize the role of sampling error when making conclusions based on a random sample taken from a population.	1	2	3	4	5	6
20. Describe numerically the strength of association between two variables using linear models.	1	2	3	4	5	6
21. Explain the differences between two or more groups with respect to center, spread (for example, variability), and shape.	1	2	3	4	5	6
22. Recognize that a sample may or may not be representative of a larger population.	1	2	3	4	5	6
23. Interpret measures of association.	1	2	3	4	5	6

24. Distinguish between an observational study and a designed experiment.	1	2	3	4	5	6
25. Distinguish between "association" and "cause and effect."	1	2	3	4	5	6
26. Recognize sampling variability in summary statistics such as the sample mean and the sample proportion.	1	2	3	4	5	6

Open-Ended Question B:

Please review your responses to **items 12 – 26 (on the previous page only).**

a) Choose one item from **items 12-26 (on the previous page only)** that you indicated feeling **LEAST** confident about teaching high school students. Think about the reason(s) you feel this way. Use the space below (and the back of this paper, if necessary) to identify the item number and explain your reason(s).

b) Choose one item from **items 1 – 11 (on the previous page only)** that you indicated feeling **MOST** confident about teaching high school students. Think about the reason(s) you feel this way. Use the space below (and the back of this paper, if necessary) to identify the item number and explain your reason(s).

Using a scale of (1, 2, 3, 4, 5, 6) where 1 = not at all confident, 2 = only a little confident, 3 = somewhat confident, 4 = confident, 5 = very confident, 6 = completely confident, please rate your confidence in ***teaching high school students*** the skills necessary to complete the following tasks successfully:

	Not at all confident					Completely confident
27. Describe characteristics of a normal distribution, such general shape of distribution, symmetry, how standard deviation influences shape, and area under the curve.	1	2	3	4	5	6
28. Estimate percentages via the empirical rule (i.e., percentage of observations within 1, 2, or 3 standard deviations from the mean) using the mean and standard deviation of a dataset which has an approximately bell-shaped distribution.	1	2	3	4	5	6
29. Estimate a specified area under the normal curve using technology or a statistical table.	1	2	3	4	5	6
30. Summarize categorical data using two-way tables (i.e., contingency tables, frequency tables).	1	2	3	4	5	6
31. Calculate and interpret relative frequencies using two-way tables (i.e., contingency tables, frequency tables).	1	2	3	4	5	6
32. Find conditional and marginal frequencies from two-way tables (i.e., contingency tables, frequency tables).	1	2	3	4	5	6
33. Fit an appropriate model (e.g., linear, quadratic, or exponential) using technology for a scatterplot of two quantitative variables.	1	2	3	4	5	6
34. Assess the fit of a particular model informally by plotting and analyzing its residuals.	1	2	3	4	5	6
35. Identify the slope and y-intercept	1	2	3	4	5	6

coefficients of a linear model and interpret them in the context of the data.

Using a scale of (1, 2, 3, 4, 5, 6) where 1 = not at all confident, 2 = only a little confident, 3 = somewhat confident, 4 = confident, 5 = very confident, 6 = completely confident, please rate your confidence in ***teaching high school students*** the skills necessary to complete the following tasks successfully:

	Not at all confident					Completely confident
36. Calculate, using technology, the correlation coefficient between two quantitative variables.	1	2	3	4	5	6
37. Evaluate whether a specified model is consistent with data generated from a simulation.	1	2	3	4	5	6
38. Explain the role of randomization in surveys, experiments and observational studies.	1	2	3	4	5	6
39. Describing purposes and differences among surveys, experiments, and observational studies.	1	2	3	4	5	6
40. Evaluate how well the conclusions of a study are supported by the study design and the data collected.	1	2	3	4	5	6
41. Estimate a population mean or proportion using data from a sample survey.	1	2	3	4	5	6
42. Develop a margin of error for an estimate of a population mean or proportion using simulation models.	1	2	3	4	5	6
43. Compare two treatments from a randomized experiment by exploring numerical and graphical summaries of the data.	1	2	3	4	5	6
44. Determine if the difference between two population means or proportions is statistically significant using simulations.	1	2	3	4	5	6

Open-Ended Question C:

Please review your responses to **items 27 – 44 (on the previous 2 pages).**

a) Choose one item from **items 27 – 44 (on the previous 2 pages)** that you indicated feeling **LEAST** confident about teaching high school students. Think about the reason(s) you feel this way. Use the space below (and the back of this paper, if necessary) to identify the item number and explain your reason(s).

b) Choose one item from **items 27 – 44 (on the previous 2 pages)** that you indicated feeling **MOST** confident about teaching high school students. Think about the reason(s) you feel this way. Use the space below (and the back of this paper, if necessary) to identify the item number and explain your reason(s).

Appendix D: SETS Demographic Questions

Demographics: Teaching and Learning Statistics Name_____

(1) Did you take AP Statistics in High School? Yes _____ No _____

(2) From the following list, which level degree will you receive at the conclusion of your teacher preparation program? (Circle ONE response)

Bachelor's Master's Both a Bachelor's and Master's

(3) Upon completion of your teacher preparation program, which grade levels will you be certified to teach? (Circle ONE response)

middle school high school middle and high school

(4) Including this semester/term, how many classes have you taken at a college or university in statistics (Circle ONE response)

None 1 class 2 classes 3 classes 4-5 classes 6-7 classes 8 or more classes

(5) Indicate the courses within your degree program in which you have learned instructional strategies for teaching statistics topics. Check all that apply.

☐ A course focused entirely on teaching methods for statistics
☐ A course on teaching methods for mathematics that contained units or lessons focused on teaching statistics
☐ A statistics course that had lessons or assignments focused teaching methods for statistics
☐ Other:_____
☐ None

(6) How comfortable are you using the following statistical technologies?

	completely uncomfortable			somewhat comfortable			sufficiently comfortable			completely comfortable
Fathom	1	2	3	4	5	6	7	8	9	10
TinkerPlots	1	2	3	4	5	6	7	8	9	10
Core Math Tools	1	2	3	4	5	6	7	8	9	10
TI 83/84 Stat Package	1	2	3	4	5	6	7	8	9	10
TI Nspire	1	2	3	4	5	6	7	8	9	10

Stat Package										
StatCrunch	1	2	3	4	5	6	7	8	9	10
JMP	1	2	3	4	5	6	7	8	9	10
SAS	1	2	3	4	5	6	7	8	9	10
R	1	2	3	4	5	6	7	8	9	10
Minitab	1	2	3	4	5	6	7	8	9	10
Other	1	2	3	4	5	6	7	8	9	10

(7) Which technological tools do think would be useful in your own teaching of statistics?

	no opinion		not useful		somewhat useful			useful		very useful
Fathom	1	2	3	4	5	6	7	8	9	10
TinkerPlots	1	2	3	4	5	6	7	8	9	10
Core Math Tools	1	2	3	4	5	6	7	8	9	10
TI 83/84 Stat Package	1	2	3	4	5	6	7	8	9	10
TI Nspire Stat Package	1	2	3	4	5	6	7	8	9	10
StatCrunch	1	2	3	4	5	6	7	8	9	10
JMP	1	2	3	4	5	6	7	8	9	10
SAS	1	2	3	4	5	6	7	8	9	10
R	1	2	3	4	5	6	7	8	9	10
Minitab	1	2	3	4	5	6	7	8	9	10
Other	1	2	3	4	5	6	7	8	9	10

(8) How well prepared do you feel to teach the high school Common Core State Standards about statistics?

1	2	3	4	5	6	7	8	9	10
completely unprepared			somewhat prepared			sufficiently prepared			completely prepared

(9) As a secondary mathematics teacher you have to be prepared to teach a wide variety of subjects: algebra, geometry, pre-calc/advanced algebra, calculus, and statistics. Given these five different topics, algebra, geometry, pre-calc/advanced algebra, calculus, and statistics, please rank these in order of how well you feel prepared to teach them from most to least.

_____

_____

_____

_____

_____

Appendix E: Invitation Email to Interview Participants

To Whom It May Concern:

I am writing to invite you to participate in a 30 minute Skype interview regarding your responses on the Self-efficacy to Teach Statistics (SETS) survey and Levels of Conceptual Understanding in Statistics (LOCUS) test that you recently took. You will be compensated $25 for your participation in this part of the study.

If you are willing to be a participant in this research study or would like further information before making a decision, please do not hesitate to contact us. I look forward to hearing from you.

Thank you,
Jennifer Nickell
Ph.D. Student in Mathematics Education
North Carolina State University
jnickel@ncsu.edu

Appendix F: Interview Protocol

Introduction:

Hi, _____ thanks for agreeing to work through a task with me today. *Ask rapport questions about how their semester went.* Can you tell me a little about yourself? How did your finals go? Are you looking forward to student teaching next year?

I really appreciate you taking time to meet with me today. As you may know, I am interested in how prepared you feel to teach statistics. So I am going to ask you questions about the self-efficacy survey and the LOCUS test you took a few weeks ago. Keep in mind that what you say and your work will remain confidential.

Because I am interested in how you feel, it would really help me if you would expand on your answers and talk as much as you can. I will probably ask questions to be sure I understand what you are saying. I'm asking because I am really interested and want to be sure I understand what you are saying. Likewise, if I ask a question that does not make sense to you, please tell me.

I might take a few notes to help me remember what you're saying. I don't want to take too many notes. So, I would like to record our conversation. Also, I am using a program that records our conversation on the computer screen. During the interview, if you ever want me to stop the recording, please let me know. All questions are voluntary, so you don't have to answer a question if you don't want to.

SETS Questions
Provide participant their responses to several of the six open-ended questions.
Example open-ended question: Choose one item from items 1 - 11 that you indicated feeling LEAST confident about teaching high school students. Think about the reason(s) you feel this way. Use the space below to identify the item number and explain your reason(s).

Example response: I chose item 2 because I never learned about variability between individuals.

For each one ask:
Tell me more about your confidence to teach _____
Say more about that.
Please describe particular experiences that are impacting your confidence?
Probe for more details if needed.

Once the open-ended items have been addressed, ask:
From all the preservice teachers that took this survey, item _____ was commonly identified as a topic they are less confident in teaching to high school students. What do you think about that? Why do you think that is?

From all the preservice teachers that took this survey, item _____ was commonly identified as a topic they are more confident in teaching to high school students. What do you think about that? Why do you think that is?

Have prepared student's answers and overall score if they ask about it.

LOCUS Questions
Show stat cycle (below) to participants and say This is a useful picture to describe how some people think about doing statistics. Think back about your coursework to think about what experiences they have had.

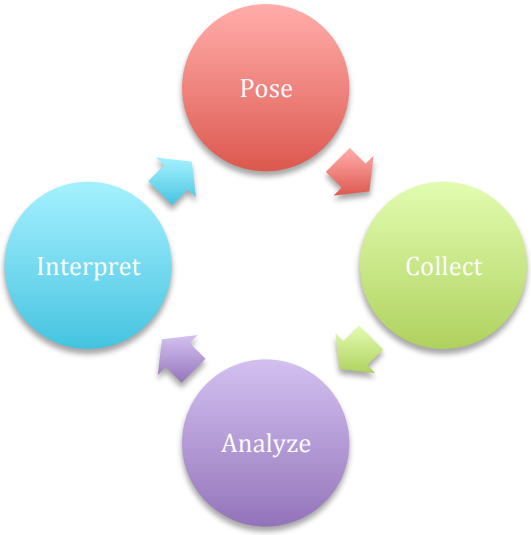


Figure 15. Statistical investigative cycle

This represents the statistical investigative cycle.
Have you seen something like this before? *If yes*, tell me about it? If no, have you heard about the different phases that one might go through in the statistics investigative cycle? What experiences have you had with the statistical investigative cycle?

Show scores of participants in the aggregate for each part of the cycle.
From all the preservice teachers that took this test, this is the percent of items correct for each phase of the cycle. Why do you think that preservice teachers are weaker in _____ part of the cycle? How does this compare to your experience?

Methods Course
To what extent did your teacher preparation courses prepare you to teach statistics? Tell me more.
Probe about lesson planning or student misconceptions about statistics.

Probe about technology. You indicated on the demographics question you did/did not have experience using technology. Tell me more about that.

Overall Questions
Throughout this interview you have talked about a variety of experiences you have had with developing your content knowledge and preparation to teach statistics. How do you think your understanding of the statistics content impacts your confidence to teach?
Say more about that.

On the survey you were asked to rank five different topics: algebra, geometry, pre-calc/advanced algebra, calculus, and statistics. Here is how you rank them _____

Tell me more about how you ranked them.
Why did you rank statistics HERE as compared to NEXT RANKED TOPIC

Probe You mentioned a lot about the content but what else do you think you will need to know to teach it well?

Closing
Thank you for agreeing to work through the task and participate in the interview with me today. Shortly, I will be emailing you a copy of my interview notes. Please review them and verify that they represented your experiences and confidence in teaching statistics.

Do you have any questions? If you have any later please feel free to contact me.

Appendix G: North Carolina State University IRB Approval

NC STATE UNIVERSITY

North Carolina State University is a land-grant university and a constituent institution of the University of North Carolina

Office of Research and Innovation
Division of Research Administration

Campus Box 7514
Raleigh, North Carolina 27695-7514

919.515.8754 (phone)
919.515.7721 (fax)

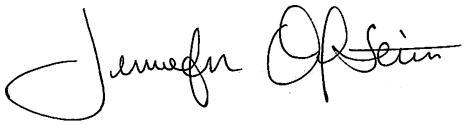
From: Jennifer Ofstein, IRB Coordinator
North Carolina State University
Institutional Review Board
Date: September 16, 2014
Title: Preparedness of Preservice Teachers to Teach Statistics IRB#: 5230

Dear Jennifer Nickell,

The research proposal named above has received administrative review and has been approved as exempt from the policy as outlined in the Code of Federal Regulations (Exemption: 46.101. b.2). Provided that the only participation of the subjects is as described in the proposal narrative, this project is exempt from further review. This approval does not expire, but any changes must be approved by the IRB prior to implementation.

NOTE:
This committee complies with requirements found in Title 45 part 46 of The Code of Federal Regulations. For NCSU projects, the Assurance Number is: FWA00003429.
Any changes to the research must be submitted and approved by the IRB prior to implementation. If any unanticipated problems occur, they must be reported to the IRB office within 5 business days. Please forward a copy of this letter to your faculty sponsor, if applicable. Thank you.

Sincerely,



Jennifer Ofstein
NC State IRB

Appendix H: Sample LOCUS Items

A 13-year study of 1328 adults randomly selected from a population carefully monitored the personal habits and health conditions of participants. Personal habits included tobacco use and coffee consumption. Health conditions include incidence of stroke. Which of the following questions about this population CANNOT be answered using data from this study?

(A) Are coffee drinkers more likely to smoke than adults who do not drink coffee?
(B) Does coffee consumption cause a reduction in the incidence of stroke?
(C) Do coffee drinkers have fewer strokes than adults who do not drink coffee?
(D) What percentage of the population are coffee drinkers?

Figure 16. Sample level C Formulating Questions item from locus.statisticseducation.org

Each member of a random sample of 1,000 adult males from the United States was asked a number of questions, including questions about height and annual income. When the responses were analyzed, it was determined that taller men had greater incomes than shorter men, on average, and the difference was statistically significant. Which of the following conclusions would be most appropriate based on these results?

(A) The study establishes that being tall causes men to have greater incomes, on average, and this conclusion can be generalized to all men in the United States.
(B) The study establishes that being tall causes men to have greater incomes, on average, but this result only applies to the men in the sample.
(C) The study establishes that being tall causes men to have greater incomes, on average, than shorter men, and this conclusion can be generalized to all men in the United States.
(D) The study establishes that being tall causes men to have greater incomes, on average, than shorter men, but this result only applies to the men in the sample.

Figure 17. Sample level C Collect Data item from locus.statisticseducation.org

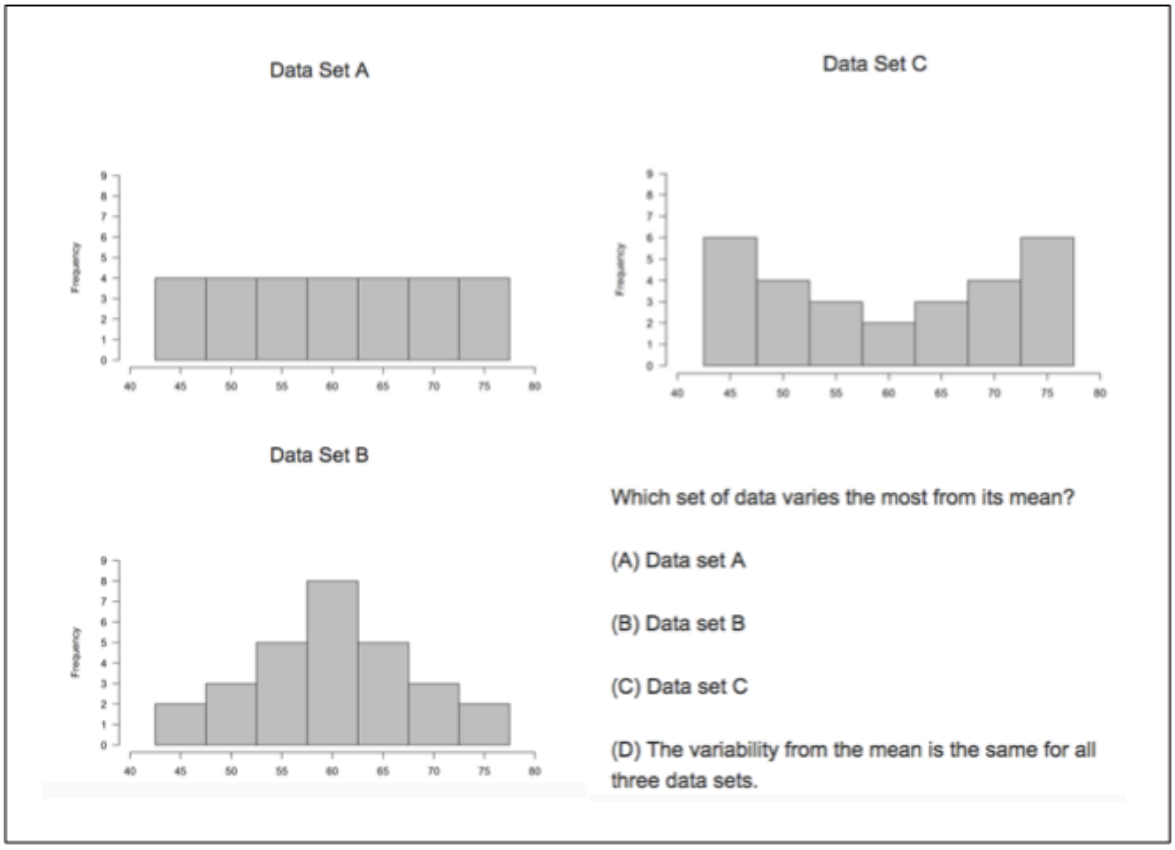


Figure 18. Sample level B Analyze Data item from locus.statisticseducation.org

Mario wondered if students who graduated from a nearby college with an engineering degree would have higher yearly salaries, on average, than students graduating with a degree in mathematics. To investigate, he randomly selected ten engineering graduates and eight mathematics graduates. He calculated the mean yearly salary to be $58,421 for the ten engineering graduates and $58,402 for the eight mathematics students. The difference in the sample means of $19 per year resulted in a p-value of 0.478. Based on this information, which of the following statements is correct?

(A) The difference in means is both statistically significant and practically significant.
(B) The difference in means is neither statistically significant nor practically significant.
(C) The difference in means is practically significant, but not statistically significant.
(D) The difference in means is statistically significant but not practically significant.

Figure 19. Sample level C Interpret Results item from locus.statisticseducation.org