How do Teachers Deal with the Heuristic of Representativeness

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Research Question

- How teachers anticipate students' difficulties with respect to representativeness
  - Conjunction Fallacy
  - Law of Large Numbers
  - Gambler's Fallacy
- What strategies teachers claim to use to help students in their thinking

Methodology

Participant*	Experience teaching statistics (in years)	Degree	Level of the class taught**
Martin	2.5	Master's Math Ed	AP
Sarah	2	Master's Math Ed	AP
Maria	8	Master's Math Ed	AP
Kelly	2 (5 in math)	Dr. Candidate Math Ed	Regular
Richard	7 (17 in math)	PhD Math Ed	AP

- Purposeful sampling
- In-depth interviews with 5 tasks previously used in research
- Audio taped and transcribed

Analysis

- Expert-novice contrast (practicing teachers)
- Interpretativism and grounded theory approaches

Conjunction Fallacy
Episode 1

Bill is 34 years old. He is intelligent, but unimaginative, compulsive and generally lifeless. In school, he was strong in mathematics but weak in social studies and humanities.
Which statement is more probable?

(a) Bill is an accountant who plays jazz for a hobby.

(b) Bill plays jazz for a hobby.

Tomado de Mountier y Houde (2003)

Episode 2

The Scandinavian Peninsula is the European area with the greatest percentage of people with blond hair and blue eyes. However, every possible combination of hair and eye color occurs there. Suppose we choose at random an individual from the Scandinavian population.

Which do you think is the most probable?

(a) The individual has blond hair.
(b) The individual has blond hair and blue eyes.
(c) The individual has blond hair and does not have blue eyes.

Tomado de Tentori, Bonini y Osherson (2004)

	Martin	Richard
Students' difficulties	The wording of the tasks could be problematic Associate his difficulties to students' difficulties Did not identify the tasks had compound events and he was mislead	The wording of the task could be problematic Do not think about the sizes of the sets Do not organize the information in a understandable way Situations are not frequent in the statistics curriculum
Teachers' strategies		Venn diagrams Relate to set and subsets Relate the task to situations familiar to the students (classes) Expose students to similar tasks

Teachers' Answers

- "… technically, there is really no proof that he is either an accountant or plays jazz for a hobby." (Martín)
- *"… students would say: 'This is not a mathematics problem there are not numbers on it'"* (Sarah)
- *"I do not think that there is a logical connection between being an accountant and playing jazz for hobby"* (Kelly)
- *"Strong in mathematics is a clue… so they would think that Bill is an accountant" (Richard)*

Law of Large Numbers
Episode 3

A hospital registers all babies born each day. Which event is more probable?

(a) Eight out of every ten newborns are girls

(b) Eighty out of every one hundred newborns are girls

(c) Eight hundred out of every one thousand newborns are girls

(d) a, b, and c have the same probability.

Taken from Tversky y Kahneman (1974)

Law of Large Numbers
Episode 4

You flip a coin a number of times, which event is more probable?

(a) Seven hundred out of every one thousand times the coin lands head

(b) Seventy out of every one hundred times the coins lands head

(c) Seven out of every ten times the coin lands head

(d) There is no difference in probability among the options a, b, and c

	Martin	Richard
Students' difficulties		Consider that equal fractions mean equal probability
Teachers' strategies		Use the binomial distribution to calculate each probability Relate the task to familiar situations for students like basketball shots Simulate using a coin or a random number generator Explore the distribution of probability of the three different options Compare empirical with theoretical distributions

Teachers' answers

Only Richard and Maria identified the potential difficulty in students

- *"I do not think that my students are going to have difficulty with this situation. They would calculate the fractions and would determine that they are equal"* (Martín)
- *"I think that they [students] have enough experience with fractions and proportions and they can recognize easily they are equivalent"* (Sarah)
- *"We could write each one of these fractions and compare them among them and then talk about what equivalent fractions mean"* (Sarah).
- *"… if I give this question to my students the majority would choose (d) because they are the same proportions"* (Richard)
- *"I think that before instruction, the students would think that they are equal because they come to statistics courses with a strong background in fractions and proportions"* (Maria)
- *"I think that my students are capable of reducing fractions. That is not beyond their understanding"* (Kelly)

Gambler's Fallacy

Episode 5

In tossing a coin six times, which **sequence** is the most likely?
("H" denotes a head, and "T" implies a tail.)

(a) H-T-T-T-T-T

(b) H-T-T-T-H-H

(c) H-H-H-H-H-T

(d) All of the above are equally probable

Taken from Kahneman and Tversky (1972) with modifications

	Martin	Richard
Students' difficulties	Expect equal number of heads and tails in a sequence	Find unusual strings of heads or tails Expects equal number of heads and tails Do not consider that the order matter Do not see independence in each toss of the coin Do not see the result as a sequence but as a group of results
Teachers' strategies	Make a tree diagram	Write out the entire sample space and group them by number of heads Explore a simpler sample space (3 heads) Do simulations to compare what happen in the long run

Teachers' Answers

- All the teachers identified the sequence H-T-T-T-H-H as the more probable one

- All teachers offered simulation as a strategy to help students

  - *"… do a real simulation with coins or random numbers generator" (Maria)*
  - *"… we would do a simulation using calculators or with a deck of cards" (Sarah)*
  - *"… we would write down all the sample space and then we would do simulations to compare" (Ricardo)*

Conclusiones

- Novice teachers tended to think using the same heuristics of thinking that students use.
- The teachers who struggle with the tasks were less capable of identifying students difficulties and offering reasonable pedagogical strategies
- Experienced teachers had more pedagogical strategies to support students

Comments, questions, suggestions?

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"Chance does not have consciousness nor memory"

Joseph Bertrand