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ENHANCING REASONING ON RISK MANAGEMENT THROUGH A DECISION-MAKING PROCESS ON A GAME OF CHANCE TASK

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In this paper, we present a design-based research of a "game of chance" task, which aims enhancing reasoning on risk management. The retrospective analysis of the data of 48 Spanish students of grade 7 (ages 12 to 14) has provided information about the evolution of the levels of reasoning on decision-making in an operationalizing risk management process. Students reasoning evolve from an initial pre-structural level to different levels of reasoning: uni-structural, multi-structural or relational. When reasoning about risk management, the predictive character of probability as allowed distinguishing between decisions "of risk" or "under uncertainty".

INTRODUCTION

"Risk management is any measurement to eliminate or control hazards at an acceptable level" (Borovcnik 2015, pp. 16). The overlapping between risk management and hazard may cause an epistemological obstacle when conceptualizing risk. In order to try to surpass this obstacle, we opt in this paper by using the concept of *operational risk management* [ORM]. We define operational risk management as a decision-making process to systematically evaluate possible courses of action, identify risk and benefits, and determine the best course of action for any given situation. Another obstacle related to the construct of risk is the fuzzy distinction between uncertainty and risk. Knight (1921) argues that on its distinction underlies the predictive character of probability. If the underlying distribution of probabilities is not known, then Knight speaks of *decisions under uncertainty*; if it is known, then, he calls them *decisions of risk*. Although some objections have been made to the naïve, subjectivist point of view and the epistemological nature of this distinction (Borovcnik, 2015), it can be considered a starting point to sustain the criteria for distinguishing between decisions "under uncertainty" or "of risk".

Latten et al. (2011) present decision of risk as one of the skills needed to develop the risk competence. Furthermore, they conceive the unidirectional relationship between risk and risk management. Nevertheless, we consider the need of a bidirectional relationship between the developing of the risk competence and improving the risk management skill. And, in consequence, we conjecture that a path to construct the mathematical notion of risk could emerge through enhancing reasoning on risk management in a systematic operational risk management process.

REASONING ON RISK MANAGEMENT

Ongoing studies on risk literacy suggest the inclusion in the curriculum of the concepts of proportions, distribution (probabilities and frequencies), conditional probabilities, Bayes' formula, and the distinction of absolute and relative risk (e.g. Eichler and Vogel, 2015). In this paper, as presented by Bakker and Gravemeijer (2004), we think on the potentialities of the interrelationship between data and distribution. From an upward perspective, in which data analysis promotes the

construction of the frequency distribution; from a downward perspective, in which the probability distribution leads to model data.

Previous investigations about reasoning on risk have used the *Structure of observed learning outcome* model [SOLO]. For example, the research on young students' mental modeling within simple situations of decision making under uncertainty (Eichler and Vogel, 2013), or on middle school students' levels of reasoning about data dispersion in risk contexts (Sanchez and Orta, 2015). We use the SOLO model to analyze the levels of evolution of students' reasoning on a decision-making process when operationalizing risk management. We present a model of five levels of reasoning on decision-making which is based on: (a) *pre-structural*, irrelevant information; (b) *uni-structural*, isolated information; (c) *multi-structural,* a set of information; (d) *relational*, using a set of information and considering an interconnected knowledge of context and statistical and/or probabilistic concepts; and (e) *extended abstract*, previous integrated whole may be conceptualized at a higher level of abstraction and generalized to the construct of risk. When reasoning about the decision making-process, if students do not recognize the predictive character of probabilities, we say that their reasoning about the decisions is "under uncertainty". Meanwhile, if students recognize the predictive character of probabilities, we say that their reasoning about the decisions is "of risk".

METHODOLOGY

Integers Addition Bingo is a game of chance where two integers randomly generated from -5 to +5 have to be added mentally and mark their appearance in a card. The task has been constructed to address the question: *how should I construct the card to minimize the risk to loose in the integers addition bingo?* Firstly, the task aimed students to understand the game through playing with cards of 10 numbers. Secondly, reasoning on the decision-making process when selecting one card from two or four, or when constructing cards to win. Finally, reasoning on risk management when answering: *how to construct the card to minimize the risk to lose*? The task is complemented with an app constructed with the dynamical multiplatform Geogebra. The app, which generates randomly integers form -5 to +5, gives the possibility of knowing the table of absolute or relative frequencies of appearance of each added pair of numbers, analyze and compare the graph of the distribution of frequencies and the binomial distribution of probabilities.

A design-based research approach was used with two teachers jointly developing the task during seven sessions of one hour. The participants reported in this paper were 48 students in two classes of grade 7 (12 to 14 years old) from Spain. The retrospective analysis consisted in a case study of the cards constructed cooperatively by the students, students' materials including their individual and cooperative reasoning about the decision-making process when selecting and constructing the cards, audio recording of the deliberative dialogues of the decision-making process. The retrospective analysis presented in this paper aims to understand: how did the reasoning on decision-making evolve when operationalizing risk management?

RESULTS

The figure summarizes the retrospective analysis of the task designed in relation with the moments of playing, reasoning on the decision-making process of comparing or constructing cards, and thinking stochastically when analyzing the distribution of relative frequencies or probabilities and comparing both distributions. On this retrospective analysis we distinguish, as designed, three

moments: understanding the uncertainty underlying the "Integers Addition Bingo", operationalizing risk management and reasoning on risk management (see Figure 1).

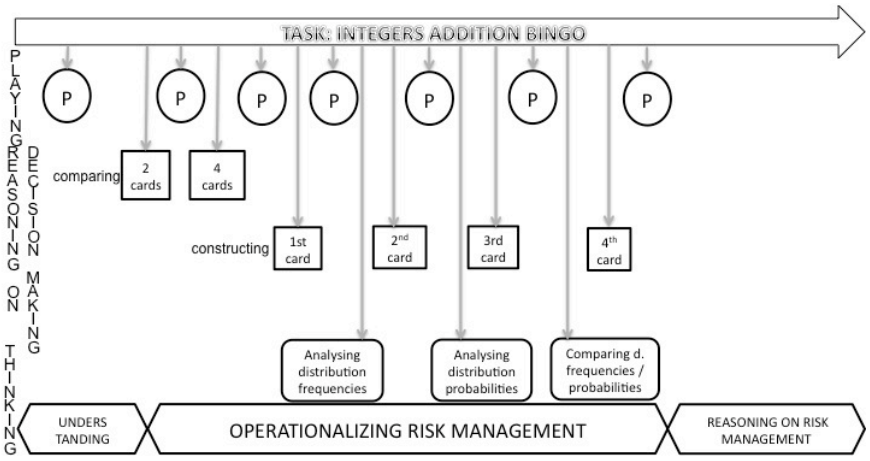

Figure 1: Retrospective analysis of integers addition bingo task development

In the moment of understanding, only some students recognized the uncertainty underlying the "Integers Addition Bingo" game. They reasoned about the uncertainty of the game with the intuitive sense of randomness (8,5%), the subjective sense of luck (2,12%), a hazard situation (4,24%) and a possibility or probability involved (4,25%).

During the operational risk management moment, students reasoning evolved from the initial pre-structural level to different levels (see Table 1).

	Selecting		Constructing				Reasoning on
SOLO (%)	2 cards	4 cards	1st card	2nd card	3rd card	4th card	Risk management
Pre-structural	100,0	58,3	25,0	0,0	0,0	0,0	0,0
Uni-structural	0,0	41,7	75,0	83,3	83,3	75,0	76,2
Multi-structural	0,0	0,0	0,0	16,7	16,7	16,7	26,2
Relational	0,0	0,0	0,0	0,0	0,0	8,3	2,4

Table 1: Levels of reasoning on decision-making

The majority of the students achieved a uni-structural level of reasoning (76,2%). They based their decisions on recognizing the modal clumps of the distribution of frequencies or probabilities. Some of the students (26,2%) achieved a multi-structural level of reasoning. They based their decisions on recognizing the modal clumps and the position of the majority of the data of the distribution of frequencies, or identifying the central position of the zero and the symmetry of the binomial distribution of probabilities. Three students achieved a relational level of reasoning basing their decisions on recognizing the modal clumps, the position of the majority of the data of the distribution of frequencies and the symmetrical distribution of probabilities.

We can distinguish students with a uni-structural level (57%) and multi-structural level (12%) of reasoning, which based their decisions exclusively on the analysis of the distribution of frequencies. We consider that their decisions were "under uncertainty". Other students, achieving a uni-structural level (12%) or a multi-structural level (14%), based their decisions on the analysis of the distribution of probabilities. In this case, we consider that their decisions were "of risk". We can mention only one student with an initial understanding of the hazardous situation of the game,

which have developed a relational level of reasoning on decisions of risk, and who distinguishes between the randomness of the experimental data obtained and the predictive character of the binomial distribution of probabilities.

DISCUSION AND CONCLUSIONS

The first investigative cycle, consisting on constructing the first card, play to obtain data, analyze the distribution of frequencies and conclude to construct the second card, has provided students the opportunity to reason on the density of the data and its skewness. The analysis of the density has provided students the opportunity to evolve from a pre-structural to a uni-structural level, coinciding this evolution with the results of Sanchez and Orta (2015). However, this upward perspective, as presented by Baker and Gravemeijer (2004), is constrained by the lack of informal or formal knowledge for analyzing the center and spread of the distribution. From a downward perspective, integrating the analysis of the skewness, the position of the majority of the data and the symmetry of the binomial distribution, has allowed students with the knowledge to reason on how to manage to win. May be, giving the opportunity to conceptualizing the risk to loose to the election of the lower expected probabilities.

The comparison between both distributions, when reasoning about the predictive character of the probability, has given information about the evolution of students' decisions "under uncertainty" to "of risk". We agree with the proposal of Knight (1921) about this distinction, although more research on this sense has to be made to surpass the obstacles presented by Borovcnik (2015).

In, conclusion the task, based on operationalizing risk management through the analysis and comparison of distributions, has allowed the evolution of the reasoning on risk management. However, more research is needed to develop tasks, which operationalize risk management as a path to integrate the enriching of the risk and stochastical literacy.

References

Bakker, A., & Gravemeijer, K. P. (2004). Learning to reason about distribution. En D. Ben-Zvi, & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and Thinking* (pp. 147-168). Dordrecht: Kluwer Academic Publishers.

Borovcnik, M. (2015). Risk and decision making: The "logic" of probability. *The Mathematics Enthusiast , 12* (1,2,3), 112-140.

Eichler, A., & Vogel, M. (2013). Principles of tasks' construction regarding mental models of statistical situations. En B. Ubuz, C. Haser, & M. Mariotti (Ed.), *Proceedings of the Eight Congress of the European Society of Research in Mathematics Education* (págs. 777-786). Ankara, Turkey: Middle East Technical University.

Eichler, A., & Vogel, M. (2015). Teaching risk at school. *The Mathematics Enthusiastic , 2* (1, 2,3), 168-183.

Knight, F. H. (1921). *Risk, uncertainty, and profit.* Boston, MA: Hart, Schaffner & Marx; Houghton Mifflin Company.

Latten, S., Martignon, L., Monti, M., & Multimeier, J. (2011). Die Fördering erster Kompetenzen für Umgang mit Risiken bereits in der Grundschule: ein Project von Riko-Stat und dem Harding Center. *Stochastick in der Schule* (31), 17-25.

Sanchez, E., & Orta, A. (2015). Levels of Reasoning of Middle School Students about Data Dispersion in Risk Contexts. *The Mathematics Enthusiast , 12* (1, article 23), 275-289.