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PREPARING TEACHERS FOR TEACHING PROBABILITY THROUGH PROBLEM SOLVING

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In this paper we discuss the role of probability problem solving in preparing teachers, introducing the idea of solving problems with didactical intention. Based on this idea, we introduce the notion "way of solving probability problems by simulation". Following this notion we will show the "way we teach the way of solving probability problems by simulation", distinguishing whether teaching is offered to future elementary mathematics teachers or secondary school teachers.

INTRODUCTION

For years, many research results in statistics and probability education have been repeatedly suggesting to teachers' trainers to carry out a better preparation of future teachers in statistics and probability in order to improve their competencies in these topics and, as a consequence, to improve competencies of the primary and secondary school students in statistics and probability addressees of their teaching. This is not an easy task, of course. How should this preparation be is an unsolved question for the community.

In general, researchers agree that training mathematics teachers must combine two types of knowledge, the content knowledge and the pedagogical content knowledge (Shulman, 1986), or as Ball, Thames and Phelps, (2008) call it, the content knowledge for teaching. Some variations of this dichotomical model include the terms such as mathematical proficiency for teaching (MPT in Wilson & Heid, 2010) and horizon content knowledge for teaching (HCKT in Fernández & Figueiras, 2014). Training programs of probability teachers should do it. But, it is not easy to think about useful tasks for future teachers in which content knowledge and pedagogical content knowledge could be presented together. Of course, generally future teacher students have many difficulties in learning and understanding basic concepts of statistics and probability, still not learnt or understood from when they were secondary school student. Therefore, how can they learn about pedagogical content knowledge if they have so many difficulties with the content knowledge? An answer to this question could be to consider the idea we want to introduce here, in preparing teachers our proposal is ~~be~~ solving probability problems with didactical intention (Huerta, 2015).

PROBABILITY PROBLEM SOLVING AND PREPARING TEACHERS

We are not discovering anything new if we affirm that mathematics teachers, both in primary and secondary school, have many difficulties in understanding basic concepts on statistics and probability in a meaningful way. So, they find great difficulties when they are tackling tasks about statistics and probability, including of course problem solving tasks. As a consequence, it is possible that all of us could share the idea that the probability problem solving should be present in preparing teachers, not only considering them as learners of mathematics (statistics and probability

students) but as future teachers that should understand what aspects relate the process of solving probability problems to the construction of statistics and probabilistic thinking in students, and also learning to become an agent of the process of solving probability problems into primary or secondary classes. Shaugnhessy (1992) already talked about this, claiming that teaching probability is teaching problem solving.

Santos (2012) formulates himself some questions about what problem solving activities should be considered in a program intended for preparing teachers, both as learners and future professionals. In the following questions he is referring to mathematics, but however we should read them as if they are referring to statistics and probability:

- What mathematics problem solving activities help teacher students to construct and develop the probabilistic and statistic content knowledge for teaching?
- What kind of problems facilitate the construction of students' probabilistic/statistic knowledge?
- How can we distinguish and characterize the mathematics thinking that future teachers have to display in their experiences in solving probability problems and in the design of activities for teaching?

Having this in mind, we have considered the idea of solving probability problems with *didactical intention* in order to prepare (primary and secondary school) teachers to teach statistics and probability. That is to say, we propose probability problems that have to be solved but, at the same time, when future teachers students are doing that, they have to pay attention to the didactical potential of problems. This means that students explore the possibility that the problems they are solving become useful to be solved by students at primary or secondary school and, if so, what transformations must the problems suffer, if so needed, in order to be appropriated for teaching. Hence, we place teacher students in a double condition: first, as solver of probability problems, getting knowledge about the content needed for solving them, and also as a future teacher that he/she would like to teach problems like those, provoking the need to get pedagogical content knowledge in order to infer potentialities, difficulties and so on. Thus, for example, future teacher may become conscious of what difficulties they and their classmate have found in solving the problems. Lastly, solving problems with didactical intention, like those we are going to show in this paper, may allow future teachers to get both basic content knowledge and pedagogical content knowledge about statistics, probability and probability problem solving.

SOLVING PROBABILITY PROBLEMS WITH DIDACTICAL INTENTION IN PREPARING TEACHERS. AN EXAMPLE.

Many researchers (see for example the modelling point of view in Chaput, Girard & Henry, 2011) and also some programs for preparing teachers (see Rossman`s (2013) interview of Mike Shaughnessy, who talks about MET II suggestions to elaborate preparation programs) suggest that simulation be present in the teaching of probability. We will use this suggestion in order to show

what we understand by solving problems with didactical intention, that is, the *way* of solving probability problems by simulation (Huerta, 2014).

An old problem (Shaughnessy, 1983) is retaken and reformulated for our teacher students. This is the original problem: "A cereal company has put plastics models of 5 Star Wars characters in boxes of SPACY-0's. The company puts one character in each box. How many would you expect to have to buy in order to get all five characters?" (p. 340). We have only changed the context of the problem, to one more current context, and also the number of figures of the collection, 6 figures in order to facilitate students in the searching and choosing of appropriated random devices. Theoretically it may be modelled by an absorbent Markov's Chain (Gordon, 1997). Of course, this model is far away from the content knowledge that we can expect in every mathematics teacher. Only its knowledge would be desirable from a MPT point of view in preparing secondary school teachers and maybe not from others. However, intermediate and concrete theoretical models can also be available in Engel (1975). So we need to consider this problem in two differentiated scenarios: one for preparing elementary mathematics teachers and another for future secondary mathematics teachers. Both scenarios share the way the problem is going to be solved, by simulation, but they differ in their didactical intentions. We have the intention to explain this way to solve probability problems and the associated didactical intentions in each scenario.

THE WAY OF SOLVING PROBABILITY PROBLEMS BY SIMULATION

We understand "the way of solving probability problems by simulation" as a resolution method, with heuristic content, for solving probability problems that appeals to simulation during the resolution process of the problem.

We say that a problem has been solved by simulation if, during the resolution process of the problem, the posed problem (the *original problem*) has been transformed in another (the *simulated problem*) by means of one random device, so that the simulated problem is probabilistically equivalent to the original one. A solution of the simulated problem may be given, with the hope that it could be translated as a solution for the original problem (see Figure 1). A complex process that needs to be analyze.

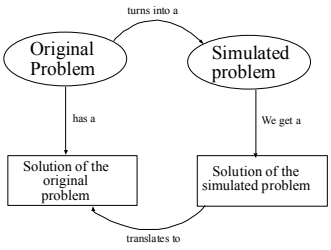

Figure 1: Basic diagram of the process of resolution of a probability problem by simulation

So, solving a problem by simulation requires considering transformation tools from the original problem to the simulated one, so that both problems are probabilistically equivalent. Didactically, what is interesting in those tools is their heuristic character and their heuristic potential of exploration and discovering.

THE WAY TO TEACH THE WAY OF SOLVING PROBABILITY PROBLEMS BY SIMULATION.

It is not the same to propose to solve a problem by simulation to elementary teacher students that to the secondary ones. The reason may be obvious: because of the different initial preparation in mathematics. For the former, simulation has to be proposed as a method for solving probability problems that allows students to go from realistic situations to process of modelling, working with the problem into concrete models using statistics for solving simulated problem and next to get back to the original one. However, for the secondary school teacher, the proposal has to be the opposite. That is to say, from the model to the realistic situations by means of simulation. Both samples of students share a space when they are using statistics for solving the simulated problem.

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