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DATA ANALYSIS IN THE UNITED STATES: CURRICULUM PROJECTS

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Abstract

The National Council of Teachers of Mathematics Curriculum and Evaluation Standards reinforced by the Quantitative Literacy Series from the American Statistical Association have had an impact on the curriculum in schools in the Untied States, particularly in the area of data analysis and statistics. Textbooks now include many topics from this area, and teachers are beginning to look for ways to include the concepts in their mathematics programs. Implementation issues, however, are prevalent. Teachers lack the background knowledge to teach the content and often omit it. Many of the texts focus on product answers for example "What is the mean?" and do not build process nor conceptual understanding. Teachers are confronted with many new topics and new ways to teach; they have to make choices in order to "cover" their curriculum. A lack of statistical understanding and erroneous thinking about statistics is evident in many published materials. The National Science foundation has funded a variety of curriculum projects for primary, middle level and secondary students that include data analysis as an appropriate and meaningful part of the curriculum. These projects exemplify in different ways how the tenets of the Standards can be put into practice. Examples selected from several of these projects are used to give some indication about the direction data analysis seems to be taking in the United States and how this is reflected in the schools.

Introduction

In 1989 the National Council of Teachers of Mathematics (NCTM) published the *Curriculum and evaluation standards for school mathematics,* the first in a set of three standards documents, followed by *The professional teaching standards* (NCTM, 1991) and the *Assessment standards* (NCTM, 1995). For the first time in K-12 mathematics in the United States, clearly defined content strands in the area of statistics and probability were designated as important parts of the curriculum. Statistics and probability formed a single strand for grades kindergarten to four and were delineated as separate strands for grades five to eight and grades nine to twelve. The focus of much of the content standards in statistics was on data analysis. The teaching standards and the assessment standards both gave further credibility to the importance of the strands by using examples from data analysis and probability to illustrate how standards based teaching and assessment should look in the classroom.

At the beginning of the 1990's, the National Science Foundation (NSF) and other funding agencies funded large scale curriculum development projects for elementary, middle school and high school, and gave each a period of approximately five years to produce curricular materials that would be in keeping with the NCTM standards. How do data analysis and statistics fare in these new projects? What is the role of data analysis within a grade level? What statistical content has been stressed and what omitted? What problems can be identified as these new curricula are implemented? The following discussion is based on an examination of materials from several curriculum projects, in particular Mathematics in Context, Data Driven Mathematics and Core Plus Mathematics Project.

Mathematics in context (MIC) is a curriculum for students aged ten to thirteen with ten units per grade level, each part of a distinct content strand with strong connections between the strands. *Contemporary mathematics in context* (Hirsch et al., in press) is from the *Core plus mathematics project* (CPMP) and is a core curriculum for students aged thirteen to sixteen. At each grade level a section of the textbook is devoted to statistics. The other content sections include problems from statistics as part of the integration across content strands. *Data driven mathematics* (DDM) is a series of ten modules designed to be used as replacement units for traditional mathematical topics for students at different levels in secondary school. In each module, standard mathematical concepts are motivated by the use of data. The units that pertain directly to data analysis in each project are listed in the references at the end of this discussion.

The materials in each project were developed by teams of mathematics educators, mathematicians and curriculum developers with input from practicing K-12 teachers. In most cases, statisticians were linked to the projects in an advisory capacity; in Data Driven Mathematics, statisticians were part of the development and writing teams.

General trends

Exploring data is a content strand in all of these programs

Initially in all of the projects, students collect much data from measurements on their own bodies (height, arm span, foot length) or from their own personal experience (ways they travel to school, number of letters in their name, telephone numbers). As students get older, they do less collecting and analyzing of their own data and more working with data sets that are given. There appears to be less exploration and more focused analysis of the data. There is little use of technology as a way to help investigate data before grade nine. Scientific calculators are used for some of the computation, and an occasional reference is made to software. Both CPMP and DDM rely heavily on graphing calculators to do number crunching and as a way to investigate certain situations.

Integration of data analysis techniques is used to provide different ways to approach some standard concepts

The use of average motivates the study of ratio in the MiC unit *Rates and ratios* (Keijzer, et al., MiC, in press). In *Exploring symbols: An introduction to expressions and functions* (Scheaffer et al., DDM, in press), students learn about variables by investigating situations that involve answering questions such as "In which state is it safest to drive?" given data on licensed drivers, motor vehicle registrations and traffic deaths for a sample of the states in the United States. Students use formulas and evaluate expressions to calculate crime ratings or education ratings for cities based on census data. Matrices are introduced as a way to organize and manipulate data, ranking baseball hitters and using multivariate regression techniques to predict college success in *Advanced modeling with matrices* (Witmer et al., DDM, in press).

Number sense is developed by working with real data, interpreting numbers and thinking about numerical summaries

In some instances, the focus is directly on a data analysis topic such as "Analyzing and Comparing Data" and in others, data analysis is embedded in another topic such as *Exploring symbols* (Scheaffer et al., DDM, in press) and *Mathematics in a world of data* (Cifford et al., DDM, in press) both have as an underlying theme the use of real numbers and how to interpret them. Students think about accident and death rates, investigate the number of people in different age brackets in proportion to the entire population, and analyze slugging percentages in baseball.

Data and situations based on data are used to provide practice with computation and manipulation skills

In *Rates and ratios* (Keijzer, et al. MiC, in press) students use ratios as a way to compare the availability of telephones in countries in Africa. They investigate the difference in conclusions made from the absolute number of phones in each country and those based on the ratio of people per phone in each country. They also practice computation with decimals based on organizing the times for track events. In the CPMP (Hirsh et al., in press) algebra sections, students practice writing equations as models for the relationship between the health expenditures in the United States over time, the mass and volume for olive oil, and the relation between fat and calories in restaurant food.

Statistical content - some examples

The mean can be confusing. In some of the pilot work, students, actually in grade nine or age fourteen, were given a histogram without any labels on the vertical axis and asked to estimate the mean for the distribution as shown in Fig. 1.

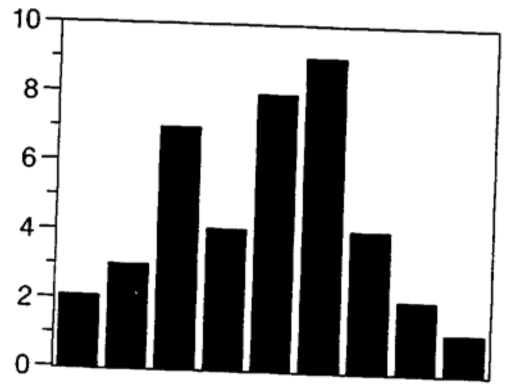

Figure 1. Histogram for estimating the mean

Almost half of the class "balanced" the bars, as did teachers in a workshop, indicating that they did not understand clearly what a histogram represented and further that they were not at all clear about when they could find the mean using a compensation approach or when they could use a calculation.

Graphs are used as a way to communicate the results of student thinking, as an end in themselves (particularly true of most traditional texts), or to investigate data by making different displays. Scatterplots are not used just to find a relationship between paired variables, but also as an analysis tool to help understand data. Students reflect on tables and plots and number summaries to make interpretations. The MiC unit *Dealing with data* (de Jong et al., MIC, in press) centers around different ways in which you can analyze the heights of 1064 fathers and of their sons (using the 1903 Karl Pearson and Alice Lee data) and what each of these ways will add to your understanding: box plots are used to compare the heights of fathers and sons; histograms are used to investigate the distribution of heights of fathers and to make an estimate of the mean height; scatterplots are used to determine whether there is any relation between the heights of fathers and their sons. In *Exploring symbols* (Scheaffer et al., DDM, in press), students compare the information they obtain from a line plot of SAT scores over the years to the information they obtain from a scatterplot. A great emphasis in all of the statistics sections in CPMP is placed on beginning from data distributions and moving between numerical summaries, tables and graphs.

None of the projects encourage students to create their own displays. In *Insights into data* (de Lange, et al., MiC, in press), students conduct an experiment growing bean sprouts in different solutions. In one class, careful instructions had been previously given about how to make box plots, stem-and-leaf plots, and number line plots. Students had not learned how to make plots over time nor how to handle multiple measurements. The task was to summarize and display the final lengths of the sprouts grown in different solutions and to show how the sprouts grew over the time period. The children's displays of the final lengths were exactly those that had been taught, see Figs. 2 and 3.

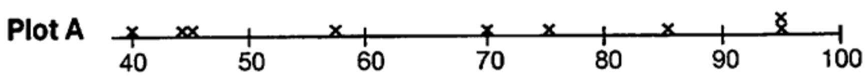

Figure 2. Number line plots of lengths of bean sprouts in tap water

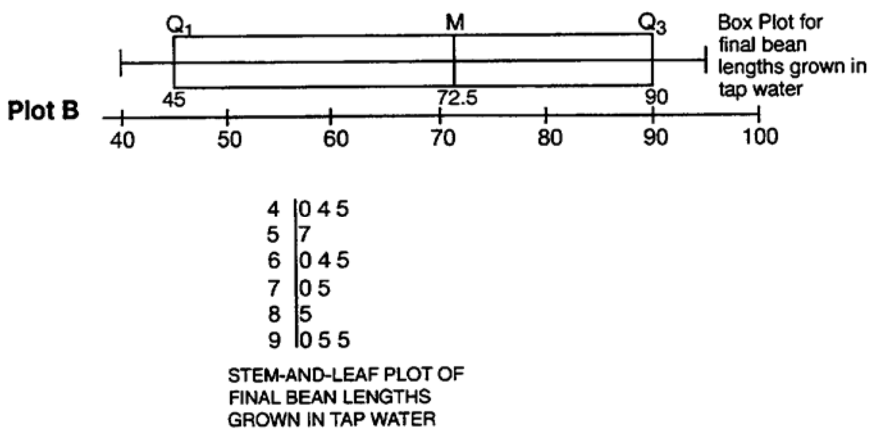

Figure 3. Plots of final lengths of sprouts grown in tap water

The graphs of the sprouts growth over time were creative and unique, demonstrating that the students really did have a feeling for what changes had taken place, see Figs. 4 and 5

The first graph reveals the variability in growth as well as the trend. The second shows what happened to each individual sprout over the 7-day period. The variability here is not as obvious, but both individual and collective growth over time is very visible. One sprout grew tremendously over the weekend which illustrated for the students the dangers of extrapolation. Left to themselves, students found very diverse and appropriate ways to display their data. This illustrates a fundamental pedagogical dilemma: *how to provide students with a standard set of statistical tools without stifling their creativity and their instincts about appropriate solutions.*

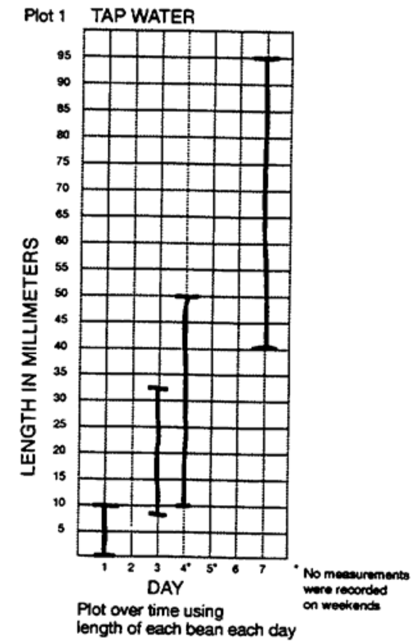

Figure 4. Plots over time using range of lengths each day

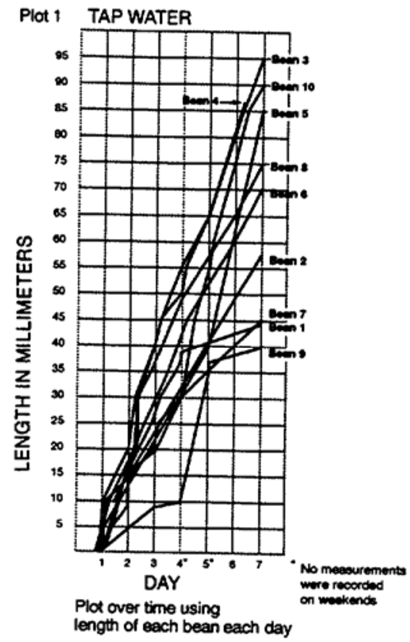

Figure 5. Plots over time using length of each bean each day

Collectively, the curricula make an effort to promote critical thinking, a skill not part of most traditional materials. Students investigate data sets where they have to make choices about what kind of summary is most appropriate: What value would you chose to describe the number of protected animals in different parts of the world? In this case, the maximum is more useful than any measure of center. When is the sum of the data more useful than the average? In *Dealing with data* (de Jong et al., MiC, in press) students consider which of the arguments is the most convincing based on 1064 pairs of father son heights. Some answers were:

> *"Sons are taller than their fathers because the total height of all the fathers is 72,033 inches. For the sons, the total height is 73,125.7 inches. Tiwanda*
>
> *"Sons are taller than their fathers because sons beat their fathers in height 664 times out of 1064. There are 19 ties. " Huong*
> *Sons are taller than their fathers because more than half of the sons are taller than their fathers." Anita*
>
> *"Sons are taller than their fathers because the tallest son in the data set is taller than the tallest father." Dustin*

In the chapter "Patterns in Association" in year two (Grade 10) of CPMP (Hirsch et al., in press), students learn to think critically about the use of correlation as a measure of linearity and as a criteria for curve fitting. They evaluate sample surveys and compare voting methods to sampling methods in year three (Grade 11), "Modeling Public Opinion." CPMP (Hirsch et al., in press),

Sampling and Representativeness

Statistics might be called both an art and a science. Art is defined as "skill and performance, acquired by experience." In this sense, data analysis is an art; experience helps you chose from many different options ways to express a data set that reveal patterns and relationships. Different "artists" can make different choices and often reveal different information. But statistics is also a science, "accumulated knowledge systematized and formulated with reference to the operation of general laws." The techniques of data analysis help to develop the art forms; when data serve as the foundation for making conclusions and drawing inferences, however, there is a science that guides the procedures, general laws that shape the reasoning process. It is important to use correct procedures, proper terminology and the science that makes the techniques legitimate.

Sampling is the process of selecting a subset of a population in order to measure some characteristic and eventually to make some statement about this characteristic either in reference to the sample or in reference to what it indicates about the population as a whole. Certain "truths" or expectations are known about random behavior (the science). If you take random samples of size 50 from a population where 30% have a given attribute, you can expect to generate a distribution centered around 15 with a standard deviation around 3. Thus, when you work with random samples you can have a probabilistic assessment of error, based on what you would expect. In order to get random samples, you have to use some selection method that depends on chance, not on any subjective factor. Human judgment and choice usually show bias, while blind chance is impartial.

A great deal of emphasis is placed in some curricula on having a "representative" sample, arguing that such a sample is necessary in order to make statements about the population. When sampling is necessary, there is generally very little known about the population so that a judgment on the representativeness of the sample cannot be made by any objective criteria because we do not know enough about the population to know on what criteria we want to be sure we are representative! A representative sample based on gender may contain hidden factors you did not consider that may make the sample now more "unbalanced" than if you had used a random sample. If gender balance is important, there are statistical ways to manage this without introducing another factor that may bias your results. (Freedman, Pisani, & Reeves, 1980 page 12)

Questions contained in the materials such as "Was the sample representative?" focus students on the wrong issue and create the impression that their subjective judgment is going to produce a better sample than one that is obtained by using random processes.

For most situations, a better frame of reference might be to use the concept of bias: Did the sampling procedure systematically leave out any segment of the population? Was there selection bias or non-response bias? *Projects* (Scheaffer et al., DDM, in press), *Insights into data* (de Jange et al., MiC, in press), and "Modeling Public Opinion" from year three (grade 11) of CPMP (Hirsch et al., in press) raise issues concerning bias. In *Great expectations* (Roodhardt et al., MiC, in press) there is a unique attempt to help students visually understand the random sampling process using a calculator random number generator. Several different random samples chosen from a population of 100 are displayed, and students asked to observe how the samples differ.

Articulation - some observations

The development of concepts as students proceed through the K-12 curriculum has not yet been clearly carried out nor perhaps well thought out. Each project has started from their own platform to build student knowledge of statistical concepts. The mean and median have been introduced in elementary school, but each is introduced again in lower secondary school and again in upper secondary school, usually with no assumption of prior knowledge.

Data situations reoccur over and over again throughout the grade levels. Thoughtful problem construction must be done at each level to ensure that learning is cyclic not circular. Students collect the measurements of their heights at nearly every grade level. Collecting body measurements occurs in all of the curricula, and students are asked to describe the relation between height and arm span in three different grade levels. Temperature data are used in nearly every curriculum, often repeated from year to year. Plots over time (part of the algebra strand, not the statistics strand in MiC) begin early and the activities are repeated with little change over several years. Students collect information to describe a typical student several times. Lessons at each level should be designed to bring out increasingly complex underlying statistical concepts and promote growth in student understanding when situations are revisited. Care must be taken, as well, that students do not become bored with the repetition.

Variability on the other hand is nearly ignored until upper secondary school with emphasis on range and in specific instances on quartiles. Even then center and dispersion are rarely treated as a pair of summary statistics that together help you understand the distribution. Outliers are alluded to only briefly. There is a lesson on the mean absolute deviation in "Patterns in Data" (Hirsch et al., CPMP, in press) in grade 9 and intensive work on the standard deviation in grade 11 in "Patterns in Variation" (Hirsch et al., CPMP, in press). In *Exploring symbols* (Scheaffer et al., DDM, in press) for grade 9 students, a section is devoted to standard deviation and another to using the standard deviation to find a z score to help standardize data sets for comparison purposes. Because the topic of variability is difficult but fundamental, it seems it should have more of a foundation in the early grades.

Most of the curricula examined encourage students to do projects, although the structure is informal and does not specifically address designing an experiment. Does watching television lower student performance in school? Without any data, it is tempting to respond based on personal knowledge of one or two individual cases. To produce an answer that is not subjective, however, you need either to obtain available data from some source or design an experiment to collect information that will help you answer the question. Designing experiments is an important way to collect data and recommended in the CSE's Guidelines for the K-12 curriculum. There are very few such experiments in the curricula, besides a mung bean experiment (de Lange et al., MiC, in press) and bungee jump (Hirsch, et al., CPMP, in press) gives the topic a thorough treatment and includes activities to help students understand what is meant by experimental design and how to put it into practice.

Care must also be taken to ensure that terms and concepts are used appropriately. Bar graphs and histograms appear almost indiscriminately in some places with seemingly little thought given to whether they are appropriate in terms of the data and of concept development. In several instances the term bar graph is used to refer to a histogram (Class heights are graphed in a "bar graph.") It is not clear whether the mode should be actually treated as a measure of center; the projects are not even internally consistent in this respect. (Is the mode a typical result?) Careful distinction between discrete and continuous data has not always been made. For example making line graphs for discrete data can imply continuity where none exists.

Conclusion

While the progress in introducing statistical concepts into the K-12 curriculum has been steady and shown a great deal of growth, there is still much to be done. There is need for further research, better professional development of teachers about data analysis; and more careful thinking about the curriculum from a global perspective.

> Numerical literacy, the ability to follow and understand numerical arguments, is important for everyone. The ability to express yourself numerically, to be an author rather than just a reader, is a vital skill in many professions and areas of study. The study of statistics is therefore essential to a sound education.

> We must learn how to read data, critically and with comprehension: we must learn how to produce data that provides clear answers to important questions; and we must learn sound methods for drawing trustworthy conclusions based on data. (Moore et al., 1989 page xvii)

The curriculum projects are producing materials that seem to be on the right course. The CSE *Guidelines* might be revisited as benchmarks to evaluate where we should go in the future to ensure that we do have materials that will educate students to read data, produce data and use them to make sound conclusions.

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Notes on references:

1. Data Driven Mathematics (DDM) is the name of a project. Each booklet in this project is separate, authored by a different set of authors and available independently. There is no overall editor.
2. Math in Context (MiC) is the name of the project and of the curriculum. It consists of a series of booklets, each authored by different sets of authors, individually packaged and available independently.
3. Core Plus Mathematics Project (CPMP) was the project of a set of high school of three books, one for grade 9 (year 1) one for grade 10 (year 2) and one for grade 11 (year 3). Each book consists of separate chapters that have names such as "Patterns in Data." Chris Hirsch is the overall editor but each chapter is written by a different team of authors.