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EMERGING RESEARCH ISSUES IN THE TEACHING AND LEARNING OF PROBABILITY AND STATISTICS

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Abstract

This paper identifies and discusses some research issues in the teaching and learning of probability and statistics that a) have already emerged, b) that are beginning to emerge, and c) that we hope will emerge in the near future. Prominent issues that have emerged include i) people's naive conceptions and beliefs of probability and statistics concepts, and ii) the impact of instruction on these conceptions and beliefs. Issues that are just beginning to emerge include i) research on students' understanding of graphs and visual displays of data, and ii) the importance of investigating teachers' own conceptions and beliefs about probability and statistics. Fertile issues for future research that have yet to be investigated include i) research on the importance of building connections between probability concepts and data handling concepts, and ii) research on cross cultural differences in understanding probability and statistics. There is now quite a body of literature on naive beliefs and conceptions about stochastic concepts. Some researchers are beginning to embark upon developing a 'theory of graphicacy' to describe students' understanding and interpretation of graphs. Research in the other areas mentioned above is either currently rather dormant or still needs to emerge.

Introduction

About twenty years ago there was practically no research being done on the teaching and learning of probability and statistics by mathematics or statistics educators. Any exceptions were doctoral theses, mostly conducted in the late 1960's or early 1970's. Everything was an "emerging issue" for research in probability and statistics, because the slate was fairly well empty. Two decades later we are in a position where research in the teaching and learning of probability and statistics has begun to develop a enough of a history that we can identify existing and emerging fertile research issues. In this paper we will identify and discuss some research issues that have already emerged, some that are beginning to emerge, and some that we hope to see soon emerge in probability and statistics. Two important issues that we will acknowledge, but not deal with in this paper, are research on alternative

assessment and research on the use of technology in teaching and learning probability and statistics. Research on assessment and the use of technology are paramount for all areas of mathematics education at this time. In this paper we intend to concentrate on several issues that are particularly important for probability and statistics.

Issues that have emerged

Prominent issues that have emerged in research on the teaching and learning of probability and statistics include i) people's naive conceptions and beliefs of probability and statistics concepts, and ii) the impact of instruction on these conceptions and beliefs. In the past two decades there has been substantial work on naive conceptions, while only a little on the impact of instruction.

Naive conceptions and beliefs

Research on common naive conceptions of probability and statistics has dominated much of the early research in stochastics, as can be witnessed by reviews of the literature in Garfield & Ahlgren (1988), Shaughnessy (1992), and Greer & Ritson (1993). Intuitive heuristics such as representativeness, availability, and conjunction (Kahneman and Tversky, 1972, 1973; Tversky & Kahneman, 1983) or the 'outcome approach' (Konold, 1989, 1991) that have been identified by cognitive psychologists have given us one possible framework to analyze students' responses to probability and statistics tasks. They have provided us with a partial picture, but certainly not the whole picture, of the maze of peoples' naive conceptions and beliefs about chance. The variety of verbal explanations offered by subjects dealing with probability tasks suggests that there are competing conceptions and belief systems at work (Scholz, 1991; Borovcnik & Bentz, 1991). Similarly, research on students' naive conceptions of statistical concepts such as the mean (Pollatsek, Lima and Well, 1981; Mevarech, 1983) and their understanding of the role of the cells in a contingency table (Batanero, Estepa, Green, and Godino, 1996) has suggested that certain statistical concepts are also distorted by naive conceptions and beliefs. Landwehr (1989) summarizes several common misunderstandings about elementary statistical concepts. People tend to:

1. Believe that any difference in means between two groups is significant
2. Have unwarranted confidence in small samples
3. Have insufficient respect for small differences in large samples
4. Mistakenly believe there is no variability in the 'real world'

Certainly the area of people's intuitive conceptions and beliefs will continue to be a fertile research ground in probability and statistics for a long time. On the other hand, the companion area of the impact of instruction

on people's beliefs and conceptions of probability and statistics is receiving much less attention, and is potentially more important for our teachers and students.

The impact of instruction on conceptions and beliefs

Experiments that carefully document changes in student's stochastic conceptions, beliefs, and attitudes over a long period of time are needed to obtain a clearer picture of students' cognitive and affective development in stochastics in an instructional environment. This is particularly important at a time like the present when there is such a concentrated effort to reform curriculum and instruction in mathematics in so many countries (Cockroft, 1982; NCTM, 1989; LOGSE, 1990; Greer & Ritson, 1993). What kind of impact, if any, do some of these new curriculum materials, or new pedagogical strategies, have on student's conceptions and beliefs in probability and statistics?

There are few examples of past research in this area. Shaughnessy (1977) found that although a small group, problem solving approach to probability and statistics did bring about big changes in some students' beliefs about chance, there was still resistance to much change in those beliefs among other students. (Nisbett, Fong, Lehman and Cheng, 1987) found that students could reason about basic statistics concepts, like the law of large numbers and generalize these concepts to a wide range of areas after instruction. However, the Nisbett study did not include probability or decision making tasks of the types that other researchers have used to evoke entrenched beliefs or preconceptions about chance.

More recently there have been several teaching experiments that have attempted to document changes in students' conceptions after instruction. The National Center for Research in Mathematical Sciences Education (NCRMSE) at the University of Wisconsin and the Freudenthal Institute in the Netherlands conducted a comprehensive five week teaching experiment to introduce data visualization in secondary school. The report on this teaching experiment (de Lange, Burrill and Romberg, 1993) contains a list of goals, examples of the materials that were used, as well as a discussion and comparison of the philosophy and pedagogy used in their curriculum. Reflections on the results of this teaching experiment were provided from various perspectives: a curriculum developer, a funding contractor, and a secondary teacher. These reflections concluded that a corresponding pedagogical revolution must accompany the type of curriculum revolution contained in materials such as their Data Visualization unit. Success in implementing innovative materials in data visualization rises or falls on the ability of teachers to use them as envisioned by the developers.

In another teaching experiment conducted with lower secondary age students (11-16) by Biehler and Steinbring (1991), data analysis was introduced as detective work. Teachers gradually provided students with a data 'tool kit'

consisting of tasks, concepts, and graphical representations. The experimenters observed over 100 class sessions, documenting episodes with notes, teacher diaries, student portfolios, student tests, and small group work on computers. Biehler and Steinbring concluded that all students had success in acquiring the beginning tools of EDA, and that both the teaching and the learning become more difficult as the process becomes more open. There appeared to be a tension between directive and non-directive teaching methods in this study.

These two case studies provide a starting point for the extensive work that is needed in documenting the growth of student understanding of probability and statistics concepts over time, and the effects of pedagogical interventions in teaching stochastics. Both these studies looked at whole classes, and reported summary learning experiences of the students, and difficulties encountered by the teachers. A fruitful path for future research on the influence of instruction on student growth in probability and statistics might be long term case studies of individual students. Careful documentation of students' thinking over time, and how it changes, will provide us with a better road map of the conceptual struggles and the problematic teaching areas in probability and statistics. This information can in turn be useful in revising our curriculum and instruction practices, as well as in uncovering interesting questions for further research.

Research issues that are beginning to emerge

Research on the understanding of graphs

A body of empirical and teaching studies is beginning to accumulate on the understanding of graphs, in an attempt to develop what some researchers are calling a 'theory of graphicacy.' Curcio (1987) and Aberg-Bengtsson & Ottosson (1995) have studied students' general understanding of graphs. Curcio distinguished three difficulty levels in the comprehension of graphical data:

- Reading the data without any interpretation, attending only to facts explicitly represented in graphs
- Reading within the data which requires comparisons, mathematical concepts and skills
- Reading beyond the data which requires extension, prediction, or inference.

Students' understanding of particular types of graphs has been investigated by a number of researchers. Bargraphs (Pereira-Mendoza & Mellor, 1991), bar charts and line graphs (Aberg-Bengtsson & Ottosson, 1995), stem-and-leaf plots (Dunkels, 1994; Pereira-Mendoza & Dunkels, 1989), box-plots (Carr & Begg, 1994), and scatterplots (Estepa & Batanero, 1994) have all been the subject of individual research investigations. More generally, the entire subject of Exploratory Data Analysis (EDA), has been taken up by Biehler & Steinbring

(1991) and by Dunkels (1992). In general these studies suggest that visual representations of data such as stem-and-leaf, box-plots, & scatter-plots are appropriate to introduce to middle school age students. On the other hand, care must be taken to emphasize the understanding and interpretation of such plots, and to focus on the substance within visual displays of data rather than on the methodology of constructing them (Carr and Begg, 1994). Discussions with students on which types of plots are more useful or which ones convey better information in different data situations are very important.

Pereira-Mendoza (1995), discussing graphing at the elementary level, suggested that children should:

- Explore the assumptions underlying the classification of data and interpretation of the meaning of data.
- Discuss and explore the possibility of alternative representations.
- Predict from the data.

Such activity at the elementary level will provide a sound basis for secondary teachers to build upon when looking at statistical ideas.

Students' understanding of graphs is more complicated than one might think upon casual observation. Recently I observed a class of middle school students struggle with locating modes and medians within their own visual representations of data. These students were quite competent at finding medians when given raw data, but their visual model for the mean interfered with their ability to find the median from a graph. The students were accustomed to using stacked cubes to represent data values in columns, where each column represents one piece of data. For example, while considering the number of pets in a household, a column of 4 cubes represents 4 pets in one of the households, a column of 7 cubes would be 7 pets, another column of 4 cubes represents a second household with 4 pets, and so forth. The mean number of household pets can then be visually displayed as a 'leveling' of all the columns, making "cuts" in the cubes if necessary. This visual display represents the mean as a vertical height. On the other hand, in many visual displays of data, such as frequency distributions, the mean, as well as the median, is visually located as a horizontal value. Furthermore, when students built a frequency distribution for household pet data, and then tried to find the median from the graph, many of them identified the median as the middle 'column' of data rather than the middle piece of data.

This type of incident raises all sorts of questions about what students need to explore and what we need to research. What types of graphs are the most helpful for demonstrating centers and spreads? How do students actually use graphs to represent or interpret concepts? Students need experiences wrestling with the meaning that is, or is not, represented in various graphs. Situations that challenge their conceptions of graphs, translating information

back and forth between raw data and various visual representations of that data, could be critical to developing their understanding.

Bertin (1983) and Tufte (1983) present systematic analyses and general design principles for graphical displays of statistical data. A list of suggestions by Tufte may provide a starting point for the empirical study of the psychology of graphical representations of data. It may also provide a blueprint for a philosophy of teaching data visualization for curriculum developers. Graphical displays should:

1. Show the data
2. Induce the viewer to think about substance rather than merely about methodology
3. Present many numbers in a small space
4. Make large data sets coherent
5. Encourage the eye to compare different pieces of data
6. Reveal the data at several levels of detail
7. Serve a reasonably clear purpose, such as description, exploration, tabulation, or decoration
8. Be closely integrated with the statistical and verbal descriptions of the data.

A thorough review of the literature on graphicacy, as well as a first attempt at building a theory of using and understanding graphs, is presented by Bright, Curcio, and Friel (1996). This paper operationalizes some of the components of "graph sense", identifies some issues in understanding graphs from the literature, and presents a list of researchable questions on the learning and teaching of graphs.

What are teachers' conceptions of probability and statistics?

Another researchable area that is emerging involves the beliefs and conceptions that our teachers hold about probability and statistics. Teachers lack experience in carrying out the juggling act of trying to teach concepts, skills, and graphical techniques in probability and statistics, while trying to encourage students to make their own decisions about analyzing raw data. Many teachers do not themselves have strong backgrounds in these areas. Besides, teachers are apt to have the same naive conceptions and beliefs about probability and statistics as their students.

On the other hand, successful implementation of the recommendations for student experiences in probability and statistics that have recently come forth (Cockroft, 1982; NCTM, 1989; LOGSE, 1990; Greer & Ritson, 1993) ultimately depends upon our teachers' ability to pull it off. What can we do to help change teachers' conceptions and beliefs about the nature of probability and statistics?

Research studies are needed to gather information about teachers' conceptions at both the preservice and inservice levels. Such studies on teachers' beliefs and conceptions can provide information to help us develop experiences that will allow teachers to grow in their own understanding of probability and statistics, while sensitizing them to the array of conceptions and beliefs that they are likely to encounter among their own students. This issue has emerged because it is clear that there will be a continual need for more and more inservice experiences for teachers in probability and statistics to effectively carry out the recent recommendations from the curriculum reform movements (See for example, Biehler, 1990). To date, however, I am not aware of any research that has documented teachers' conceptions or beliefs, or how they change while teaching a unit or a course on probability and statistics. This area is important, and it is wide open.

Research issues that still need to emerge

Connections between probability and data handling

What is the relationship between probability and data handling in an introductory statistics course? There is disagreement on the importance of probability in data handling. Some authors and instructors wish to remove probability as much as possible from their treatment of data handling. Others feel that it is important to systematically make connections between probability and data handling wherever appropriate. My own opinion leans strongly toward the benefits of integrating probability with data handling, building important connections between them. For example, it seems particularly appropriate to build connections between visual displays of data and probability concepts during a treatment of sampling. What is a 'likely' spread of values in a data set if it is drawn from a population with known characteristics or parameters? Given a sampling distribution, is it likely or unlikely that it was drawn from a given population? How do we begin to get a handle on quantifying this "likely" idea? As soon as we begin to treat, however informally, the notion of a confidence interval, we must be prepared to deal honestly with questions of probability.

Biehler (1994) compares the cultures of EDA and probabilistic thinking and argues that there are definite benefits to considering probability distributions when teaching data analysis. The question is, how much probability and for which audiences? This is certainly an important curriculum development issue, but it is also an important research issue. What do we gain, or sacrifice for our students, when we do, or do not, systematically make connections between probability concepts and statistics concepts, particularly with visual displays of data? This is certainly very difficult work, but we need to begin to research the benefits and/or drawbacks of making these connections.

Cross cultural differences

Are there differences in peoples' conceptions or beliefs about probability and statistics among various cultures? Most of the psychological research that has been done in decision making under uncertainty has been done in a few western countries, principally the United States, Israel, the United Kingdom, and Germany. Likewise, the mathematics education research on students' conceptions and beliefs about probability and statistics has been done predominantly in these same western countries. Are phenomena like judgmental heuristics or naive conceptions of probability and statistics concepts artifacts of western culture, or do they appear across many cultures? Eisenhart (1988) discusses the advantages of conducting research in mathematics education from an educational anthropologist point of view. Ethnostochastics is an area that has not been investigated yet by researchers in probability and statistics. Cross-cultural comparative studies, particularly case studies of individuals working on decision making tasks and probability estimation tasks, will give us information on just how wide spread throughout the human race these naive conceptions and beliefs of probability and statistics really are. I know of no work in this area in Africa or Asia or many parts of the middle east. And yet, games of chance are found in almost all cultures throughout the world. Cross cultural conceptions of chance is another fertile area for future research, one which I hope will emerge in the near future.

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