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## STATISTICAL INDEPENDENCE - ONE CONCEPT OR TWO?: IMPLICATIONS FOR RESEARCH AND FOR CLASSROOM PRACTICE

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### Abstract

Statistical Independence is usually defined as  $\text{pr}(B|A) = \text{pr}(B)$  or  $\text{pr}(A \cap B) = \text{Pr}(A)\text{pr}(B)$ . The topic is not well understood by students, and it is often claimed that this difficulty leads to the well-known "Gambler's Fallacy", which denies the obvious fact that a coin has no memory. This paper will argue that there are in fact two quite different types of statistical independence. These will be defined, and it will be shown that such a classification helps to remove some of the common logical and pedagogical difficulties. The paper will then look at some well-known research results in the light of such a reclassification, and present other data which suggest that there may be more complex influences on people's predictions than have previously been recognised.

### Introduction

Freudenthal (1973) made two statements about probability which seem to have escaped close attention from mathematics educators but which help to clarify the idea of independence and suitable ways to teach it.

[Independence is]...an undefined fundamental concept as are points and lines in geometry... [W]hat matters in probability is almost never one single probability field, but rather interrelatedness of many probability fields ...(pp. 612 - 613).

In this paper we claim that there are in fact two quite different definitions of independence in statistical thinking which are not always well distinguished. We make the distinction clear and re-examine some well-known pieces of research and common classroom practices in its light.

### **The classical definition**

It is usual for two events A and B to be defined as independent if and only if  $\text{pr}(A \cap B) = \text{pr}(A) \times \text{pr}(B)$ . An alternative definition,  $\text{pr}(A|B) = \text{pr}(A)$ , is almost equivalent. (It does not include the case where  $\text{pr}(B) = 0$ ) and is probably pedagogically more satisfying, (Freudenthal, 1973). Feller's (1968) standard text will be used as a basis for indicating the difficulties of the conventional approach, but many other well-regarded texts might equally well have been chosen. He argues in the following way.

Two true dice are thrown. The events "ace with first die" and "even face with second" are independent since the probability of their simultaneous realisation  $\frac{3}{36} = \frac{1}{12}$ , is the product of their probabilities, namely  $\frac{1}{6}$  and  $\frac{1}{2}$  (p. 125).

Earlier he stated that if A is "ace with first die" and B is "even face with second die" then their "simultaneous realisation" is represented symbolically by  $A \cap B$  (Feller, 1968, p. 16).

What is meant by  $A \cap B$ ? Intersection of sets can only be defined on subsets of the same universal set. Feller implies that he agrees with this, but in the case of the two dice mentioned above he does not indicate at all what universal set he is considering. His claim that  $\text{pr}(A \cap B) = \frac{3}{36}$  seems to rest on the argument that because there are six equally likely outcomes from tossing the first die and six from the second, so there will be 36 equally likely outcomes from tossing both dice and three of these constitute the simultaneous realisation of A and B. Is this legitimate?

No! Consider two iron dice which have been magnetised and are being tossed in an environment free from significant magnetic field influence. Each die is fair when thrown individually but each interacts with the other when thrown close together and some pairs of outcomes will be more likely than others. Or consider two fair dice connected by a string (Freudenthal, 1973). The shorter the string the greater the likelihood of interaction between them and the greater the likelihood that some pairs of outcomes will be more likely than others. In both of these cases all 36 points of the possibility space are possible, but not equally likely.

It is the generally unrecognised circularity of Feller's (1968) argument which, in our opinion, has led to much of the confusion associated with the teaching of independence. This issue has attracted the attention of some statisticians, and we shall now summarise ways which have been suggested for overcoming this difficulty.

### **Set-theoretic attempts to resolve the circularity of the definition**

In dealing with compound probability functions Feller (1968) has defined the idea of "cylinder subsets" and "rectangular subsets" (pp. 128 - 132). For

two finite sets,  $S_1, S_2$  we may consider their Cartesian product,  $S_1 \times S_2$ . If  $A_1 \in S_1$ , then the cylinder set  $A$  may be defined as  $A_1 \times S_2$ . Similarly, If  $B_2 \in S_2$ , then the cylinder set  $B$  may be defined as  $B_2 \times S_1$  (see figure 1)

This definition overcomes the objection that  $A$  and  $B$  must be members of the same set for intersection to be defined. But his claim that the experiments are independent if the outcomes satisfy  $\text{pr}(A \cap B) = \text{pr}(A) \times \text{pr}(B)$  leads to an inconsistency. The expression "ace with first die", might refer to  $A_1$ , or it might refer to  $A$ . However, the expression "simultaneous realisation" must refer to  $A \cap B$ . When he refers to a probability of  $1/6$  he must be referring to  $\text{pr}(A_1)$ , not  $\text{pr}(A)$ , because the probability of an ace with the first die has been established without reference to the second die, but  $A$  is defined in terms of the outcome of the second die. So when his argument is converted into standard notation— $\text{pr}(A \cap B) = \text{pr}(A) \times \text{pr}(B)$ —the symbol " $A$ " has two different meanings in the one equation.

A similar approach was taken by Borovcnik, Benz & Kapadia, (1991, p. 49). It is rigorous, but does not lead to the standard definition of independence. Fine (1973) distinguished the two meanings of independence and argued that it is possible to axiomatise independence between experiments and also independence between events so that one may be deduced from the other. Even if he were right, he would be overlooking the fact that in practice, as we shall show, independence of experiments is usually decided *subjectively*, unlike independence of events, which is a deterministic concept. So the rigour would be of little use in practice.

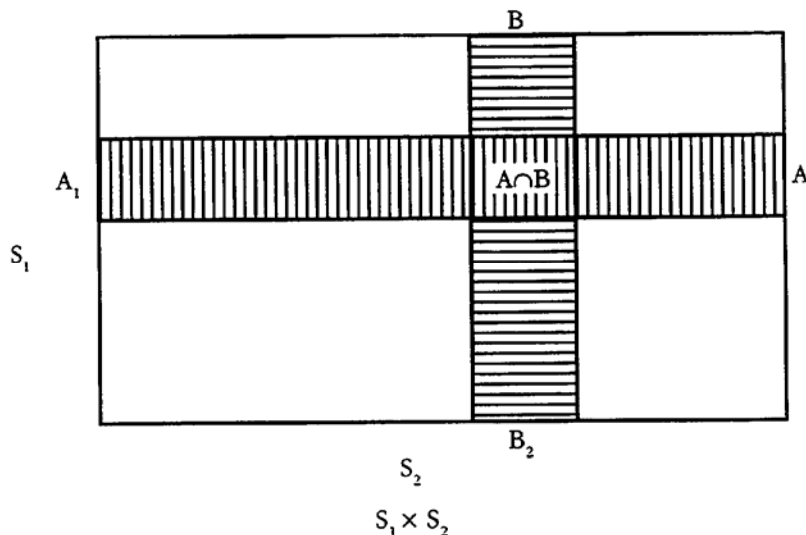


Figure 1 Feller's "cylinder subsets"

### Some other suggested solutions

There have been several approaches to this problem which are summarised briefly here to acknowledge our indebtedness to other thinkers.

The idea of a "collective" (a long sequence of identical observations or experiments which are random and have limiting frequencies) with corresponding "labels" (roughly equivalent to events) was proposed by von Mises (1964) as being a fundamental idea in probabilistic thinking. Collectives may be combined to form new collectives with labels being formed from the Cartesian product of the original labels. Such an approach is based on a stochastic structure whose primary concern is randomness rather than set-theory.

Heitele (1975) proposed that "... the idea of independence is even more fundamental [than that of conditional probability]. It is a fundamental idea to consider chance experiments *with no physical bond*, [italics added] as stochastically independent" (p. 196). Such an approach requires subjective judgement about whether a physical bond really exists. Indeed, Page (1959) stated clearly that a decision that two events are independent is really a subjective one which is analogous to the decisions made by mathematical modellers as to whether their model is good enough. However, Page and Heitele differ on one important point—Page is discussing events, Heitele is discussing experiments.

### Distinguishing events and random trials

We argue that once it is appreciated that "independence" can refer to either events or to random trials, and that it has different meanings in the two contexts, then the logical difficulties outlined above can be overcome and many pedagogic difficulties also disappear.

Maury (1985) distinguished "chronological independence" from "territorial independence", (p. 283, quoting Labin, 1969) but her term "chronological" is unnecessarily restrictive. Borovcnik *et al.* (1991) have argued that what they call "stochastic independence" arises from a natural extension of the concept of independence for events to an analogue for random variables (p. 53). However, their argument retains Feller's circularity, and does not emphasise the subjective nature of stochastic independence.

### The two types of independence defined

We now define the two types of independence and go on to use the definitions as a basis for discussion of some research results.<sup>1</sup>

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<sup>1</sup> A more detailed treatment of the definitions may be found in J. Truran (1992, pp. 41 - 48).

### ***Classical independence***

Classical independence refers to events which are subsets of the possibility space of a specific random generator. Two events A and B are said to be independent if and only if  $\text{pr}(A \cap B) = \text{pr}(A) \times \text{pr}(B)$ . This form makes it clear that independence is symmetric, and the alternative form of this definition,  $\text{pr}(A|B) = \text{pr}(A)$  makes it clear that independence means that the probability of obtaining A from a subset B of the possibility space is the same as the probability of obtaining A from the possibility space itself. The independence of A and B does not imply that A causes B, that B causes A or that some third event C causes either A or B. The common suggestion that "A has no effect on B"<sup>2</sup> as an equivalent definition of independence is at the heart of the confusion between classical independence and trial independence.

### ***Trial independence***

It is possible to express the consequences of trial independence in the form suggested by Feller using the Cartesian product of the outcomes of the two random generators. If two trials X and Y having m and n possible outcomes called  $A_m$  and  $B_n$  are trial independent, then, for all meaningful i and j, X and Y are trial independent if and only if  $\text{pr}(X \rightarrow A_i, Y \rightarrow B_j) = \text{pr}(X \rightarrow A_i) \times \text{pr}(Y \rightarrow B_j)$ . We have already shown that such a definition is incompatible with the definition of classical independence. But this formal definition begs the critical question.

If X and Y are trial independent, then X has no effect on Y, and symmetrically. It is here that cause and effect are important. A decision about whether X and Y are trial independent might be based on an analysis of the probability distribution of their Cartesian product. But it is usually based on examination of the inter-relationship of X and Y and in particular on a subjective decision about whether one has any effect on the other.

So in practice the existence or not of trial independence tends to be decided subjectively. This point is usually overlooked or under-emphasised in standard statistics text-books, whose main aim is rigour. But the rigour is only of value once it has been decided that trial independence exists. This is not a trivial point. Feller overlooked in his analysis of two dice quoted above. Many people decide wrongly, as the pervasiveness of the Gambler's Fallacy in our society demonstrates. Indeed Steinbring has argued that many children are inclined to believe that "ultimately no genuine *independent* [italics as in original] drawings have been carried out". (Steinbring, 1991, p. 513).

While it is certainly possible to make a statistical assessment of the validity of such a judgement, this is outside the skills of most members of society, and, in particular, of students in schools and tertiary institutions.

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2 *Vide* discussion and examples in J. Truran (1992, pp. 125 - 126)

## **Summary of the mathematical claims being made**

In this paper we are making several claims:

1. The classical definition of independence of events cannot be accurately applied to independence of trials.
2. The fact that both of these forms of independence involve multiplication explains why the same word is used for both, but the existence of multiplication gives no guide to the type of independence which is being discussed.
3. Trial independence is in practice usually decided subjectively.
4. Ideas of "cause and effect" are only relevant to trial independence.
5. The distinctions between the two forms are usually not emphasised in textbooks, and this leads to both logical and pedagogical difficulties.

It is trial independence which is concerned with Freudenthal's "interrelatedness of many probability fields" and with Page's concern to find adequate models. Classical independence is concerned with the interrelatedness of probability outcomes—a much simpler and more objective idea. We shall now re-examine some pieces of stochastic research from the point of view of these two types of independence.

## **A reconsideration of some research results**

Little research has been conducted into children's understanding of classical independence. Of course, this topic is usually presented in a formal way, and as such is relevant only to older children. But it might be possible to see if the understanding is possible for younger children. Understandably, most research has focused on outcomes of chance processes. People want maximum success, so they are interested in random outcomes, especially infrequent ones (Goodnow, 1955). But because the research has concentrated on outcomes it has sometimes not been clear that it has been concerned with trial independence, not outcome independence. As a result the interpretations made have sometimes lacked sharpness, as may be seen from the examples discussed below.

### ***Prediction of outcomes***

Research projects asking subjects about the outcome of a random generator tend to take two forms. One requires subjects to make some form of prediction of the next outcome of a random generator. Logically, such questions are meaningless. It has been argued, however, that the subject's responses when the structure of the random generator is unknown do indicate that they have

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3 This is the phenomenon of "probability matching behaviour" described in Fischbein (1975, Ch. 4).

some intuition of the nature of the generator.<sup>3</sup> The second form asks whether one outcome is more or less likely than another outcome (Green, 1983). We do not question the validity of these responses within the constraints defined by the experiments.

However, there is substantial evidence that children do not see random generators as having constant properties and that their replies vary according to the environment in which the generator is operated (J. Truran, 1992, pp. 54 - 56, 1994; K. Truran, 1994). For example, when 300 primary school children (K. Truran, 1995; in press) were asked about the behaviour of random generators, about half focussed less on outcomes than on what "motivated" the behaviour of the generator. They considered that a random generator has "a mind of its own", or may be controlled by outside forces. These forces might be spiritual:<sup>4</sup>

I                    Is there anything you can do to get the number you want to come up?

KA (M, 12: 4) Pray.

I                    Do you think that works?

KA                  Sometimes.

Or the forces may be physical:

I                    Is there any way that you can make a coin land on the face you want?

PG (M, 8: 11) I remember where the numbers are before I throw it.

I                    How does that help you make it land on the face you want?

PG                  It comes on the number opposite the one you started on.

I                    Does that always work?

PG                  Yes.

Such views can be very resilient:

I                    Why didn't it work?

JP (F, 8: 8)        Because I'm probably holding it wrong. You're meant to hold it on number twelve and number six.

I                    Is that between your thumb and finger?

JP                  Thumb and middle finger. Your middle finger is meant to be on twelve and your thumb on six. [Throws Die.]

I                    Did that work?

JP                  Well, no, it takes practice and practice and practice; it takes pretty much time to get it on the number you want.

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<sup>4</sup> In the transcripts from interviews presented here, I stands for Interviewer, the letter string is a code for the subject whose gender and age in years and months is given in parentheses.

An alternative approach is to ascribe everything to luck, and not to see the statistical regularity of chance events:

- I                    Is there anything that you can do to get the number you want to come up?  
 BC (F, 9: 6)      No, it's just luck.  
 I                    How much luck?  
 BC                  [shows with her fingers] About half an inch.

Many other significant features have been reported—the way a generator is operated, its size, its physical configuration. Furthermore, decisions about stochastic processes tend to change when there is some prospect of reward (Cohen, 1964; Fischbein, 1975, pp. 37 - 41; Peard, 1995; Teigen, 1983; K. Truran, 1994).

If children's appreciations of a single random generator are subject to so many variables, it is not surprising that their understanding of trial independence is also likely to be influenced by a number of non-mathematical factors. This needs to be acknowledged in any programme designed to teach probability, but it is rarely made. Indeed, until trial independence is seen as a form of independence in its own right, any such acknowledgement is unlikely.

#### ***Random generators operating simultaneously***

Fischbein and his collaborators have shown that some children do not regard the simultaneous operation of random generators (e.g., tossing three dice together) as mathematically equivalent to their consecutive operation (e.g. tossing three dice one after the other) (Fischbein, Nello & Marino, 1991). This result clearly shows that children do not see trial independence as constant. Fischbein found that children believed that they had more control over successive tosses; in our experiments children aged 8 - 12+ years were more likely to prefer successive tosses because "If you throw the dice together they rub against each other, and the numbers change." (K. Truran, 1994). The two types of response are similar, but different.

Such strong beliefs in the physical behaviour of dice frequently over-ride any understanding of randomness or independence. Not only do random generators have no constancy of existence in the mind of many children, but when they are operated together, they are seen to lose any independence they might have had.

#### ***Assessment of a sequence of runs***

Finally, subjects have been presented with sequences of runs from a random generator, and asked which was the most likely. Both children and adults tend to chose a sequence which appears to be more "representative". (Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993). Thus, from six tosses of a coin, "HTTHTH" is seen as more likely than "HHHHTH". Given that most

naïve students do not have the combinatorial skills to assess these probabilities, such a response is not surprising. It is related to the "gambler's fallacy" which argues that outcomes will in the long run balance each other out and that therefore a long run of Heads is highly likely to be followed by a Tail. Outcomes are determined to some extent by previous outcomes, not solely by the independence of the random generator.

We would argue that before drawing such a conclusion it is also necessary to understand what subjects think about the mechanism of the random generator and whether this understanding is taken into account when they make their decisions. Traditional responses have been described as arising from an "outcome approach to uncertainty" (Konold, 1989, p. 61). While we do not dispute that the validity of the findings, or their interpretation, we suggest that the findings of such "outcome oriented" research need to be balanced by "trial oriented" research undertaken with the same subjects. We submit that an understanding of subjects' knowledge of the nature of trial independence is a relevant aspect of research in this field which has largely been overlooked.

### **Implications for classroom practice**

Steinbring (1991) has posed a critical question for researchers concerned with the influence of their work on classroom practice.

How can the epistemological structure of mathematical knowledge and the structure of teaching-learning processes be made compatible in such a way that knowledge is not simply reduced to mere methodological conventions (p.518).

He answers his own question by stating that it is necessary "to embody a proper knowledge-epistemology in the social interaction" and that theoretical knowledge is especially valuable in a subject with a high need for subjective decisions (Steinbring, 1991, pp. 518 - 519). Our own close examination of the mathematics of independence is in harmony with Steinbring's views, and we now present some examples from current classroom practice in order to illustrate the point more precisely.

### ***The use of trees***

Trees are a common diagrammatic aid in classroom teaching. The usual format emphasises the events being discussed. The probabilities corresponding to each branch are multiplied, usually with some statement that the events are independent.

Consider drawing two balls without replacement from an urn containing 3 Red and 2 Blue balls. This, in our country, is commonly illustrated in the following way (refer figure 2).

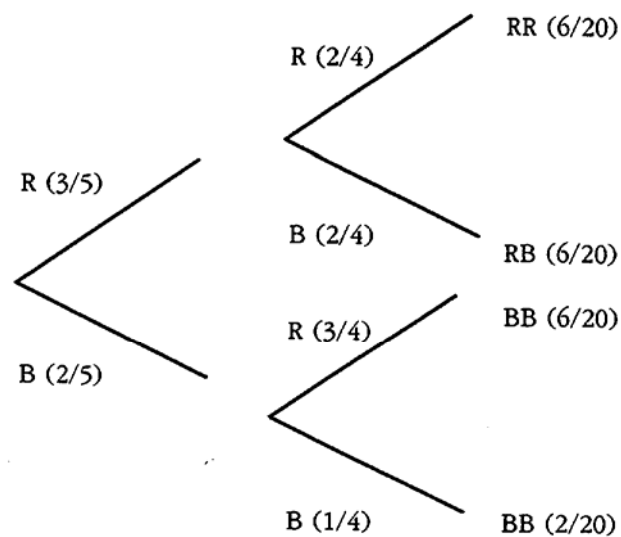


Figure 2. Commonly used form of tree diagram

At first sight there seem to be three values for  $\text{pr}(R)$ . Of course, two are conditional probabilities. But if it is conditional probabilities which are being multiplied, how can we be using the classical definition of independence to justify the multiplication? What we have done is to make a *subjective* decision

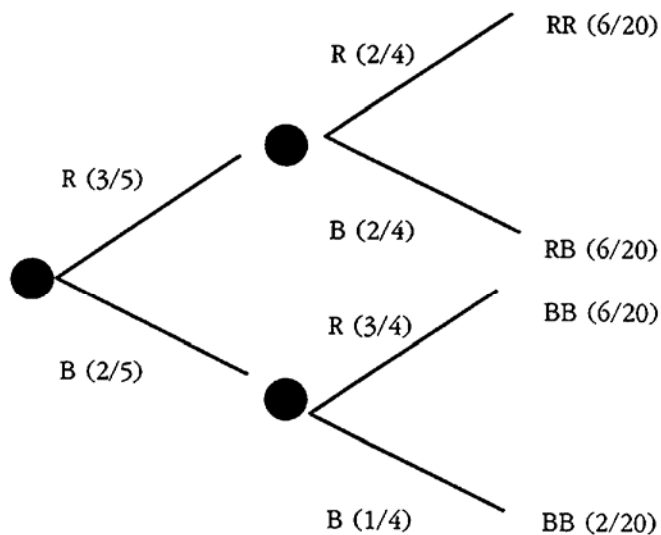


Figure 3. Proposed form of tree diagram which emphasizes logical structure

that the random generators are *trial independent* and to multiply on the basis of this assumption.

The traditional tree omits the most significant aspect of its logical structure. One of us has proposed an alternative form of a tree to overcome this deficiency, while conceding that it does not overcome the problem of conditional probabilities ( J. Truran, 1989).

The solid circle provides a model for the process of operating the random generator. Once this model has been incorporated into the structure it is easy to emphasise the need to decide, on subjective grounds, whether the random generators are trial independent or not.

### ***Questions about classical independence***

In Australian textbooks, questions on classical independence often take this form:

A card is randomly selected from a pack of 52.

A is the event of getting an Ace.

B is the event of getting a Spade.

C is the event of getting the Ace of Hearts.

Which of the following are independent?

(a) A and B

(b) A and C

(c) B and C

(Adapted from Haese, Harris, Haese, Webber & Danielsen, 1984, p. 138)

Sometimes the probabilities are given, and sometimes more support is given to assist the student to calculate the probabilities, perhaps by providing or suggesting a Venn Diagram.

It is clear that this situation involves only one random generator, and requires the use of the formula for classical independence. The question is mathematically valid. But, in our experience, it is not usual for teachers to emphasise that this situation is an analysis of outcomes and not of random generators. Indeed the opening line places emphasis on the random generator. Nor is it usual to emphasise that this analysis is totally deterministic, requiring no subjective decisions at all. We submit that once the two types of independence are accepted as different, then the wording of questions will need some modifications which will help students to appreciate the distinction. One simple such modification would be to rewrite the first sentence in the form 'Some of the possible outcomes of drawing a card at random from a pack of 52 cards are as listed below.'

### ***Questions about concurrent operation of random Generators***

One question in a recent Australian probability textbook presents a photograph of three dice and asks "What total is most likely to occur from rolling three dice?" (New South Wales. Board of Studies, 1993, pp. 52 - 53).

This question is accompanied by significant notes for the teacher, there is no hint that a significant number of children might consider consecutive tosses to be different from concurrent ones. If mathematics education is really to become a science as well as an art, then such issues should be automatically included in any pedagogic support materials. We might produce many other such examples where text-book questions mask a host of well-known misconceptions by students.

### **The dilemma**

The major purpose of this paper has been to argue that there are two distinct, but usually unrecognised forms of independence, and that many pedagogic difficulties disappear when this distinction is acknowledged. It is further argued that much research in this field has concentrated mainly on outcomes and has placed insufficient emphasis on children's understanding of random generators.

But there remains a real dilemma. Trial independence is usually arrived at subjectively. There is a received wisdom among mathematicians which is that certain situations are most reasonably seen as trial independent. Many children do not share this received wisdom, yet it is one which most mathematicians believe it is worth communicating with them. There are two ways in which children might be given experiences of trial independence which might in due course lead to their accepting the received wisdom. One involves an examination of outcomes from random trials. This is the traditional approach, and it is extraordinarily difficult to do in a simple way. The other is to examine the generating mechanisms. This is at first sight a much simpler way, but it is rarely practised. Frequently issues of the symmetry of the random generator interfere with the issue of the independence of its operation.

If Freudenthal (1973) is right in saying that independence is a fundamental concept in stochastic thinking, it is essential that we in mathematics education present a stochastic knowledge-epistemology which distinguishes the two types of independence in order that the subjective form is seen to be quite different from the well defined classical outcomes form.

May we conclude with a challenge. We suspect that for some "independence" is synonymous with one or more of "fairness", "symmetry" or "mutually exclusive". A formal examination of this hypothesis and of its pedagogic implications could be another fruitful field of research.

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