Co-constructing Statistical Knowledge

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1. Introduction

In the literature means, modes and medians are referred as measures of central tendency and they are important concepts in data handling and analysis. Some authors (Batanero et al., 1994; Carvalho, 1996, 1998; Cudmore, 1996; Hawkins, Jolliffe and Glickman, 1991) also stress that students have difficulty with these basic concepts and to some of them these concepts can be reduced to a computation formula. The main goal of this study was to analyse peer interactions in order to understand their role in pupils' performances when they were solving statistical tasks. A deep analysis of their discourse makes clear the way they construct an intersubjectivity (Wertsch, 1991) that facilitates the choice of the solving strategies and helps pupils to undertake their mistakes.

2. Theoretical background

All those who work in a school context are aware of the importance of social interactions in knowledge apprehension and in skills acquisition. In the 70s, the first introductory studies by Doise, Mugny and Perret-Clermont (1975, 1976) lightened up the importance of peer and group interactions in cognitive development. Using piagetian tasks they verified that children working in peers or small groups progressed more than those who worked individually.

Later on, the importance of contexctualized studies became a reality and notions as the zone of proximal development (ZPD) and the need of a more competent peer (Vygotksy, 1962, 1978) were explored by many authors who studied social interactions (Moll, 1990; Van der Veer and Valsiner, 1991). Anyhow, the most important step in pedagogical terms was to recognise that children were able to implement their sociocognitive development and their school achievement in Maths not only when they were interacting with a more competent peer, but also when they were interacting with a peer that was as competent as themselves or even less competent (CÈsar, 1998, 1999; CÈsar e Torres, 1997). This means that peer interactions are very powerful and that they can be an effective way of

promoting pupils' school success.

When children learn scientific concepts at school they already have spontaneous ones (Vygotsky, 1962, 1978) and for some time this may cause cognitive conflict. Nunes (1995) stressed the importance of the nature of the task and tools we provide children with, in the way they construct and organise their knowledge and acquire skills. The concepts they appropriate are mediated by their social interactions and by the tools they use.

3. Method

3.1. Subjects

We used pupils from the 7th grade attending a secondary school in Lisbon (n = 214). The case we choose is representative of many of those dyads.

3.2. Instrument

The task we used was adapted from Abrantes (1997). Looking to the table data about salaries pupils should be able to discuss how to calculate the mean, the median and to choose by which statistical parameter this distribution was better represented.

Task

In a factory, five of the workers were randomly chosen to do a study about the salaries. The salaries were:

Worker	A	B	C	D	E
Salaries (escudos/month)	54 000	42 000	60 000	48 000	180 000

1.1 - Do you think that the five workers agree if we say that the great majority of them have a salary equal to the mean? Why?
1.2 - If you are the worker D how does you fill? Why?
1.3 - If instead of the mean we choose the median what you think was the worker's opinion. Why?
1.4 - Do you choose the mean or you choose the median to represent the salaries of the workers. Why?
2- The mean of four numbers is 25, three of those numbers are 15, 25, 50... What is the missing number?

3.3. Procedure

Due to technical difficulties we decided to audio tape these peer interactions in a small room at school. Children came in peers at an extra-curricular time

previously scheduled. We asked peers to discuss their solving strategies, and to write them down only when both of them agreed that they had found a solution for that problem. Each peer only had a sheet of paper for their answers. We also asked them to write everything they thought and not only the solution itself. We asked them to avoid erasing and to cut the things they wanted to change.

4. Results

4.1. Co-constructing statistical knowledge: The case of Hugo (H) and Elsa (E)

H - We must find the mean.
E - We must start adding the numbers.
H - 54 thousand plus, plus what?
E - I must work it out. (Hands the calculator over to H). Plus 42 thousand, plus 60 thousand, plus 48 thousand, plus 180 thousand. Add it all up.
H - (Shows E the result on the calculator). Now, it must be divided.
E - By five.
H - We divide the result by five. (Shows the calculator to E).
E - 384 thousand divided by five equals seven hundred and sixty eight. (Shows the operation).
H - Now we can introduce the mean equal to this (Shows the calculator).
E - Yes. What do you think? Do you think it is all right?
H - No, I don't.
E - Don't you? Why not?
H - Is this the amount they receive? (Points to the mean amount).
E - It is not, it's the mean.
H - Oh well, then I think it is.
E - Do you? Why?
H - Because they earn the mean, I think. I am not sure.
E - But they do not earn the mean. They earn less.
H - I think that is it.
E - Then it is not, because they earn a salary below the mean (Writes down). If you were employee D, how would you feel about your salary?
H - I would feel bad.
E - Why?
H - Because it would not be enough.
E - Because he earned less than the mean (Writes down). If, instead of saying that the majority of employees in that company earn a salary equal to the mean, we say that the majority of employees in that company earn a salary equal to the median, what do you think would be the employees' opinion, and why?
H - They say they are underpaid.

E - Ah!
H - The employees would say they were underpaid and that they have children to feed.
E - Now it is your turn, why?
H - I think the salary is not enough because they have children to feed and a rent to pay.
E - Is their salary equal to the mean or to the median? What do you think would be the employees' opinion?
H - They would say they did not earn enough.
E - They would say the salary was not enough, for what?
H - To keep a home.
E - To pay the rent. (Writes down) Would you choose the mean or the median?
H - The median.
E - But what is the median?
H - I don't know.
E - Well. We have to put this right and then it will be the middle one... 54 thousand.
H - I would choose the mean because it has...
E - Mean because it is low. Not higher.
H - It has 20 thousand in excess.
E - Because the mean has a higher value. The mean of four numbers is 25, three of those numbers are 15, 25, 50... What is the missing number?
H - 30.
E - 30? Wait.
H - 15 + 25 is 40.
E - This adds up (Values given) to 90.
H - Add it all up and then divide by 25.
E - 3.6. I am not sure this is right.
H - I am not sure either. Do the other one.

4.2. Discussion of the Case

Hugo (H) and Elsa (E) are 7th grade students. Hugo is classified by his teacher as a poor student, Elsa is classed as an average student who could get better marks if she worked more. At the pre-test, they both obtained a low performance (level 1), but they improve with the post-test for maximum marks (level 3).

On the cognitive development test (ECDL), Elsa went as far as was expected for her age - Intermediate level. Hugo, like all unsuccessful students (did not succeed at 3rd and 7th grade, at first) reveals a poorer performance on the first

ECDL application - concrete level. At the end of the school year, Hugo was at the Intermediate level.
Elsa thinks mathematics are difficult and that it requires a lot of studying but that sometimes you need not study so much as, for example, with statistics, because it is like what you see on TV and read in the papers... Graphs and means... when the teacher brought these up I didn't know how to deal with them, but had heard about them.

Hugo feels that to like mathematics is to like the Maths teacher... When I like the teacher, I think it is fine... This year I like it, but usually I don't. Hugo begins by saying that one must calculate the mean and Helen gives a definition of an arithmetic mean. This is a co-elaborated answer in which each element of the pair adds something and completes the other's answer. None of the elements leads the interaction, this being very balanced. The reason may be that both elements have similar mathematical performances at pre-test.

When the subjects solve the algorithm of the mean, Elsa, in a non-directive pedagogical attitude, requests Hugo's opinion without revealing right away what her opinion is, waits until Hugo thinks it over, giving him some cues, helping him to reformulate and work out a richer answer from a mathematical and psychological point of view than if he had just given his answer. The former situation might have induced Hugo to accept his colleague's answer without elaborating significantly on his own, just imitating the answer. Hugo's uncertainty as to the validity of his own answers makes him feel insecure. This reveals some leadership on the part of Elsa, as Hugo is not so sure about his answer.

For the next question, requesting the subjects to compare a salary with the total sum of salaries, we note that their answer is mediated by Hugo's social representations concerning living cost level. The solution found is based on a comparison not with the total salaries, but with obtaining that salary in the subjects' day-to-day life. We note that Hugo's resolution is influenced by the spontaneous concepts he has, which is quite usual in pupils that are still in the concrete level of cognitive development.
Elsa, although being able to calculate a median, lets herself be influenced by Hugo. In fact, the choice between the mean and the median was due to the quantitative and concrete aspect of the average value and not to any mathematical criterion as, for example, which of the two values best expresses the reality of those salaries. This is to say that the social influence present all along this task may have induced the subjects to give an incorrect answer.

For the next question, both subjects answer on a trial and error basis, with Elsa

trying to find a solution based on arithmetical strategy and Hugo inducing Elsa in error. She is unable to see she is doing a mistake and is left in doubt as to whether this would be the best solution. Hugo's leadership at this time is quite clear. Elsa who, at the beginning of the resolution, requested time to think, cannot stop Hugo who, although in doubt about the validity of the answer, knows from their didactic contract (in the Maths class) that, once a solution is discovered, one can go on to the next question and, even when faced with Elsa's doubt, does not try to listen to her and, eventually, work out a more consensual and positive solution. Elsa, faced with Hugo's leadership, is unable to impose her need for more time to solve the situation.

3. Final remarks

The way pupils solved this task is a quite good example of how they understand the situations that they are confronted to during their school activities. Although they were doing Maths many of them did not face this task as a typical mathematical task and they used much everyday common sense knowledge in their answers.

The most common way to solve this task was to compare the mean value with all workers' salaries and the median value with the mean. They often did not discover any mistakes until this is pointed out to them by their peer. That happens when they have to discuss between the two of them until they agree on a solution. These results made us believe that the study of protocols, child dialogues, tasks and instructions presented to pupils are essential to the comprehension of the way they acquire their knowledge and skills.

Peer interactions were a powerful mean of implementing new solving strategies and a better performance among most peers. The analysis of some of their protocols shows how the interaction that is taking place makes children clarify their way of reasoning, find out new strategies, respect the other ones rhythm of work, be better prepared to assume their own points of view or how that interaction takes a positive role in the construction of a more positive self-esteem.

Above all, peer interaction was an effective way of apprehending knowledge and acquiring new skills not only for those who had more difficulties in Maths but also for the ones who were more competent. This was even stronger than in previous studies (CÈsar, 1994) as we implemented a final discussion with all peers. This fact is one of the most interesting aspects of this study, because it lights up the power of peer interactions as a common practice in the classroom.

References

Abrantes, P. (1997). *Projecto Mat 789*, Lisboa: A.P.M.

Batanero, C. et al. (1994). Errors and difficulties in understanding elementary statistical concepts, *Internacional Journal of Educational Science and Techonology*, *25*(4), 527-547.

Carvalho, C. (1996). Algumas questões em torno de tarefas estatísticas com alunos do 7º ano. *Actas do Profmat 1996* (pp.165-171). Lisboa: APM.

Carvalho, C. (1998). *Tarefas estatísticas e estratégias de resposta*. *Actas do VI Encontro de Educação Matemática* (pp.127-134) Portalegre: SPCE-SEM.

César, M. (1998). Social Interactions and Mathematics Learning. *MEAS 1 Proceeding* (pp.110-119). Nottingham: Nottingham University.

César, M. & Torres, M. (1997). Pupils' Interactions in Maths Class. *CIEAEM49 Proceedings* (pp.76-85). Setúbal: ESE de Setúbal.

Doise, W., Mugny, G. & Perret-Clermont, A.-N. (1975). Social interaction and development of cognitive operations, *European Journal of Social Psychology, 5*(3), 367-383.

Doise, W., Mugny, G. & Perret-Clermont, A.-N. (1976). Social interaction and cognitive development: further evidence, *European Journal of Social Psychology, 6*(2), 245-247.

Hawkins, A., Jolliffe, F. & Glickman, L. (1991). *Teaching statistical concepts*. London: Longman.

Moll, L. C. (1990). *Vygotsky and Education*. Cambridge UK: Cambridge University Press.

Nunes, T. (1995). Sistema de signos e aprendizagem conceptual. *Quadrante*, 4, 1, 7-24.van der Veer , R. & Valsiner, J. (1991). *Understanding Vygotsky: A quest for synthesis*. Oxford: Blackwell.

Vygotsky, L. S. (1962). *Thought and Language*. Cambridge MA: MIT Press.

Vygotsky, L. S. (1978). *Mind and Society: The development of higher psychological processes*. Cambridge MA: Harvard University Press.

Wertsch, J. V. (1991). *Voices of Mind. A sociocultural approach to mediated action.* Hemel Hempstead: Havester Wheatsheaf.

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