# A CURRICULUM OF PROBABILITY AND STATISTIC IN ITALIAN SECONDARY SCHOOL (11 - 14)

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In Italy school is compulsory for all its citizens from 6 to 14 years old. In the field of education teaching programs are decided by a central authority (The Ministry for Public Education) and apply to all over the country.

In 1977 two laws were introduced to bring about substantial modification to the only existing school and, at the same time, the planning of all subjects matter. These latter having to rise to the needs of the basic education of every single citizen besides providing him with those subjects already taken to be basic to the continuation of his studies. In that occasion we have for the first time the introduction of statistics and probability using the following methods:

*Themes. 3. The certain events and probable events mathematics.*

*a. . . . . (Omission)*

*b. statistical surveys and their graphic representation (histograms, aerograms . . . ), frequencies, averages.*

*c. Random occurrences; notion of probability and its application.*

*Guide to Reading of Contents.*

*The introduction of elements of descriptive statistics and the notion of probability provides a fundamental instrument for the development of a mathematical awareness of considerable interdisciplinary value. Notion of probability arise not just as a natural conclusion to the arguments on statistics but also from simple experimentation of a random nature. The teacher in attempting to avoid a formal definition of probability will take care to put his/her students on their guard against the most common misunderstandings regarding both the interpretation of mathematical data as well as the adoption of probability in the forecasting of events. However they must not go beyond the calculation of probability in very simple situation connected to concrete problems (like genetics, economy, games).*

However it must be said that following an introduction of these limited subjects from the point of view of programs no real introduction into the classroom was really ever put into effect. The reasons for this are manifold. Teachers to whom the task of teaching statistics and probability is assigned, are in effect these who also have to teach mathematics, the natural sciences, physics and chemistry. In Italy there are no degree courses of a widely-varying subject matter . . . , for which reason teachers come from

degree courses of a much more specific nature such as mathematics, physics or chemistry.

The legislator in face of such difficulties has sought to enforce the obligation of including such arguments more strongly in linking one of the three final questions in the exams of mathematics in the school to the subject matter of statistics and probability. The result being that in more cases than one, the theme has been treated in a forceful and distinctly formal manner. The teacher who desires to fulfill the real teaching role will principally summon help from text books. The situation however is getting no better, publishers have in general simply added to those chapter already written with a certain logic that did not foresee objectives related to the subject. In certain other cases (more than can be mentioned) the treatment of these arguments gets a rather confused and erroneous handling in the book.

The teacher therefore refers to university research centres responsible for teaching methodology which have over recent years elaborated and experimented on the proposals put forward on the teaching of such subject matter.

The limitations of such proposals are in their concrete application.

Thus we have tried to present the problem and bring the solution into prospect which, if clear and correct at the level of subject matter and didactics, would also in addition possess the characteristics for concrete realisation and hence generalisation for the greatest possible number of situations and teachers. Experimentation has also been carried out on the proposed curriculum in order to test not just certain aspects of content but also methodology and those aspects related to "time-linked resources".

## Probability

A. Educational and didactical objectives.

Every human action is undertaken and takes place, whoever relevant the modalities in an environment where a state of uncertainty is present.

The probability, understood as pertaining to a specific moment, that is, the mathematization of uncertainty for that moment, constitute a fundamental form of knowledge for each somewhat cultured citizen. A citizen who has constantly move within an environment in which uncertainty has the role of a presence which can't be eliminated, needs to go to a school which educates him in the necessity of understanding and dealing a bit rationally with uncertain phenomena.

B. Disciplinary Objectives

The principal disciplinary objectives connected to the teaching of notions of probability can be: i) The student can distinguish between certain, impossible, and uncertain situations and knows how to evaluate events. ii) He can qualitively evaluate the uncertainty of events using available notions and information. iii) He can evaluate in concrete terms

the uncertainty in straightforward situations. iv) He has the capacity to use a few basic tools for calculating probability.

## Statistics

### A. Educational and Didactic objectives

The study, measurement, analysis, representation, management and utilisation of variability are primarily statistical tasks. Statistics, however, is eminently the formal discipline for the study of the problems connected to the variability of data. The educational and didactic objectives of statistics therefore are principally connected to the capacity to perceive from a basic point of view, on the part of the student, the variability of the data involved together with the ability to find one's bearings with the information with which they are presented in concise form (they therefore have immediately some kind of synthesis which could have made them lose some important informative elements). Objectives can be summarised in a kind of literacy of statistics. To such objectives must be added, again from a basic point of view on the part of the student, other considerations such as some form of numerical ability for an ability in the process to make some elaborations on the data as well as simple calculations and other simple representations.

### B. Disciplinary Objectives

The principal disciplinary objectives connected to the teaching of statistics may be seen as: i) A capacity to plan small cognitive investigations (also of a sampling nature) with some attention given also to the problem of informativeness and an understanding of the variables. ii) The capability to devise basic direct recordings of data and to trace information,also in indirect forms/through institutional sources: bulletin published by private and public bodies, archives, etc. iii) Capability to express the data essential information in simplified form. iv) Capability to synthesize the data information with basic indicators. v) Capability to provide a basic utilisation of the indicators and representations. vi) Capacity to perceive the interrelationship between phenomena and an understanding of the dependency between variables. vii) A critical capacity to the competence and correctness of the information and the statistical details reported from sources in books, newspapers, television, etc.

## Methods

The problematic starting points are pre-eminently taken from real life. Nevertheless, and above all on account of the grand complexities of the problems presented to us in real life, the teacher must seek to introduce them in a simplified manner in order to adapt them to the maturity and understanding of the pupils. What we are talking about is favouring a form which seeks to stimulate the pupils' interest and curiosity: a type of "problem posing" rather one of "problem solving".

Not being able to describe all of the curriculum we will attempt to report on several exemplary and paradigmatic aspects which often are omitted in the text books.

A. To obtain the first primary disciplinary objective, Probability B, i), we propose an activity on reflections "in the language" and "in the substance" based on the fact that events related to our lives can be either certain, impossible or uncertain, is proposed. This quite clear although trivial affirmation, has been shown to be evident in the analysis of concrete situations on the part of pupils of eleven years.

   Thus a card for completion was presented to the pupils headed by the words; "certain, uncertain, impossible', in the following way:"it is . . . that today is Monday, "it is . . . 25th of December is a holiday", "it is . . . that I will go up at the end of the year", and so on.

   In the discussions had in the classroom we became aware of pupils on certain difficulties concerning the right compilation due to two facts: a) The subject's understanding of uncertainty and thus the sentence; b) the involvement of personal desire in respect to the objectivity of the facts (nearly all the pupils were "certain"that their own favorite teams would have won the next game). In the analysis of incomplete sentences of the type proposed above the teacher also took care to take note (even if in a more informal way and related to the age of the pupils) the causes of uncertainty as listed in Probability A i), iii), iii), iv).

B. To attain objective outlined in Probability B, ii), we went on to the compilation of a card of a more "qualitative" type. The analysis of the preceding uncertainty had in fact made us come up against a range of grades for which the word "uncertain" no longer seemed entirely satisfactory in order to complete the sentences.

   Thus, the following sentences relative to situations already defined as "uncertain" were proposed but this time with the words "very, a little, fairly, nearly", as extra qualifications.

   "It's . . . that tomorrow it will rain," "it's . . . uncertain that team X will win the match on Sunday," "it's . . . uncertain that tomorrow I'll come to school", etc.

   The completion of these sentences was done individually and consequently discussed in classroom. The teacher had concentrated her attention particularly on the reasoned use of the motivations which caused them to use one adjective rather than another.

C. To attain the objective stated in Probability B, iii) we have appealed to the concept pursued also through the teaching of mathematics in general, that "quantification" is a particularly significant moment in the learning process of real mathematics in so far as it allows us to obtain results otherwise unobtainable, but which at the same time requires a simplification of reality (see Calvino).

The selected model to go on to the next stage of quantification has been that of an urn containing balls all to the same size and form and differing in colour.

A series of transparent glass jars were brought into the classroom, and it was noted to the pupils how the glass allowed one to see how many balls there were belonging to each colour, inside the jars.

**Figure 1**

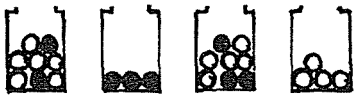


**Figure 2**

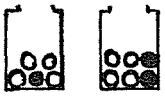


**Figure 3**

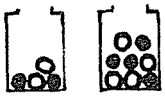
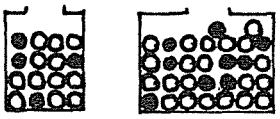


**Figure 4**



A situation as in figure 1 was proposed to the pupils who were then asked "which jar is it better for me to choose if, pulling out a black ball, with my eyes closed, wins me a prize?". The pupils had absolutely no problem, without any first trial, in saying that it was better to choose jar number 2, that in fact winning was a certainty. Here still we have not quantified the uncertainty but made observations on model which presents us with certainty, impossibility and uncertainty. Then it was asked "if you win a prize pulling out a white ball from one of these urns (fig. 2) which would you choose? Why?" The answers were: "There is no difference", "it's better to choose jar number 1". The mistake of the first answers was due to the fact of not having considered "all the balls" but only the number of white ones. Thus the question was repeated leaving the urn untouched and increasing the number of balls in urn number 2, to 15. At this point nobody had still answered "there is no difference because the white balls remained at 5".

For those who had not understood before, the conviction arises that the most fitting (major certainty of checking on an event) depended both on the number of white balls (a favourable outcome) and on the number of all the balls which do not bring you a win. Thus, the connection between the favourable cases and all the possible (not just those opposing) cases was even less clear.

A situation of type 3 was presented posing the question "in which of the 2 turns is it more appropriate to dip into, if I win a prize for getting a white ball?" Many pupils chose urn number 2 because it contained more white balls "even if there are even more black ones". The brighter pupils (having only recently encountered equivalent fractions) managed to guess the indifference of the situation observing the similarity of the relationship be-

tween favourable and unfavourable cases and between favourable and possible cases. In order to make this perception of the facts more mathematical, a situation like in figure 4 was presented that is 12 white and 4 black balls in the first urn and 20 white and 10 black in the second. Lots of students chose the second urn motivated by the fact that it contains not just more black but white balls as well, others tried to make use of their visual faculty the first seemed at a glance more reasonable. From an analysis of the preceding case the more intuitive – rational pupils managed to come up with the explanation that having the possibility to choose from among all the balls in the first case, I have 16 possibilities, 30 in the second and between these there are 12 white in the first case, 20 in the second. Putting these numbers into relationship, we obtain a proportion of 12 to 16 and 20 to 30. Which of these 2 numbers is greater than the other? At this stage the students managed to arrive at formula for measuring classic probability making deductions about the model "without having actually carried out any experiment". To go on to the measurement of associated probability this time jars (were brought to school), whose glass had been coloured, not see which or how many balls were contained inside. After having asked them "where it was more worth- while to dip your hand in order to extract a white ball", the pupils replied that "in this case' they could not give any casual answer. The need therefore arose to be able to use the formula which had just been discovered in order to know the number of all the balls as well as those involved in the game on hand. On the other hand it became clear that many real life situations are similar to those of the opaque jars. Six jars with the same composition (noted by the teacher) were brought along and experiments were carried out and recorded. When numbers began to exceed one hundred per group it was seen that the ratio between the number of white and all the balls began to be almost constant and thus an approximation of the measurement of probability had been reached, assuming as its value that of the relative observed frequency of an event.

In order to reach the disciplinary objectives relative to the subject matter of statistics, proposals already under experimentation for a long time were propounded. Activities which could be drawn from text books or instructive magazines were among those proposed to the student. However, we think it particularly worthy to mention at this point some considerations which guarded us in the formation and administration of such proposals. The situations chosen as models have always been pedagogically involving and characterised by a certain simplicity. Problems carefully chosen by the teacher and based on those principles outlined above were taken as the starting point rather than an examination of the findings, diagrams and statistics compiled by others. This work reached its conclusion after a three year period by which time the students had already arrived, with some confidence at their disciplinary objectives Statistics, B, vi), vii). It seemed to us of particular importance to spend time over the initial subject matter. In fact many text books quietly brush over one and the other without any clarification. The activity started with the findings of a qualitative series and attention was focused on the fact that in this case there was no compulsory rule for recording, but at the most one of convenience.

An effort was then made to itemize all the information that a table of frequency could provide and find out which concise data could be extracted

there. At this point, conscious of the existence of many text books on the practices of teaching, a discussion was opened up on the limitations of the "information" from such findings. After a series of qualitative findings and their analyses we went on to findings on numerical data. The data were ordered attempting to establish the diversity in respect to previous cases and in particular the chance to take note of that which the pupils normally acknowledge as "average" that is the arithmetic mean, was taken into account. In order to reach the disciplinary objectives outlined in Statistics, ii the same style was adopted paying attention to avoiding any confusion and basing the analyses on the available or collectable information.

Objectives Statistics, B, iv and v were reached contextually through the previous activities. The objectives in Statistics, B, vii were attained through qualitative consideration and after through a representation of points on the plan (two variables) and an observation of the cloud of the same.

The objective vii has always been one of relevant pursuit up to the point of creating a unifying link with other disciplines. The total activity took thirty-three hours over three years.