EDA IN THE PRIMARY CLASSROOM – GRAPHING AND CONCEPT FORMATION COMBINED

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1. Introduction

It is important to develop children's mental images of numbers parallel to their acquisition of counting and calculating skills. This is achieved by various kinds of activities which reveal, in one way or the other, the underlying structure. In school mathematics numbers are closely connected with calculations, which are, in my opinion, too much emphasized. Analyzing how numbers appear in day-to-day life shows that calculations constitute a minor portion, while the major portion is made up of comparisons, tables, orders of magnitude, rounding, estimating, arrival and departure times, prices, dates, postal codes, and similar "numerical messages". One could express this by saying that we use more descriptive statistics than arithmetic. This should be reflected in our children's studies. Of course I am not advocating new fancy names such as "descriptive statistics" in the primary syllabus. What I mean is that the emphasis should be shifted towards handling of numbers without calculations. The introduction of stem-and-leaf displays is a step towards that end. Being based on the fundamental idea of place value stem-and-leaf displays also help developing the children's understanding of number. In this presentation I assume the reader to be familiar with the simple ideas behind the stem-and-leaf display, this ingenious mixture of table and diagram, introduced by John W. Tukey in the early 70's. An excellent introduction is found in Landwehr/ Watkins (1984), see also the "classic", Tukey (1977) and NCTM (1981).

2. Background and aims

My duties as a teacher of future class teachers for the primary school also include some teaching in schools, on the average one lesson per week. During my visits to a class the regular teacher is present. Sometimes I visit the class once a week during, say, half a term, sometimes I am with the class all the mathematics lessons for a week or two. The organization is flexible. I am free to propose topics and methods to a class teacher. We discuss, modify, adapt, and carry out in co-operation. Bringing stem-and-leaf displays down to the primary level is one of the things I have tried in several classes of Grade 1 at the end of the school year. The results are encouraging and the ideas seem to be worthwhile developing and pursuing. The present study consists of such a classroom example. It should be mentioned that children start school at the age of 7 in Sweden. The only evaluation of the lessons is my own personal impression. So far I have neither quantitative nor qualitative results, such as tests and interviews, to report. The topic is new and only in the stage of gaining ground at the

primary level. Not very much is explored and tried so far. However, teachers and researchers are beginning to look in this direction, see NCTM (1981), Biehler (1982). For the secondary school level textbooks on EDA are appearing, see Landwehr/Watkins (1984), Vännman/Dunkels (1984). For the primary and middle school level I know of one book only that treats any part of EDA, viz. Johansson et al (1984), where stem-and-leaf displays are to be found.

3. The first EDA lesson

As a preparation for this lesson each child was instructed to find out the ages of his or her father and mother. We started the lesson by voting on whose age to begin with. In most classes the majority voted for mother. Each child was given a rectangular piece of cardboard of size 8 cm by 12 cm, orange in colour. The rectangle was then folded in half parallel to the shorter side. After having unfolded again each child was to write his or her mother's age using large digits (see Fig 1) one in each half of the rectangle. The cardboard piece was then cut along the midline fold. The children then started playing around with the digits without my telling them what to do. They put the cards on their desks with the digits in reverse order. During that activity I could feel that the data made the children emotionally involved. Since there was a meaning attached to the cardboard figures the reversed number was also given the interpretation "the age of my mother". Unreasonable ages, like 92, appeared, causing laughter, thus creating a positive atmosphere and an excellent basis for discussing tens and units digits and the importance of knowing which is which. I asked questions like, "Reversing the digits makes John's mother 14 years old, how old is she in actual fact?" The role of 0 could be illustrated, "Adam's mother is 4 years old with the digits reversed. How old is she?" Next each child was asked to hold the tens digit for me to see. There turned out to be 2:s, 3:s, and 4:s. I collected one copy of each and put them on the floor with a paper strip to the right, see Fig 2. We then gathered on the floor, each child bringing the units digit of his or her mother's age. "Now each one of you", I said, is supposed to lay down your units digit on the floor in the appropriate place so that we all can read your mother's age. You begin, Hanna. I did not give more detailed instructions, the idea was to give each child a chance to discover the details. Of course there was hesitation now and then, occasionally a digit was first put in the wrong place, but the children helped each other, and on the whole the number of mistakes was

Figure 1

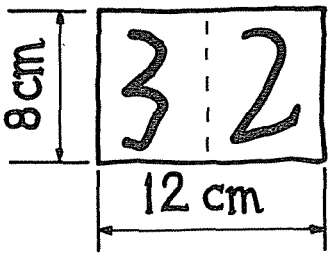


Figure 2

surprisingly low. Hanna's mother was 32, and Hanna put her cardboard piece quite correctly to the right of the strip next to the 3. I then asked the class how old Hanna's mother was, and obtained the correct answer. The next child to put down his units digit was Johan, whose mother was 34. After some hesitation he put his 4 next to Hanna's 2 as in Fig 3. We now read all together aloud in chorus the ages that were so far exhibited.

Figure 3

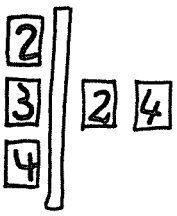
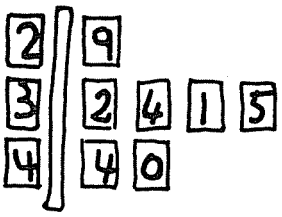
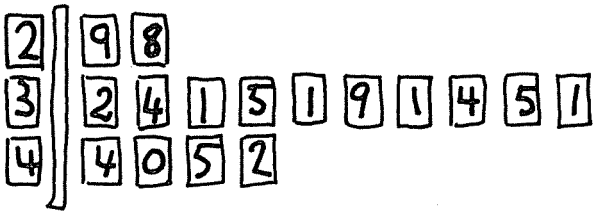


Figure 4



"Thirty-two, thirty-four", while I pointed at the appropriate digit. When we had reached the stage shown in Fig 3, by the way, someone said that the age was three hundred and twenty-four, and again there was an opportunity to discuss place value. As each units digit was put down we read through what we had got so far. I pointed, and we read aloud all together. For example, when seven children were ready the situation was as in Fig 4. And we read, "Twenty-nine, thirty-two, thirty-four, thirty-one, thirty-five, forty-four, forty." One of the children happened to read 40 as "forty- zero", and everybody laughed. I said that I thought reading "zero" was actually a good idea, more logical and consistent than saying "forty" alone. We discussed the meaning of 40, and again an interesting discussion on place value emerged. Later I encouraged "forty-zero" in our chorus reading, and I am sure that this helped several of the children to understand e.g. the difference between 43 and 403 better. When all children had put down their units digits the floor looked like in Fig 5.

Figure 5

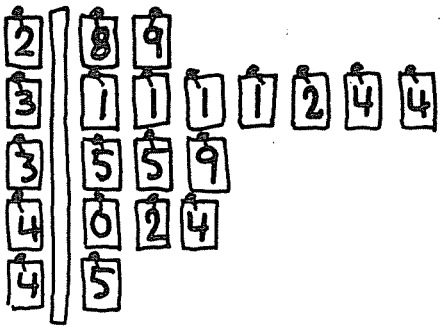


When we had read through the whole lot together the children returned to their seats in the classroom. I asked if they were contented with the diagram, and one girl said that it would have been better if the 40-row, for example, had had 0 as its first units digit. So we agreed that ordering the units digits within each row would be a good thing to do. We all found the

30-row too long and I suggested splitting it into two 30-rows. I collected new tens digits from the class, a 2, two 3:s, and a 4, and pinned them onto the notice board. See Fig 6. I also pinned a cardboard strip immediately to the right of these tens digits. We started with the 20-row. One

Figure 6

Figure 7



pupil at a time would now take the units digit from the floor he or she thought to be the next one in order and pin it in its proper place on the notice board. We all agreed that the first five units digits, i.e. 0, 1, 2, 3, 4, would be used in the first 30-row, and 5, 6, 7, 8, 9 in the second. The same would of course apply for all rows, once you split a particular tens row then you have to do it all the way. In our case we thus had to split our original 40-row as well. The result is given in Fig 7. Before proceeding we read through the new, ordered diagram in chorus. Now that we had an ordered diagram we could see all sorts of things. "How old is the youngest mother?" – "The oldest?" – "What is the most popular age? (What does 'popular' mean?)" – "Are there any ages between the lowest and the highest that are missing?" – I also wanted to emphasize what the objects of our study were, and so I asked, "How many mothers do we have on the notice board?" My point was not really the number as such. I wanted to focus on the fact that we had not pinned the actual mothers but their ages onto the notice board. The answer I received made this even more clear than I had expected, "Fifteen", said Nils. "Fifteen?" I exclaimed with honest surprise, looking at the sixteen units digit cards in our stem-and-leaf display of Fig 7. I was about to start counting in order to explain, when Nils showed that he had in fact understood the matter better than I had. He went on, "Yes, because two of the 31:s are Andreas's and Daniel's mother." Andreas and Daniel are twins. This detail shows how data belonging to the life of the children, data that is alive and can stir the children's imagination and emotions, assists in making matters easy to grasp and understand. Next the children wrote the same diagram as we had on the notice board in their notebooks. That terminated the first EDA lesson. Before my next visit the class carried out the same investigation of the fathers' ages, this time with green cardboard. So when I appeared the next time the notice board was beautifully decorated with the two stem-and-leaf displays of the parents' ages.

4. A heart-beat lesson

The object of this lesson was to use the stem-and-leaf display in a pulse rate experiment. Again we would deal with personal data. This lesson was also to serve as an introduction to the study of the human body as well as to the study of other animals. (What is the pulse rate of a hamster, a horse, an elephant?) Anyone who has been in a class of 7-8-year-olds knows that measuring the pulse rate is extremely difficult. However, the activity is great fun and really worth trying. It is hard for children of this age to feel the pulse with their fingertips on the wrist. Therefore we used 500-gram, plastic yoghurt tins with a hole in the bottom. The children worked in pairs. One child pressed the open side of the jar against the chest of the other child and put his or her ear to the hole. This simple device is surprisingly effective. After some experimenting and practicing we decided to try to count the number of heartbeats in 30 seconds. The result was to be recorded in a stem-and-leaf display on the blackboard. Fig 8 shows the result after the units digits have been ordered. Without first offering any explanations or methods of approach I then asked the children to show the number of heartbeats in a minute by constructing a stem-and-leaf display emanating from the 30-second display of ours. It should be noted that the children had not yet learnt anything about addition with carrying. My idea was that the calculation would be done mentally

Figure 8
Number of Heartbeats in Half a Minute

3	9
4	0469
5	92246
6	0

Figure 9
Number of Heartbeats in One Minute

7	8
8	08
9	28
10	0448
11	2
12	0

and that each child would get a chance of discovering the idea of carry digits. After the lesson the class teacher said that she was surprised that the children managed this task so well. She admitted that she had been worried when she heard the task, which I in fact had not planned in advance, and she had not expected the children to manage on their own. Only one child needed substantial assistance, for the rest normal classroom encouragement was sufficient. This exercise gave the idea of using the stem-and-leaf displays of the parents' ages from before in order to practice arithmetic. One can ask questions like, "What will the stem-and-leaf display of your mothers' ages be like after five years from now?" – "100 years from now?" – ""What was it like the year you were born?" – "What will it be like in 1990?" – "How old will the youngest mother be when the oldest is 50?" – In the heartbeat case one could count for 20 seconds or 15 seconds, produce a stem-and-leaf display and then construct one giving the heartbeats in one minute. There is a lot of variation to this, and gradually the children should be encouraged to formulate their own questions.

5. Closing remarks

The simplest of the EDA-methods are gradually gaining ground at school level. Books and journal articles appear treating these methods, and the importance of real data is repeatedly manifested at all levels. Working with real data, however, means that one must be prepared to improvise and to encounter unexpected difficulties. In one of the Grade 1 classes where I worked with the ages of parents one child did not know the ages and had, it seemed, forgotten to ask her parents. However, it turned out that the parents did not want to reveal their ages. "Our ages are none of the school's business", the mother later explained to the class teacher, "and that fellow from the university can make up some figures for the children's exercises. That is just as good." I strongly disagree. With invented data the emotional and intellectual involvement can never reach the heights attainable with real data from the learner's own environment and experience. I shall close by quoting the girl who came to me after the lesson where we had pinned the mothers' ages onto the notice board. She looked at the stem-and-leaf display and said, "I would like to have my mother back now."

6. References

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