CHILDREN'S UNDERSTANDING OF RANDOMNESS: REPORT OF A SURVEY OF 1600 CHILDREN AGED 7-11 YEARS

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Sample Details and Test Procedure

This paper reports on just some aspects of research carried out in early Summer 1986 using 1600 Primary school children in mixed ability classes in state schools in a small Leicestershire town. The subjects, aged between 7:9 and 11:9 were given two untimed class tests which together took from 40 to 55 minutes to administer. All questions were read out to the subjects. The first test was concerned with concepts of randomness, the second with comparison of odds.

Four test versions were prepared (A,B,C,D). Some questions were set out differently in the different versions, to reduce the likelihood or effect of copying and to investigate the influence of mode of presentation. A subjective teacher assessment of each pupil's level of general reasoning ability was obtained (A-E). The sample details are given in Tables 1 and 2. The small differences between Versions in Table 1 are not statistically significant.

Table 1: Sample by test version and level of ability

	Version				
Ability	A	B	C	D	Total
A	60	51	60	64	235
B	99	104	102	97	402
C	142	142	135	136	555
D	66	72	73	72	283
E	33	31	30	31	125
	400	400	400	400	1600

Table 2: Sample by sex and age

Year	Age	Mean age	Boy	Girl	Total
1	7-8	8.25	203	180	383
2	8-9	9.22	215	191	406
3	9-10	10.24	229	176	405
4	10-11	11.24	204	202	406
			851	749	1600

An analysis of the distribution of teacher assessed abilities showed the expected phenomenon of fewer low ability girls (Grade E:79 boys, 46 girls).

Piaget and Raindrops

In "The Origin of the Idea of Chance in Children" Piaget and Inhelder wrote:

> *Nothing is more common . . . than the form of distribution which drops of rain give when falling fortuitously at the beginning of a small shower . . . Will the subjects not have, in this phenomenon familiar to all of them, a special chance of understanding intuitively the law of large numbers?* (pp.49-50).

The author reported at ICOTS I on research with 11-16 year olds. One question related to the distribution of snowflakes landing on a sectioned roof, following the line of Piaget's work. Those results are repeated here in Table 3.

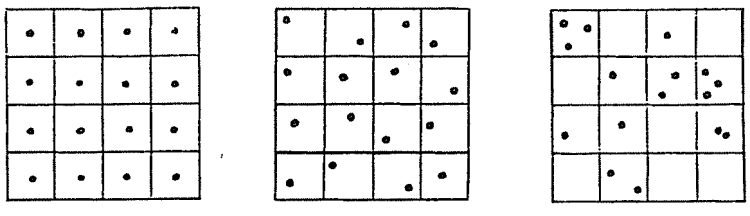
Table 3: Percentage responses for snowflake item

Ability	Version A regular	B semi-random	C random	B & C	All same
11-12	11	12	26	23	27
12-13	13	12	26	20	29
13-14	13	14	24	25	24
14-15	13	16	16	21	33
15-16	12	15	18	21	34

A puzzling feature of that item was no systematic variation with age or ability. In order to investigate this further similar items have been devised. The first is Qn. 2, shown in Fig.1 with responses in Table 4. Included are results from a Secondary school whose Third Year pupils (N=225) were given the test.

Fig. 1 Random Test Question 2: Raindrops

2. The flat roof of a garden shed has 16 square sections.
It begins to rain a little. After a while 16 raindrops have landed on the roof. Here are three pictures showing raindrops on the roof:



Which picture best shows the pattern you expect to see ?

A ☐ B ☐ C ☐ None is best ☐ Don't know ☐

Table 4: Percentage responses for raindrops item by year

Year	Age	Regular	Semi-random	Random	None best	Other
1	7-8	23	34	31	6	7
2	8-9	16	35	38	8	3
3	9-10	10	40	39	6	5
4	10-11	13	39	37	6	5
Sec	13-14	6	37	41	11	5

It would appear that although there is some decline in those preferring the regular symmetrical pattern there is no appreciable increase in preference for the random (or even semi-random) patterns. Only the youngest children (7-8) have a noticeably lower selection rate. A breakdown of responses by ability is shown in Table 5.

Table 5: Percentage responses for raindrops item by ability

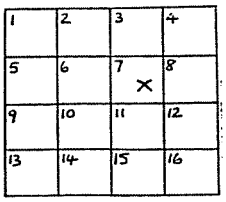
Ability	Regular	Semi-random	Random	None best	Other
A	8	40	42	9	2
B	12	41	38	6	3
C	15	36	37	7	5
D	17	36	33	6	8
E	39	25	21	5	10

Whereas the variation between the sexes is not significant that between the ability levels is highly significant ($p<0.0001$). It will be noted in particular that the least able have a much greater tendency to select the regular pattern suggesting a weak appreciation of random variation. However the equal balance between the random and semi-random response rates for all ability levels is a strong feature. Whether or not the foregoing provides evidence for a lack of a concept of randomness is a matter for debate. It depends critically upon the validity of the underlying model. When snowflakes or raindrops fall do they cluster or is there a physical property which causes them to be more evenly spread? It is clear that Piaget and Inhelder considered there to be no doubt about this! Nor about its self-evident nature! In order to make more explicit the supposed stochastic model an introductory item was set as shown in Fig. 2.

Figure 2 Random Test Question 4: Selecting counters

4. Paul plays a game using 16 counters numbered 1,2,3,4....16.
Paul puts all the counters in a tin.
He shakes the tin a lot.
Rachel shuts her eyes and picks out a counter. It is number 7.

Paul puts a cross in box 7.
The 7 is put back in the tin
and someone else picks a counter.



After Qn.4 had been read out an experiment was conducted, with the children putting crosses in the boxes to record which counters were picked. This was continued until six crosses had been entered. If no duplication had occurred by then it was made to happen without the children being aware of the intervention. Three questions then explored the children's perceptions of what might happen if the experiment were continued for 12,16 and 30 selections. A later item (Qn.9) then returned to this idea. Version A is shown in Fig.3. Versions A and C were identical, versions B and D had the same pictures but in a different order. Results are presented in Table 6.

Figure 3 Random Test Question 9: Random patterns

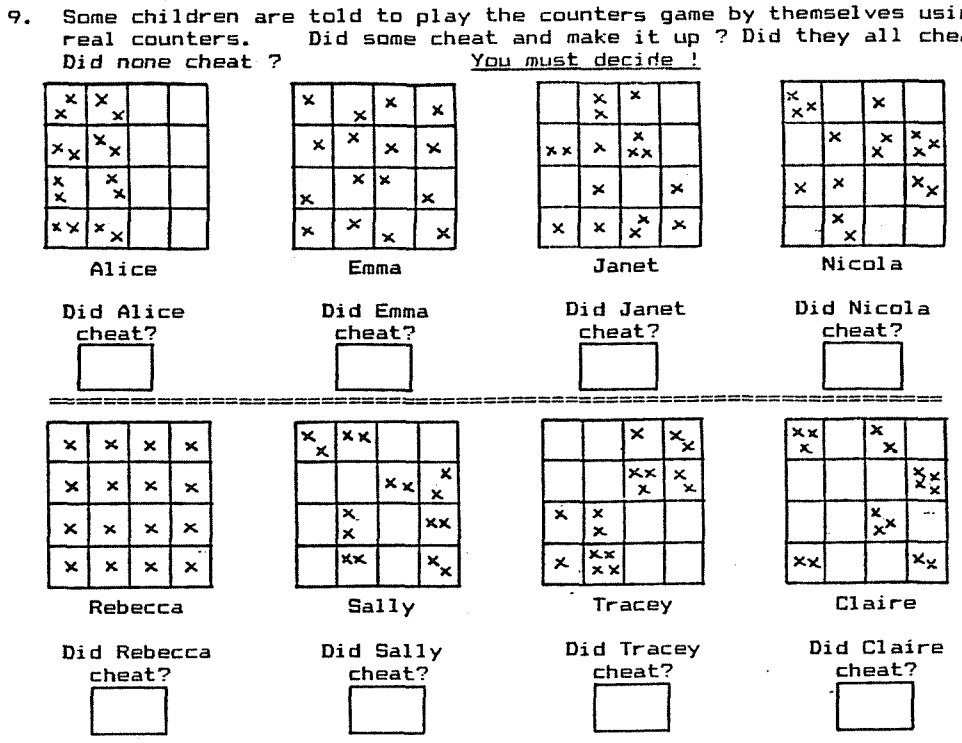


Table 6: Percentage facilities for random patterns by year

Name:	Alice	Emma	Janet	Nicola	Rebecca	Sally	Tracey	Claire
Rt. ans:	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Year								
1	67	45	70	66	70	40	53	47
2	74	52	79	67	82	40	49	48
3	84	67	86	76	90	38	53	42
4	84	67	88	81	90	43	50	48
Sec	84	74	95	90	95	40	50	41

Age and general ability are highly significant factors for the first five items. Sex is significant for none and version only for "Alice".

The three items "Rebecca", "Emma" and "Nicola" have patterns corresponding exactly with those in the raindrops item. It is evident that most subjects reject as unnatural the extremely regular pattern in both raindrops and counter selection contexts. The lower facility for "Emma" shows that the picture itself misleads about 25% of subjects, for the distributions of "Rebecca" and "Emma" are logically identical. An encouragingly high proportion of children recognise that "Nicola" has an acceptable pattern but why this should be less than for "Janet" is a mystery! The first five items are straightforward and amenable to mature logic and this is reflected in the fact the facilities rise significantly with both age and ability. The last three items are much more demanding and this leads to lower facilities with no improvement with age or ability for the 7-11 year old.

Conclusion

The high facilities for the random and regular patterns indicate that young children do have a sound conceptual awareness of randomness. The lower facility for the semi-random pattern shows that children's lack of understanding of a problem may produce responses which mislead the investigator. The results of the earlier research with 11-16 year olds and that now reported here with 7-11 year olds together point to the crucial importance of the context in which such problems are put and call into question the validity of Piaget and Inhelder's findings.

References

Piaget, J., & Inhelder, B. (1975). The origin of the idea of chance in children. Routledge & Kegan Paul.

Green, D.R. (1982). A survey of probability concepts in 3000 pupils aged 11-16 years. In Proceedings of ICOTS I (pp. 766-783). University of Sheffield.

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