Students' Concepts of Bar Graphs - Some Preliminary Findings

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1. Introduction

Today it is recognised that every adult person should be able to effectively process the large amounts of information encountered every day. Such information is frequently presented in graphical form with, for example, businesses, government, and the news media, all utilising graphs to display information. The value of the use of graphs in displaying information has been described by Weintraub (1967):

> "They present concepts in a concise manner or give at a glance information which would require a great deal of descriptive writing. They often distil a wealth of information into a small amount of space." (p.345)

Furthermore, the ease by which graphs can now be produced by computers has led to their increased use by society. This increased use means that schools should stress the development of competence in effectively utilising and analysing graphs. According to Kirk, Eggen and Kauchak (cited in Curcio, 1987) the maximum potential of a graph is actualised when the reader is capable of interpreting and generalising from the data presented.

Educational authorities have recognised that for students to effectively utilise graphs it is not sufficient for them to just be able to directly read information from a graph. The National Council of Teachers of Mathematics in *An Agenda for Action* (NCTM, 1980) called for an increased emphasis on drawing inferences and predicting from data. This increased emphasis is reflected in the recently released NCTM Standards document (1989) where graphing is included as a key component of the probability and statistics standard.

To date there has been little research on graphing, per se. As Kosslyn and Pinker (1983) stated:

> "Even a casual perusal of the literature immediately convinces one that there is a real need for research on charts and graphs, and that there is a real need for a systematic approach to the topic. Research on charts and graphs is, in a word, scanty." (p.6)

The limited research available indicates that particular features of the content and presentation of graphs can contribute to students' difficulty in understanding the information presented in a graph.

At present there is a need for research to determine specific student errors in graphing and the factors which contribute to these errors. Such information would be valuable to teachers in planning instruction and in helping students to overcome their difficulties.

2. Purpose

This paper will report some of the findings of a study of grade four and grade six students' understanding of the information conveyed by bar graphs. The total study examined the effects of various characteristics of graphical displays on students' ability to read, interpret, and predict from such displays, and discusses the results within Davis's Frame Theory (1984).

3. Theoretical model

Davis's (1984) model uses the concept of a frame - a knowledge representation structure that is stored in memory - to describe how people process information. In terms of the model, processing of information from a source starts with the selection of a cue from the information which results in selection of a frame from memory. Data is then mapped to the variables in the frame, hence the general frame information and the information from the source are brought together. This "instantiated" frame is then used as a database for decisions.

By examining the students' responses to a variety of graphical displays it is possible to create a description of a graphical frame. Errors can then be described and explained in terms of inadequacies of the frame. For example, students make errors when faced with variable scales within the context of different graphical problems. By examining students' responses on questions, on reading, interpretation, and prediction from graphs with different scale factors, it is possible to discuss whether the errors are the result of an incomplete frame which is incorrectly completed, an incorrect default evaluation, or a complete but incorrect frame. Such information, particularly when it results from the examination of performance at different grades, helps formulate a picture of the development of frames and can provide a partial basis on which to build appropriate activities to correct the errors. This "picture" can also provide a basis for the design of materials to be used to teach graphical concepts.

4. Brief comment on the literature

Many of the early studies on graphing were not designed to identify general difficulties in understanding graphs but rather to determine the relative effectiveness of different methods of representing quantitative data (Washburne, 1927; Peterson and Schramm, 1954; Feliciano, Powers and Kearl, 1963). More recent studies have measured the level of understanding of the information presented in a graph. Wainer (1980) administered a test (line chart, bar chart, pie chart) to third through fifth graders; Curcio (1981) administered a test (pictograph, bar graph, circle graph, line graph) to fourth and seventh graders; and McKenzie, Padilla and Shaw, Jr (1986) used the Test of Graphing in Science to investigate the line graphing ability of students in grades seven through twelve. These studies all showed that graphing ability increased with grade level, and that while students had few difficulties with literal reading of graphs, they were often unsuccessful in answering questions requiring higher level cognitive skills. Curcio and Smith-Burke (1982) employed task-based interviews to obtain information on the cognitive processes underlying student difficulties and employed a schemata-based model to explain why errors occurred. They found that using, or, more accurately, misusing prior knowledge, resulted in errors. They also found that students were persistent in their errors.

The only category of graphs that appear to have been studied extensively is graphs of physical phenomena (Kerslake, 1977; Bell and Janvier, 1981; Barclay, 1985; Clement et al., 1985; Mokros and Tinker, 1987). Two major categories of errors have been noted in both school and college populations; namely, confusing slope with height, and confusing the graph of an event with a "picture" of the event.

5. Sample and instrument

The population for this study consisted of students at the grades four and six levels. The sample for the study consisted of 121 students in grade four and 127 students in grade six. The interview component involved 35 students in grade four and 37 students in grade six.

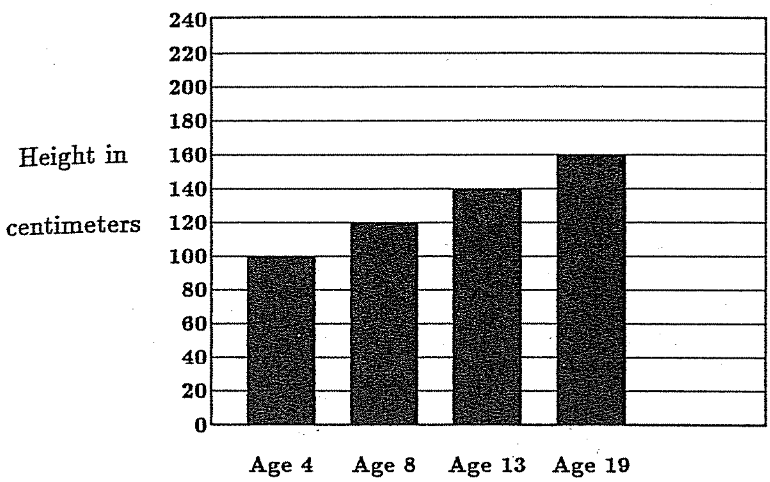
There were a total of 12 different graphs, covering real/familiar topics such as height of children, invented topics such as number of Widgets manufactured in City A, and real/unfamiliar topics such as the number of trees planted in which the trees were unfamiliar types. In addition to the different topics, the data was organised so that both patterned and non-patterned formats were presented. All graphs contained three questions; a literal question, an interpretation question, and a predict question. Figure 1 contains an example.

6. Procedure

Students at all schools were aware in advance that they would be participating in a mathematics research study. On meeting the students the researcher informed them of the purpose of the study and that it involved a written component and, for some students, a short interview at a later date.

HEIGHT OF THE SMITH CHILDREN

The following graph shows the height of four of the Smith children, ages 4, 8, 13, and 19.



1. How tall is the 4 year old? ____________
2. How much shorter is the 4 year old than the 19 year old? ____________
3. A fifth child in the family is 10 years old. Can you tell how tall the 10 year old is? Circle YES or NO.
   If you circled YES, draw the bar for the 10 year old on the graph and explain your answer. ______________________________
   ____
   If you circled NO, explain why you circled NO. ______________
   ____

FIGURE 1
Sample question

The written test was administered by the researcher. Once they had received the papers, students were told to examine each graph carefully and to answer as many questions as they could. They were advised to move on to the next question or graph if they had prolonged difficulty with any part of the test.

After the tests had been completed an initial perusal of the written responses was made and students were selected for interview. The selection was made so that a variety of different responses were chosen to obtain as complete a spectrum of results as possible. The interviews were audiotaped, did not normally exceed fifteen minutes, and were conducted within three school days from the date of the test.

At the start of the interview students were told by the researcher that they would be taken through the test and asked to explain their answers. The students' written test were then placed in front of them. If the students appeared frustrated with a question or engaged in long periods of silence they were given the option of proceeding to the next question. In some cases students were asked additional questions to those on the paper. These questions generally pertained to the reasonableness of the answer given or graphing in general.

Initially the type and nature of the errors were categorised. After this initial categorisation a frame analysis was undertaken and, finally, the research questions were analysed within Davis's frame theory.

7. Results

There were very few problems with the literal reading of graphs; the mean success rates for grade four students was 95% and for grade six students was 98%.

There were more problems with the interpretation questions; the mean success rates for grade four students was 52% and for grade six students was 78%. Most of the errors in this category can be explained by computation errors, or reading/language errors, or scale errors. In fact, the mean percentage of errors that could not be explained by these reasons was 8% for grade four and 4% for grade six, and most of the 8% and 4% were either no response or an unexplained error.

In the following discussion the examples are interpret questions.

Computation or reading/language based errors were made by students when answering literal (very few), interpret, and predict questions. Two views of these errors are that they can (1) be attributed to deficiencies in the general frames, per se, from which information is drawn to instantiate the graph frame, or (2) from errors made in mapping information from these general frames to the graph frame. For example, when students were asked how much more allowance Jane received in 1988 than in 1986, students gave answers which included either the allowance for 1988 or 1986, both allowances, or the sum of the allowances. From the written test and the interviews it was concluded that the graph frame contained incorrect information on either the reading/language or computation required to answer the question.

In another example, when students were asked for the total number of trees planted, they gave the largest number planted. In the interviews, students consistently pointed to the bar representing the greatest frequency and several used words such as most and biggest when explaining their answer. It was concluded that the graph frame contained incorrect information on the meaning of total.

Overall, it appears that while students have slots within their graphical frames for the retrieval of information from more general reading/language or computation frames, processing errors surface. Due to both the nature of study and the internalised character of a frame, it is not possible to determine whether these errors are due to problems in the generalised reading/language or computation frames, or are associated with mapping this information into the graph frame. Both possibilities explain the errors discussed under this heading.

The major problems occurred in the predict question where the level of performance was extremely low; the mean success rates for grade four students was 16% and for grade six students was 18%. If the graphs in which reasonable prediction were possible were eliminated, these percentages drop to 5 and 7 respectively. Once computational errors and errors associated with reading/language, per se, were eliminated, the analysis indicated there are four general categories of *graph-based errors*; namely, topic, scale, data arrangement (pattern), or the fact that information was not shown on the graph. Due to the length of the paper for these Proceedings, the authors will concentrate on just two categories of errors: (1) data arrangement, and (2) the fact that the information was not shown on the graph (although data arrangement overlaps with topic).

7.1 *Arrangement of data*

The effect of the visual arrangement of data on students' ability to predict from a graph was built into the study. Three types of arrangements were used: patterned data in order of magnitude; non-patterned data in order of magnitude; and non-patterned data not ordered in magnitude. The different arrangements of the data resulted in many errors in prediction. For example, when given a graph involving height the students predicted, based on the pattern in the graph, that the height of a ten-year-old child was more than that of a nineteen-year-old child. Some students even persisted with this interpretation by implying that, although this was not reasonable, it had to be true from the pattern. One student, in an attempt to protect the flawed frame, suggested the possibility of the ten-year-old being on stilts rather than change the answer.

For graphs with non-patterned data the need for a pattern was so strong for some students that this group sought and found a "pattern". This occurred even in cases where the data was not ordered in magnitude or *when any attempt to search for a pattern made no conceptual sense*. For example, when faced with non-patterned data associated with the manufacture of widgets in cities A, B, C and D, students forced a pattern for city E. Similar problems occurred when the information was real (such as allowance), but the situation did not permit prediction. During the interview, students often hesitated when describing these "patterns" but then continued on rather than admit something was incorrect. This can also be viewed as an attempt to protect the flawed frame.

Another error in non-patterned graphs where any attempt to search for a pattern made no conceptual sense, was for students to cite the absence of a pattern as the reason for their inability to predict. For example, when shown a non-patterned arrangement of a variety of different trees planted, students stated that they could not predict the number of elm trees planted because there was no pattern in the data. During the interviews students said that a prediction would have been possible if the data had been patterned.

7.2 *Information not on the graph*

Students had another common difficulty in that they said they could not give an answer since "it" was not on the graph. For example, when asked to predict a child's allowance for 1990 (the allowance for 1986-89 was given) students said that they could not predict since "1990 was not on the graph".

Overall, it appears that, while students have a pattern slot within their graphical frames, it is either incorrectly developed or not correctly linked with other components of the frame. Students tended to think that patterns *must* exist in a graph. This was true even in situations where the nature of the topic would suggest that searching for a pattern was an unreasonable activity. The search for a pattern could be thought of as the default evaluation. While it is possible that this problem has been caused by their graphical experiences, it is a major fault in the students' frames. The failure to predict because the information is not on the graph is an example of another fault in the frame. Apparently students envisaged the graph in some "complete" manner and when asked to predict "beyond the picture" were unable to do so.

8. Conclusion

In conclusion, this paper has tried to direct attention to some severe problems associated with students' graphical frames. While reading/language and computation errors account for some errors, other graph-based errors occur. In particular, a slot associated with pattern is either incomplete or incorrectly formed (either internally or in its linking mechanism to other slots such as topic, per se), causing significant problems when students were trying to answer predict questions.

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