Statistics and Mechanics

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1. Introduction

Statistics and mechanics have traditionally been viewed as alternative options, with school pupils rarely studying both these mathematical applications in post 16 courses. Indeed, in many schools (in the UK) mechanics and statistics are taught by different teachers who may be unwilling or unable to teach both subjects.

The Mechanics in Action project has been working in schools with pupils doing practical work and subsequently amassing considerable quantities of experimental data. Results for some experiments showed considerable variation. Could this variation be due to experimental error, or were some samples really different to others? The main problem was, how could that question be answered without resorting to sophisticated statistical tests? A simple method of data analysis was needed; one which could be understood by 14-year-olds. Most GCE students can handle the terms "range" and "median", and with the introduction of quartiles, boxplots seemed the ideal solution.

2. Investigation 1 - the pendulum

One of the investigations was concerned with pendulums and several ideas were to be pursued - the length of the string, the mass on the end, the angle of release of the mass, and also it was hoped to discover if the period for an oscillation changed as time progressed.

Different groups were asked to investigate different factors. All sorts of questions can arise about the data. How many times should the experiment be repeated? How should one represent the results? If the mean time for 10 swings at 10^{o} is 16.6s, and for 10 swings at 20^{o} it is 14.5s, are these the "same" (within the limits of experimental error) or not?

We encouraged the students to collect 9 or 10 values for each item of data to enable the quartiles to be easily identified and not too near the "ends" of the results, thus allowing boxplots to be drawn.

Here are the results obtained when a mass of 50g suspended from a 50cm string is swung at three different angles. We recorded the time for 10 swings and repeated this procedure 10 times.

Angle = 10°
14.57, 14.74, 14.46, 14.57, 14.56, 14.58, 14.31, 14.35, 14.44, 14.47

Angle = 15°
14.47, 14.57, 14.64, 14.62, 14.43, 14.40, 14.51, 14.35, 14.60, 14.45

Angle = 45°
14.98, 15.06, 14.89, 15.28, 15.11, 15.18, 15.19, 15.07, 15.04, 14.96

What we wish to know is: Are these three sets of results basically similar (with slight differences caused by errors of measurement)?

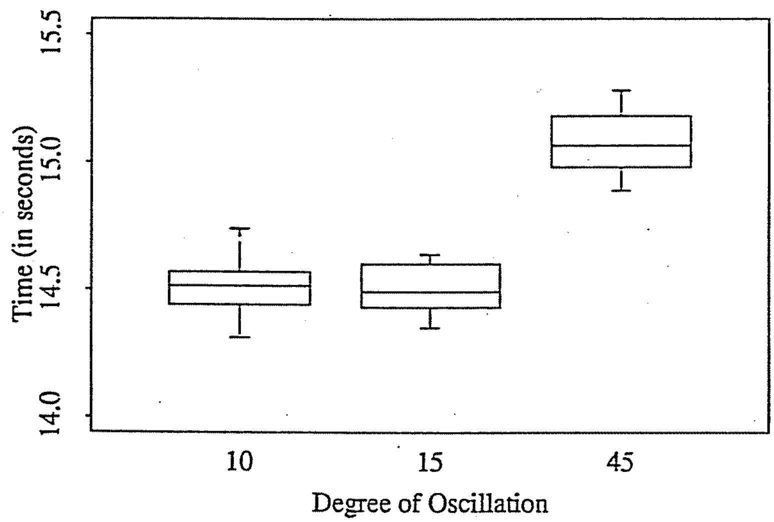

FIGURE 1

Simple boxplot showing times for 10 swings for pendulums oscillating at different angles

The boxplots show quite clearly that the results obtained at 10° and 15° are very similar, while those at 45° are different. The distribution of results at 45° does not overlap with either of the other two sets of results. We conclude therefore that the time of oscillation for 10° and 15° is the same, but that it is different for 45°. If we wish to be more certain that these differences are major differences, we can consider the idea for "outliers".

A boxplot can be modified to show unusually high or low values in a set of results. These unusual observations are called outliers and may be caused by errors in measuring or recording the results. If that is so, we want to detect and correct these errors if possible. Alternatively, they may indicate that one (or more) result is very different from the rest, and actually does not belong to that set. It may in fact indicate that it has come about due to different experimental conditions.

One "rule of thumb" for identifying outliers is to look for those results which are more than 1.5 times the interquartile range below Q_1 or above Q_3. These limits are called "fences". The whiskers are drawn out to the last points *just inside* the fences and any outliers are shown as individual points.

The position of the "fences" are not usually shown on boxplots, but in this case we believe it is helpful to do so. Figure 2 shows the same plots but with the fences inserted. In all cases the fences lie beyond the extreme values of the sample, suggesting that none of the observations are outliers. Be careful with your results, though, if you have any outliers. In this case, the whisker will be shorter (and will be drawn only as far as the next result in).

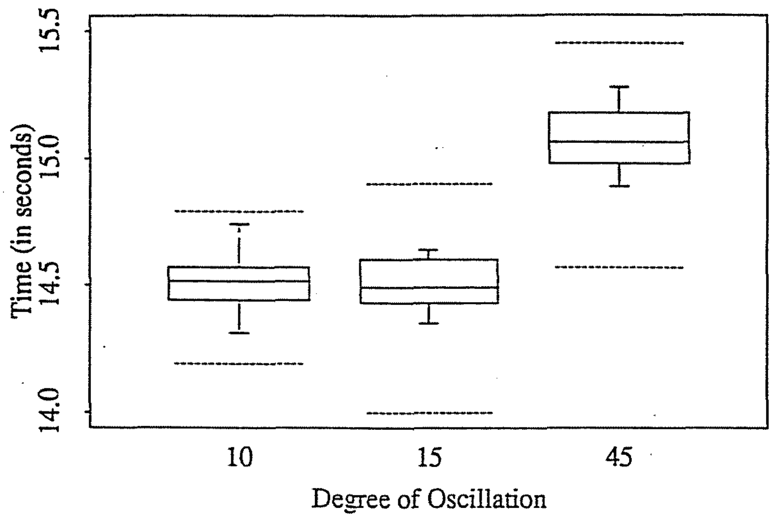

FIGURE 2
Simple boxplot showing times for 10 swings for pendulums oscillating at different angles, with fences inserted

With the fences drawn onto the boxplot, it becomes quite clear that the results obtained for 45^o are way outside the acceptable range for the other two sets. If we tried to incorporate any of the results obtained for 45^o into either of the other two sets, they would appear as outliers.

3. Investigation 2 - friction

A concept of friction is studied today by many pupils taking courses in Technology Physics and also in A Level Mathematics where the syllabus includes mechanics. We have found that simple practical work on friction can be used to verify the standard Newtonian model and also to introduce or develop some important statistical ideas.

The earlier sections of this work have been used by 14-year-old pupils while later sections are suitable for A Level students.

The practical work for this project is simple. Place a wooden block on the end of a plank of wood and raise that end of the plank until the block just begins to move. Figure 3 shows the standard Newtonian model for this situation.

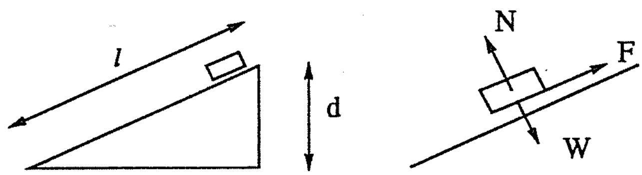

FIGURE 3
Forces on a sliding block

It can be shown that the coefficient of friction is calculated by:

$$\mu = \tan\theta = d/\sqrt{l^2 - d^2}.$$

If the experiment is repeated several times though, considerable variation in the heights, d, will be observed.

For demonstration purposes a large plank can be used. For group work, metre rulers are quite adequate.

Here are the results of one demonstration exercise. The length of the plank l = 1.82m and the value of d (height raised) were measured to the nearest cm.

Height raised (d):
56, 69, 64, 64, 73, 66, 76, 57, 69, 71, 54, 62, 54, 63, 68

Here a stem-and-leaf diagram could be used to represent the data. This diagram orders the data, groups it into class intervals, and gives a visual image of the data without losing the details of the raw results. For GCSE pupils some diagrammatic representation and the calculation of a mean or median for d would be expected.

The range of the results 76 - 54 = 22cm is quite large and at this stage it does seem optimistic to expect a constant value for μ to emerge. Can we assume that our range of results is due to experimental errors? Do these experimental errors form a normal distribution? For a large set of results (using the same block and plank) a histogram could be drawn to investigate this assumption. For a small set of results, normal probability graph paper can be used to test whether a normal distribution is a feasible model.

The next problem for an A Level group might be to estimate the coefficient of friction (given earlier). When the values of d are substituted into that equation, a range of values of μ is obtained

$$0.31 \leq \mu \leq 0.46$$

The mean value of μ is 0.38, and the variance is 0.002, giving a standard deviation of 0.046. As our values of d appear to come from a normal distribution, it is not unreasonable to assume that the corresponding values for μ will also.

Thus a 95% confidence interval for μ (for large samples) can be constructed as

$$\bar{x} \pm 1.96 \times \frac{\sigma}{\sqrt{n}}.$$

This gives a range of

$$0.36 \leq \mu \leq 0.40$$

If for example the true mean was 0.41, the percentage error would be 7% or 8% which is still within the ±10% range which experts on friction seem to regard as reasonable.

There are a variety of further investigations which can be done with this idea. The surface of the plank (or ruler) may be changed by sticking on a piece of sugar paper - the difference is surprising.

One colleague suspected that the range of values for μ would increase as μ increases. Our results supported this hypothesis to some extent, but not convincingly so. Perhaps you would like to investigate this.