Integrated Real Life Themes and the Teaching of Probability and Statistics at 11-16 Level

Alan Rogerson - Melbourne, Australia

1. Introduction

This paper re-evaluates the basic notions of probability and statistics and discusses their introduction through integrated real-life themes, a method already successfully tested in schools for students aged 11-16. In effect, this approach provides a viable alternative to the great majority of school/university courses which teach probability and representational/parametric statistics virtually as an extension of pure mathematics.

2. Probability

Why is it that our students' expectations, based on common cultural ideas of "chance", "luck" and "likelihood", often seem to be at odds with our formal teaching of probability as a purely deductive system based on set theory? It is quite common, for example, to find textbook introductions to probability demonstrating that a Football Pools or Tattslotto win is very unlikely, that a long string of losses doesn't increase the probability of either a win or a loss next time, that getting a number *close* to a winning number has no significance, and so on. All of this "killjoy" information is justified using models of "unbiased" coins, dice and cards. When we restrict ourselves to these situations, the mathematical argument seems unassailable - even though this does not always appear to convince our students. The problem that has rarely been examined, however, is *why* our students' expectations arise in the first place, and why everyday experience does not seem to readily confirm elementary probability theory (Rogerson, 1981).

Even in a theoretical sense, our grounds for belief in the applicability of the mathematical model for probability are less secure than we might think, and this is precisely why probability theory has created its own safe domain (or deductive system) to work in. Put another way, how long would we adhere to the *theory* if a dice constantly turned up 5 or 6: at what point would be suspect it was "biased"? This points to the inherent circularity in our justification of the mathematical model. As Hacking makes clear, this difficulty has bedevilled attempts to establish the "law of averages", "law of large numbers" or the "long-run frequency rule" (Hacking, 1976; Epstein, 1977).

Generations of teachers have rather explained their students' erroneous expectations by assuming that there was some fault or deficiency in their students' socialisation. An alternative explanation, however, is that the fault may lie in the *application* of our theoretical models to everyday life. What would happen if, instead of the artificial examples we choose in probability classes, we looked at everyday situations that *do* reinforce our students' expectations, contrary to the theoretical model? For example, when there are many accidents at a particular road junction, we call it a *black spot* and regard accidents as more likely there. When social clubs go to the same place every year on their annual outing we expect it to be the same this year. When a newspaper does a feature on different teams every week, it becomes more and more likely our team will feature next week. When we see two number 78 trams (or buses) pulling away from a stop - it's *less* likely there will be another for some time, and so on.

It is these situations that are typical of the myriads of everyday events that have gradually formed our students' intuitive notions of chance, luck and likelihood. In everyday life we naturally come to expect *causes*, *reasons* or *motives* to predict accidents, annual outings and newspaper features. We very rarely find that repeated events in real life are *independent*, on the contrary they tend to be more or less probable according to the situation. Intuition, or socialisation, has not therefore mislead our students, in fact it has reinforced their *correct* interpretation of their total life experience. Hence it is the theoretical models for "unbiased" dice and coins used to introduce probability that are artificial and "unreal" in stressing *independence*, usually because it is mathematically easier to deal with.

The moral of this discussion is that we should base our *introduction* to probability on our students' already formed intuitive ideas, using everyday real-life situations and ideas. Afterwards we can develop the special and simpler cases of independent probability. Of course the lack of reinforcement of independent events is only one characteristic of our students' "pre-knowledge". Our introduction to probability should comprehend and build on as much of their previous experience as possible, and this represents a specific *didactic* problem. Such an approach has already been worked out in some detail (Rogerson, 1981 and 1983, SMP 7-13 Unit 6).

3. Statistics

Statistical techniques are widely used and applied in the sciences, in politics and economics and in educational research. In all of these areas, decisions must be taken about the collection (i.e. selection) of evidence, the size and scope of samples and the framing of questionnaires and surveys. How much freedom does this give us, and how

malleable are the results of statistical evidence and analysis? When Disraeli coined the phrase "Lies, Damned Lies and Statistics" he highlighted a popular conception of statistics as selectively manipulating and distorting real-world data. Unfortunately, this is not merely a popular misconception, there are serious deficiencies in the assumptions underlying the application of parametric statistics to real-world data (Lindley, 1972; Lakatos, 1976; Hacking, 1976).

How do these problems affect the introduction and teaching of statistics at school level? Most school (and many tertiary) courses ignore the difficulties altogether by teaching statistics as though it were a branch of pure mathematics. For example, many textbooks introduce measures of central tendency with questions such as "Calculate the mean, mode and median of 3,5,5,7,9,11". Not only is this a precursor to the worst kind of cook-book methods, but it ignores the fact that each separate average is appropriate only for use in *specific* contexts. How then can we avoid the dilemma of either teaching statistics in a purely mathematical or cook-book fashion or, on the other hand, trying to apply statistical techniques to real-life problems (Rogerson, 1987).

A practical solution to this dilemma was developed in the successive series of SMP texts (SMP, 1966-). The main feature was the gradual *integration* of probability models and statistical techniques into the real-life contexts of problems. This implies emphasising not only the purpose of a particular problem, and the reasons for choosing one technique rather than another, but also the *application* of statistical results through real-life decisions such as the siting of traffic lights, the building of new supermarkets, the use of new drugs, the choice of certain seed types to cultivate, and so on. The key to this approach is to emphasise the relevance of the *total social context* which determines the initial selection and collection of data, the choice of models and techniques to analyse that data and the vital final stage of decision making on the basis of the statistical results.

This approach effectively "demystifies" the *apparent* scientific authority of statistical results and replaces the blind acceptance of significance levels, correlation coefficients and so on with the human power of decision making (Rogerson, 1982). This also helps to avoid the manipulation and distortion of statistical results which stems primarily from treating statistical results as though they had the status of physical laws. Although this approach to statistics teaching originated during the early developmental period of the School Mathematics Project (1960-1975) it has since influenced many other projects, especially the Mathematics in Society Project.

4. The Mathematics in Society Project (MISP)

MISP was formed in 1980 by an international group of teachers from the UK, USA, Australia, Italy and Spain, who felt that the key to a positively motivated view of mathematics lay in its use in society. This meant looking at mathematics in a completely different way: not as a skeleton list of skills and concepts, or explicit syllabus items, but rather as a collection of real themes which *implicitly* represented the living body of mathematics. For example, two teachers in Canada had already written and tested themes on Housing, Driving, Eating, Competing, etc. In the UK, the SMP 7-13 Project Units 5 and 6 contained thematic ideas on cooking, navigation, architecture and even science fiction. A group of Italian writers had tested a three-year middle school

mathematics course with historical and economic themes. All of these materials have been modified or translated for testing as part of MISP. For full details on this project see Rogerson (1980, 1983 and 1986).

What is it that characterises the MISP approach? It seems to us that mathematics education has moved from a subject-centred to child-centred to society-centred approach during the past thirty years.

Different styles for the same content

Subject-centred	*Child-centred*	*Society-centred*
Taught as at University	Content revised and selected	Selection to suit use in society
Illustrative examples	Multi-paced to suit different abilities	Creation of situations from society
Re-structured content - from particular to general	Revision of content based on student feedback	Re-creation of what mathematics means in society

To illustrate this development, take the case of probability and statistics. Prior to 1960 if it was taught in schools at all it was considered a branch of pure mathematics. During the 1960s there was an attempt to construct more "realistic" examples for school use. This led in turn to the teaching of probability in a more child-centred way using practical apparatus such as dice and coins, while "real-life" data was increasingly used in statistics, with practical experimentation being introduced into both areas. In order to make probability/statistics society-centred, MISP has had to discover ways in which it is used implicitly in society, and has therefore produced themes based on social, historical and economic statistics (Romberg, 1982). This in turn has modified our view of how much, and which, content should be taught in probability/statistics.

When MISP began in 1980 the idea of linking mathematics with society seemed a new and radical initiative. It is clear now, however, that mathematics and society has become a dominant focus within mathematics education and MISP materials are now being translated into Italian, Polish, Greek and Hungarian and have already been widely tested in English.

5. Conclusion

The world is full to overflowing with practical examples of the use of probability theory and representational and parametric statistics to compress, summarise or interpret large quantities of real data, in order to make decisions that affect everyday life in our society. Both probability and statistics can be introduced and taught *implicitly* within integrated real-life themes which take into account the total social

context of the problems to be solved. This thematic approach demonstrates that statistics and probability models and techniques serve a *functional* or *instrumental* role in society, a role that should always be controlled and mediated by ethical values.

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