## Introduction to Research Papers on University Students

Much of the published research on how college students (or adults in general) understand probability has been done by psychologists in the area referred to as "judgements under uncertainty" (Kahneman et al., 1982). The main contribution of this group has been to identify and describe common errors that adults make in reasoning about uncertain events. Kahneman et al. attribute these errors to the use of intuitive strategies (heuristics) such as "representativeness" and "availability" and specific misconceptions related to them.

Researchers in statistical education have investigated the prevalence and age-distribution of these hypothetical "heuristics". Like the psychologists, the educators typically use problems that involve coins, marbles, or balls drawn from urns, and gender of baby births; all situations involving equiprobable outcomes that can be easily modelled using basic laws of probability. The educators have been particularly interested in the relation of reasoning about probability to other mathematical ideas and in instructional strategies that might ameliorate the errors in reasoning (e.g. Shaughnessy, 1977).

Those research papers at ICOTS 3, by psychologists and educators alike, represent a second phase of research on how well these heuristics actually explain students' responses to probability problems. The four studies described here closely examine inconsistencies in student responses that are not explained by the hypothesised heuristics.

Garfield and delMas, Jolliffe, and Konold et al. all describe different kinds of empirical studies of university students. The first involves testing students before and after special instruction, the second involves written surveys of different groups of students, and the third involves in-depth student interviews. Borovcnik's paper, although building on interviews with students, draws on theories of intuition to explain students' difficulties in understanding probability.

Garfield and delMas constructed an instrument to assess errors related to the representativeness heuristic. They looked for evidence that students are guided by misconceptions such as the law of averages and the law of small numbers. Their test was given both before and after use of a computer program designed to confront these misconceptions via simulations of coin tosses. They investigated the consistency of students' patterns of response both before and after the intervention and looked at changes attributable to the simulation program. They found several inconsistencies in students' response patterns that indicate students' use of representativeness heuristics varies with the problem context. The reasoning strategies used by students appeared to be resistant to the instruction, even though the percentage of correct answers increased from pre- to post-test.

Jolliffe also investigated the effect of problem context on student responses. She reports the results of using similar versions of a written survey to groups of university students with varied experiences in studying probability. Although on some items most students appeared to demonstrate an understanding of basic probability ideas, on other items most students failed to indicate that different sequences of coin tosses are equally likely. They also showed a lack of understanding of the effect of sample size in different problem contexts. Surprisingly, students with more previous coursework in statistics gave more incorrect answers.

In contrast to the usual "heuristic" view, Konold et al. drew on earlier work hypothesising that an "outcome orientation" may be leading students to respond inconsistently to similar items (Konold, 1989). Their study involved 20 college students who were interviewed about various aspects of probability. Problems similar to those used as in the Garfield and Jolliffe studies were followed by additional interview probes. Results corroborated the previous studies, indicating that, although large percentages of students can give correct answers to basic probability problems such as those used in large-scale educational studies, they still appear to be misunderstanding the basic ideas of probability. This study suggests that students reason from a variety of perspectives and that these perspectives may change as students respond to different types of questions. The authors also believe that students' basic beliefs about coin flipping differ from what instructors expect them to believe. These beliefs and perspectives may hinder the effectiveness of instruction deliberately designed to overcome errors in understanding and reasoning.

Borovcnik's paper describes the intuitions, both primary and secondary, that interact and conflict with the theories of mathematics and probability students are taught. His ideas about how items are "loaded" may explain why students' answers vary for similar types of problems. Action-oriented items are distinguished from reflection-oriented items, the latter being more theoretical and therefore more difficult to solve. Borovcnik believes that teachers need to help students build their intuitions (i.e. secondary intuitions) in ways that will lead them more easily to a theoretical understanding of probability. To do this, he encourages teachers to stop asking students questions that ask them to make a specific prediction about an uncertain event, and instead to involve them in action-oriented activities. Borovcnik also encourages teachers to interview students to better understand their intuitions and to reveal their misconceptions.

All four research papers illustrate different approaches to understanding why students' responses to probability problems are inconsistent. The papers are in agreement that teaching university students probability concepts is a challenging and complex endeavour, and that student understanding cannot be sufficiently measured using the typical multiple-choice probability problems. The papers in this section indicate a need for further research in understanding the perspectives and intuitions students use in reasoning about probability, and in determining how these are affected by different instructional approaches. It is important that researchers and teachers spend time listening to students talk about their ideas of probability, either through interviews or as students work in small groups. Only by listening to students express their ideas and beliefs will we be able to assess the impact of instruction on understanding probability.

**References**

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