Actuarial Statistics - An Australasian Perspective

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1. Introduction

The theme of this paper originally encompassed an Australasian perspective of actuarial statistics and its place in the university curriculum. Its scope has, however, since been tacitly broadened. It was felt that one could only consider actuarial statistics in the context of actuarial studies generally, having regard to the major changes currently taking place in the financial world, and the changing role of actuaries in that world.

After a general description of the current situation in the financial world, the subject "Actuarial Statistics" is discussed. Actuarial education, and the extent to which it should take place at university, then comes under the spotlight. Finally, the situation regarding actuarial education in Australasia is considered.

Reference to actuarial examinations in this paper will mean the professional actuarial examinations of the Institute of Actuaries in England. The first six of these subjects deal with statistics, economics and specifically actuarial topics such as life contingencies, and are also taken by actuarial students in Australia and New Zealand. Of the total of ten actuarial examinations, the final four bear directly on actuarial practice, and are run separately in Australia and in the UK.

2. Current changes in the financial world

The financial world is in a state of flux, with sweeping legislative and institutional changes occurring constantly, all taking place against the backdrop of an explosion in computing power. At the same time, distinctions between banks, building societies, brokers and insurance companies etc. are becoming increasingly blurred. Concomitantly there has been an enormous concentration of firms in the service industry, and where there is still a recognisable difference between services provided, they are increasingly being provided by various subsidiaries of the one organisation.

This lowering of barriers in the financial world is mirrored by what has happened to the actuary. Actuarial consultancy has broadened to incorporate management consultancy, and is frequently allied with accounting, legal and personnel management services. The actuary who values the pension fund is now asked to solve other problems of a more or less financial or mathematical nature, while the life actuary is increasingly involved with product development and profit testing in an ever more complex commercial environment.

Together with such comprehensive changes in the business world itself there has arisen the further need to cope with ever more complex legislation, resulting altogether in a large and growing demand for individuals who are not only literate but also numerate, with good communicative skills. The range of mathematical abilities sought varies enormously, with the actuary towards the upper end of the spectrum. It is important to realise that the call for a greater number of actuaries is only part of a wide search within the financial sector for a highly skilled numerate workforce.

3. The subject Actuarial Statistics

Even within the framework of the formal UK actuarial examinations there are several possible interpretations of the title "actuarial statistics". In this paper, for instance, we do not consider Subject 6, which is concerned with the estimation of decremental and durational rates, and includes *inter alia* the study of fertility, demography and population projection. In addition to this subject, there are two statistics papers *per se*, of which we discuss only the second, more advanced paper, called "Actuarial Statistics".

The three substantive areas of the syllabus of this latter paper are risk theory, credibility theory and loss distributions, on which areas we elaborate in turn.

Risk theory: We can separate risk theory into two portions. The first comprises the more basic areas, such as the compound Poisson process. While one can suggest improvements on the approach taken by Bowers et al. (1986) (e.g. introduction of the cumulant generating function and omission of much explicit calculation of convolutions), the approach in the textbook is reasonable, and the topic of paramount importance.

One has stronger reservations about the second half, which broadly considers the ruin of an insurance company over an infinitely long time horizon. While not denying the value of an intuitive idea of the risk process and the probability of ruin, and readily admitting the historical interest of the topic, the excessive algebraic manipulation required to derive even the simplest of formulae is both offputting to students and a formidable barrier to effective examination. Questions tend to be either trivial, or tediously long and algebraically complicated, requiring substantial rote learning.

The problem arises because an advanced topic is being tackled by elementary means. As Gerber (1979) observes, many risk theoretic results are readily derived by more advanced tools, such as martingale theory, which are considered too advanced for actuarial courses. As another example, the intuition underlying the basic concept of the adjustment coefficient can really only be obtained through consideration of defective renewal equations; this topic also ties in neatly with stochastic birth processes, of substantial potential interest to actuaries, but renewal equations are again considered too difficult for inclusion in actuarial education (Gerber, 1979, p.121). Finally, the severe assumptions required for this second portion of risk theory are not discussed in any depth: these assumptions are in fact sufficiently strong that it is difficult to conceive of direct application of the theory to practice.

Credibility theory: Credibility theory is an important component of the course, all the more so in view of the fact that it is in heavy use in North America, while actuaries in Australasia and the UK have little knowledge of it. Nevertheless, prolonged exposure to the excessively awkward formulae of the empirical Bayes' formulations of credibility theory would seem counter-productive; one could place greater emphasis on simple Bayesian updating, and incorporate recent treatments of Bayesian credibility (e.g. Klugman, 1990).

Loss distributions: While loss distributions is a vital area of actuaries' statistical education, it has often been presented merely as an *ad hoc* collection of families of distributions. It is important that principles be emphasised: transformation of variates; compounding of distributions, including one provenance of the generalised Pareto as a compounded gamma distribution, in concert with the negative binomial distribution as a compounding of the Poisson; complementary descriptions of distributions via density function, hazard function etc.

Summary: Overall the subject Actuarial Statistics cannot be said to cohere well. Although to some extent the compound Poisson process forms a common element to the three areas, the subject is treated as little more than a collection of disparate topics, drawn together for convenience. This lack of coherence and the inappropriateness of some of the contents would not pose such a problem if there were not so much statistical material which could usefully be inserted into actuarial training: time series and structural models, including forecasting and the use of the Kalman filter; further econometrics; and diffusion processes, with special mention of the Black Scholes formula, and related statistical background for dealing with futures and options markets (the incorporation of which would itself necessitate increased exposure to finance within actuarial education) do not by any means exhaust the possibilities.

These comments notwithstanding, however, the three areas of Actuarial Statistics discussed above, especially with the incorporation of the suggested changes into the risk theory component, are quite suitable for university teaching. Each of these areas fits snugly into about the third year level of statistics courses, and is of substantial interest in its own right, with an enticing mixture of theory, practice and rich historical perspective. Incorporation of the more advanced tools of martingales and renewal equations would make risk theory an excellent statistics course at about the fourth year level.

We have not addressed the problem of the lack of coherence of the subject "Actuarial Statistics". Rationalisation of this subject needs to be undertaken in the context of a wholesale rethinking of the entire statistics component of actuarial education, including apportionment of material to the various statistics papers, whether compulsory or not. Such detailed considerations are well beyond the brief of this short paper.

4. Actuarial education

Skill levels in the financial world: In order to provide a framework for consideration of actuarial education, we consider the following gradation of mathematical and statistical skills, and discuss the capabilities of individuals in the financial sector.

Mathematics

M1. Basic arithmetic skills.

M2. Manipulative skills; ability to comprehend simple mathematical models, and to perform a deterministic simulation by writing a computer program.

M3. Familiarity with elementary calculus and analysis, say at first and second year university level.

M4. More advanced levels.

Statistics

S1. Data handling.

S2. Descriptive statistics, including some idea of location and dispersion.

S3. Elementary probability and "cook-book" statistics, say at first year university level.

S4. Moderate knowledge of theoretical statistics, say at second year university level, but lack of ability to apply this theory to real-life problems.

S5. Competent theoretical or applied statistician.

Individuals in the financial sector are in the main firmly positioned at level M1. Amongst dealers and brokers in the money and stock markets, and even within the ranks of other "numerate" professional groups such as accountants, very few individuals feel comfortable at level M2.

Traditionally, actuaries in the UK and Australasia have been predominantly qualified at mathematical and statistical level 3. Nowadays most newly qualified actuaries will have a mathematical level exceeding M3, but this is by virtue of recruitment from the ranks of mathematical graduates rather than because of any advanced mathematical requirements of the actuarial examinations themselves. Of the UK actuarial examinations, only Subject 5, "Actuarial Statistics", requires a mathematical level above M3. Because of the standard of statistics in Subject 5, most younger actuaries now have a theoretical statistics base superior to level S3, but fall firmly into category S4, not S5. Most actuarial work, on the other hand, is at level M2 (mainly because it involves so much deterministic simulation, e.g. profit testing), and, apart from general insurance, does not make statistical demands at a level exceeding S2.

Statistical requirements for actuaries: In view of the changes in the financial world and the wide choice of statistical material which could be inserted into actuarial education, increased flexibility within actuarial education is clearly a pressing requirement. There needs to be a broad spectrum of statistical expertise within the actuarial community, the minimum requirement being that an actuary know enough to be able to liaise with a more knowledgeable statistician: relatively few need themselves possess an advanced stage of statistical expertise. One would like a wide *de jure*, not just *de facto*, spread of statistical skills amongst actuaries. There should be substantial choice as to the level of statistics one takes in one's training: apart from the wide variation in an individual's interest in more theoretical studies, the need for statistics depends strongly on the field in which one is to work. One must also be given the opportunity to update and improve one's statistical expertise after qualification as an actuary.

Prior to 1977, the UK actuarial examinations contained an advanced optional statistics paper: the reintroduction of such a specialist paper is long overdue. In the absence of a sorely needed thorough reorganisation of statistical education for actuaries, the more elementary parts of time series and structural models could be placed into Subject 5; more advanced areas of those topics would be inserted into the putative new advanced statistics subject, along with diffusion processes and the cognate areas mentioned above.

An additional advantage of the introduction of this extra material is a closer synchronisation with continental European actuarial practice when much political control in Europe passes to the European Community (EC) in 1992. While there has been substantial homogenisation of the legal environment for the provision of insurance and financial services, there has been no attempt to coordinate actuarial training within the EC. Despite the very different traditions of the various national actuarial bodies, most strikingly contrasted between the mathematically disinclined British and the highly statistically orientated Scandinavians, it seems inevitable that some *de facto* standardisation of actuarial training will take place within Europe, although how institutionalised this will become remains to be seen. An important impetus towards such conformity would be the requirement for one country of the EC to accept actuaries qualified in another country, the point being that in most countries qualified actuaries have a role defined in legislation. An increased and more flexible statistical component of the actuarial examinations in the UK will facilitate standardisation of actuarial training throughout Europe and Australasia, assuming that this latter area continues to use the UK examinations.

5. The use of universities for actuarial education

A first objection to the teaching of any vocationally orientated material at universities is that university lecturers are in the main employed more to carry out research than to teach, and have a relatively light lecturing load in comparison with polytechnic lecturers or teachers at specialised institutions. In the UK, for example, the Chartered Insurance Institute runs its own specialised College to teach its own courses to the insurance industry, and the accountants have their own equivalents to such a College for cramming during the months prior to the professional accounting examinations. On the face of it, then, why not transfer actuarial teaching to more teaching orientated institutions?

There are several advantages of teaching vocational courses at university. First is the provision of a recognised framework for teaching and examining; the second is that the learning environment is away from the pressures of the commercial world. Thirdly, a multi-disciplinary education can be provided: given the broad sweep of the actuary's work, involving as it does some knowledge of economics, finance, accounting and legislation, this point is of vital importance. Finally, a direct benefit of providing actuarial training at universities is the close links thereby fostered between the business and the academic worlds. There is a need for the commercial world to have access to academic research, and academics are in any case becoming increasingly involved with consulting. The teaching of actuarial courses by academics in both mathematics and finance/economics areas provides an automatic connection between the worlds of academia and commerce, of great potential profit to both parties.

Ultimately, though, actuarial subjects fit readily into the university environment because they are academically respectable. The specialist actuarial subjects, such as life contingencies and the mathematics of finance, possess the additional advantage of offering interesting, self-contained choices as outside options for undergraduates, with the ancillary benefit of making the actuarial world better known to the community at large. Statistics courses profit from some slant towards the insurance and actuarial worlds, which can provide many interesting examples to which the theory can be applied. Moreover, there are close links between insurance on the one hand; and demography, pensions and other social science areas, and economics and finance on the other. These links can be exploited in the teaching of several disciplines.

That said, not all actuarial training can take place at universities: there is also the need for substantial "on the job" training. Actuaries require a good feel for the business and financial worlds, as well as adequate technical knowledge of insurance and of the commercial world generally. In the UK, exemptions from the final four papers are not available from university study. The Australians will soon be demanding a minimum of two subjects to be passed by taking professional examinations when working as an actuarial trainee in order to qualify as an actuary. At the other end of the spectrum, the actuarial qualification in Scandinavia is a university qualification, with practical experience received later when working in the commercial world. The papers in this session by Norberg and Klugman provide more extended discussions of the situations in Continental Europe and in North America respectively.

6. Actuarial education in Australasia

There has been an actuarial science degree at Macquarie University in Sydney for some twenty years, and Melbourne University is soon to introduce an actuarial science major into its Commerce degree, the aim being in each case exemptions from the first six professional examinations.

Further details are provided in the comments by John Pollard which follow, and we shall therefore confine our remarks to the situation in New Zealand.

The Diploma in Financial Mathematics recently introduced at Victoria University in Wellington places actuarial education firmly within a financial framework. As well as providing the New Zealand financial sector with a steady supply of actuaries, one hopes to increase the number of individuals working in the financial area who have a relatively strong mathematical and statistical background, together with some knowledge of actuarial work. The supply of such individuals should not be seen as draining the available pool for the supply of actuaries, but as a complement to the actuarial community within the financial sector. A further benefit to the finance industry is that to a certain extent we can cater for the student with a weaker mathematical background than that of the typical entrant to the actuarial programmes. From a broader perspective, the diploma complements the external education programme of the Institute of Statistics and Operations Research, other areas of which include lower level mathematics courses for individuals working in the financial sector.

A brief comment on the title of the diploma is perhaps in order. The name was chosen to reflect a desire to place actuarial education within a financial framework; it is also the title of the actuarial subject which used to be known as "Compound Interest", and which is now labelled "Mathematics of Finance". Both the previous and the current name for this subject are misleading: the former because the subject comprised much material other than compound interest, the latter because those who have studied finance (e.g. the capital asset pricing model, portfolio models, gearing or leverage etc.) will find only a relatively small proportion of the actuarial subject within what they consider to be the "Theory of Finance". In spite of the diploma's name, then, there is a need for explicit insertion of subjects from the theory of finance.

After a transitional period, the diploma will contain a core of actuarial papers, including Life Contingencies and the Mathematics of Finance, which will be compulsory unless they have previously been passed, either at the undergraduate level or through the professional examinations. A total of five subjects must be passed to complete the diploma. A broad range of options includes those papers of the first six professional examinations not included as core subjects; other options are topics from the theory of finance, investment, demography, operations research, applications of time series and diffusion processes to the financial markets, and of course further mathematics and statistics. In addition, an option further removed from the diploma's area, such as an introductory law subject, could be considered if a student expressed a strong desire to take such a subject. It is further the intention that actuarial subjects from the diploma be made available to third year undergraduates having the appropriate pre-requisites.

Amongst postgraduate diplomas providing courses in actuarial science (two in the UK, one soon to commence in Australia), the diploma at Wellington is the most broadly based, with maximum flexibility in the choice of options. By specific exclusion of the subject "Actuarial Statistics" from the core subjects, entry into the diploma is allowed for those with a weaker mathematical background than that of the typical entrant to actuarial studies, although clearly any such students wishing to become actuaries will wish to hone their mathematical skills during the diploma year by suitable choice of options.

References

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