BASIC MULTIVARIATE THEMES AND METHODS

Lisa L. Harlow
University of Rhode Island, United States
LHarlow@uri.edu

Much of science is concerned with finding latent order in a seemingly complex array of variables. Multivariate methods help uncover this order, revealing a meaningful pattern of relationships. To help illuminate several multivariate methods, a number of questions and themes are presented, asking: How is it similar to other methods? When to use? How much multiplicity is encompassed? What is the model? How are variance, covariance, ratios and linear combinations involved? How to assess at a macro- and micro-level? and How to consider an application? Multivariate methods are described as an extension of univariate methods. Three basic models are presented: Correlational (e.g., canonical correlation, factor analysis, structural equation modeling), Prediction (e.g., multiple regression, discriminant function analysis, logistic regression), and Group Difference (e.g., ANCOVA, MANOVA, with brief applications.

INTRODUCTION

Multivariate statistics is a useful set of methods for analyzing a large amount of information in an integrated framework, focusing on the simplicity (e.g., Simon, 1969) and latent order (Wheatley, 1994) in a seemingly complex array of variables. Benefits to using multivariate statistics include: Expanding your sense of knowing, allowing rich and realistic research designs, and having flexibility built on similar univariate methods. Several potential drawbacks include: needing large samples, a belief that multivariate methods are challenging to learn, and results that may seem more complex to interpret than those from univariate methods.

Given these considerations, students may initially view the various multivariate methods (e.g., multiple regression, multivariate analysis of variance) as a set of complex and very different procedures. This perspective could lead to confusion and frustration when trying to understand the basic features of each method. I believe that students find it easier to learn the material on multivariate statistics if they are taught to notice some common trends across the set of methods (Harlow, 2005). In a multivariate course that I teach, I present students with a number of themes that tend to run through many multivariate statistics and highlight how they apply to several different kinds of methods (e.g., Correlational, Prediction, and Group Difference).

To provide a consistent structure, I suggest a set of questions for students to ask when trying to understand a variety of different methods:

- What is the multivariate method and how is it similar to other methods?
- When would I want to use this method?
- How much multiplicity is encompassed with this method?
- What is the model or basic equation(s) for this method?
- How are variance, covariance, ratios and linear combinations involved?
- How would I assess the model at a macro- and micro-level? and
- What is a possible application of this method?

Each of these questions will be addressed for approaching the following kinds of multivariate methods that can be seen as direct extensions of 3 basic univariate methods:

- Correlational Methods (extending correlation)
    - Canonical correlation (CC)
    - Factor Analysis (FA)
    - Structural Equation Modeling (SEM)
- Prediction Methods (extending regression)
    - Multiple Regression (MR)
    - Discriminant Function Analysis (DFA)
    - Logistic Regression (LR)
- Group Difference Methods (extending ANOVA)
    - Analysis of Covariance (ANCOVA)

- Multivariate Analysis of Variance (MANOVA)

In the next sections, I present the set of 7 questions and discuss how they relate to these three kinds of multivariate methods, recognizing that there are other multivariate methods not discussed here (e.g.,. classification methods such as cluster analysis, and graphical methods such as multidimensional scaling). Several excellent books address these and other multivariate topics (e.g., Grimm and Yarnold, 1995, 2000; Johnson and Wichern, 2002; Hair *et al*., 2005).

HOW ARE THESE MULTIVARIATE METHODS SIMILAR?

Multivariate statistics are simple extensions of basic univariate methods (Tabachnick and Fidell, 2001). They often build on one or more of the following concepts:

- Correlation, *r*, for showing the strength of relationship between 2 variables,
- Combining variables in linear combinations (e.g., to form prediction equations),
- ANOVA to assess group differences between means, and
- Explaining variation and covariation among variables.

WHEN WOULD YOU USE THESE DIFFERENT MULTIVARIATE METHODS?

- Use *Correlational Methods* (e.g., CC, FA, SEM) to examine relationships among several variables (Cohen, Cohen, West and Aiken, 2003).
  - *CC* relates two *sets* of variables (e.g., assess the relationships among a set of high school (HS) performance variables of standardized achievement test (SAT) scores, HS grade point average (GPA), study hours with college performance variables of graduate record exam (GRE) score, college GPA, and number of projects or presentations).
  - *FA* examines the underlying structure in a set of correlated variables (e.g., examine whether there are 2 underlying factors assessing high school and college performance, respectively).
  - *SEM* relates any # of predictors, mediators, and DVs (e.g., investigate whether SAT score, HS GPA, and study hours predict a mediator of self-efficacy, which in turn predicts college outcomes of GRE and college GPA).
- Use *Prediction Methods* (e.g., MR, DFA, LR) to predict an outcome from a linear combination of predictors.
  - *MR* predicts a single continuous dependent variable (DV) with several predictors (e.g., predict college GPA with SAT, HS GPA, and number of study hours).
  - *DFA* predicts a categorical DV with several predictors (e.g., predict success or not in college with HS GPA, number of study hours, and self-efficacy).
  - *LR* predicts the odds of falling into a reference group of a categorical DV with several predictors (e.g., predict the odds of success in college with HS GPA, number of study hours, and self-efficacy).
- Use *Group Difference Methods* (e.g., ANCOVA, MANOVA) to assess whether means are different across 2 or more groups on 1 or more DVs.
  - *ANCOVA* assesses the difference between 2 or more group means for 1 DV after controlling for the effects of 1 or more covariates (e.g., examine whether the mean performance differs for students in a traditional lecture class versus an applied learning class, after controlling for SAT scores).
  - *MANOVA* assesses the difference between the means for 2 or more groups on a set of 2 or more DVs (e.g., examine whether the means differ for students in a traditional lecture class versus an applied learning class for 2 DVs of exam scores and project scores).

HOW MUCH MULTIPLICITY IS ENCOMPASSED WITH MULTIVARIATE METHODS?

I define multiplicity as the ability to address multiple considerations when analyzing data with a multivariate method. Most multivariate methods can embrace multiple theories and hypotheses; most can analyze several independent variables (IVs), and some allow several DVs (e.g., CC, SEM, and MANOVA). Some allow the examination of several groups or samples (e.g., SEM, DFA, LR, ANCOVA, and MANOVA); and many allow the analysis of multiple time points (e.g., examining the IVS initially with the DVs at later time points). Further, some allow multiple controls (e.g., covariates in ANCOVA, as well as MR, DFA, and LR; and manipulated IVs in ANCOVA and MANOVA). When a method can address several aspects of multiplicity, I believe that there is greater potential that the results will be more rigorous, generalizable, reliable, and valid (Anastasi and Urbina, 1997; Harlow, 2005; Pedhazur and Schmelkin, 1991).

WHAT ARE THE BASIC EQUATIONS FOR THESE MULTIVARIATE METHODS?

The methods presented here can be described by one of the following equations:

- *Correlational methods* (CC, FA, SEM) focus on the strength of relationship among variables assessed by: $(\sigma_{ij}) / \surd (\sigma^2_{ii}) (\sigma^2_{jj})$, where σ_{ij} refers to the covariance between variables *i* and *j*, and σ^2_{ii} refers to the variance for variable *i*.
- *Prediction methods* (MR, DFA, LR) focus on weighted combinations of variables such as: $Y = bX_i + ... + bX_j + E$, where *Y* is a DV, *X* is a predictor, b is a regression weight showing the change in *Y* for a one-unit change in *X*, and *E* is prediction error.
- *Group difference methods* (ANCOVA, MANOVA) decompose variance for a DV, *Y*, by assuming that Y= Grand M + τ + E, where Grand M is the mean of a DV across all participants, τ is a group mean, and *E* is error variance. The goal is to assess whether there is more variance between group means than variance within groups.

HOW ARE CENTRAL THEMES OF VARIANCE, COVARIANCE, RATIOS AND LINEAR COMBINATIONS INVOLVED IN THESE MULTIVARIATE METHODS?

- Many methods, multivariate and univariate, are globally focused on explaining the variance in 1 or more DVs with 1 or more IVs.
- Many multivariate methods take into account the covariance among variables, realizing that most IVs are not orthogonal, and that when analyzing multiple DVs, we would like to recognize any correlation or covariance among them.
- Many methods, univariate or multivariate, entail between over within ratios of variances and/or covariances (e.g., correlational methods involve the covariance between variables over the product of variances within variables; group difference methods involve a ratio of between group variance over the average variance within each group, as with an *F*-test).
- Many multivariate methods involve the formation of 1 or more linear combinations of variables to summarize the information from sets of variables.

HOW DO YOU ASSESS MULTIVARIATE METHODS AT MACRO AND MICRO LEVELS?

Many statistical methods can be assessed at an overall, macro level, as well as a specific, micro level. It is important to initially make sure that a multivariate analysis has some merit at the global level before examining further which aspects of the analysis are affected at the specific level. At both the macro and micro levels, it is useful to examine not only the significance of an effect, but also the magnitude of an effect (e.g., Kirk, 1996; Kline, 2004).

- Macro assessment entails the examination of a significance test (usually F or χ^2) that assesses whether the results are significantly different from chance. After finding significance, it is important to assess the magnitude or size of the effect. Effect size (ES) is often a measure of shared variance (e.g., R^2, η^2) showing how much overlapping variance there is between (linear combinations of) variables. Shared variance ESs are evaluated as small, medium, and large for values of .02, .13, and .26, respectively (Cohen, 1988).
- Micro assessment involves the examination of possible significance tests (e.g., *t*-test) or criteria (focusing on standardized weights > |.3|) for specific group means or specific relationships between variables. Effect sizes at the micro level can take several forms (e.g., standardized β-regression weight, *r*, Cohen's *d*), with values of .1,

.3 and .5 for small, medium and large β-weight or *r*, and values of .2, .5, and .8, respectively for small, medium and large Cohen's *d* (Cohen, 1988).

WHAT ARE POSSIBLE APPLICATIONS OF MULTIVARIATE METHODS?

It is helpful to provide an example of how to apply multivariate statistics to each of the 3 kinds of methods (i.e., correlational, prediction, and group difference). Below, I present 3 diagrams depicting 3 possible applications – a canonical correlation between 3 IVs and 2 DVs; a multiple regression with 3 IVs and 1 DV, and an ANCOVA with 1 IV, 1 covariate and 1 DV.

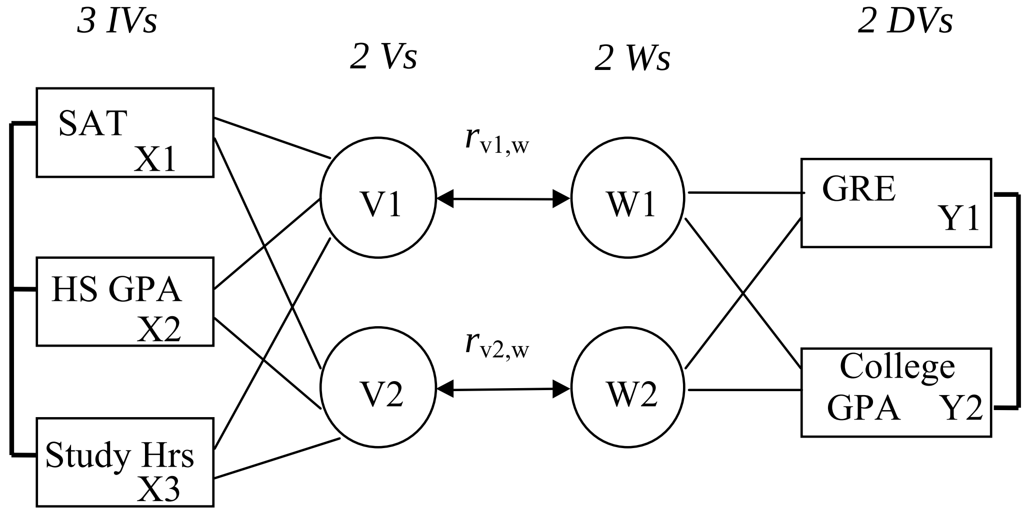

Figure 1: Depiction of a Canonical Correlation Application with 3 Independent Variables (IVs), 2 Dependent Variables (DVs), and 2 Sets of Canonical Variate Linear Combinations (Vs for *X*s or IVs and Ws for *Y*s or DVs), between which are canonical correlation *r* values

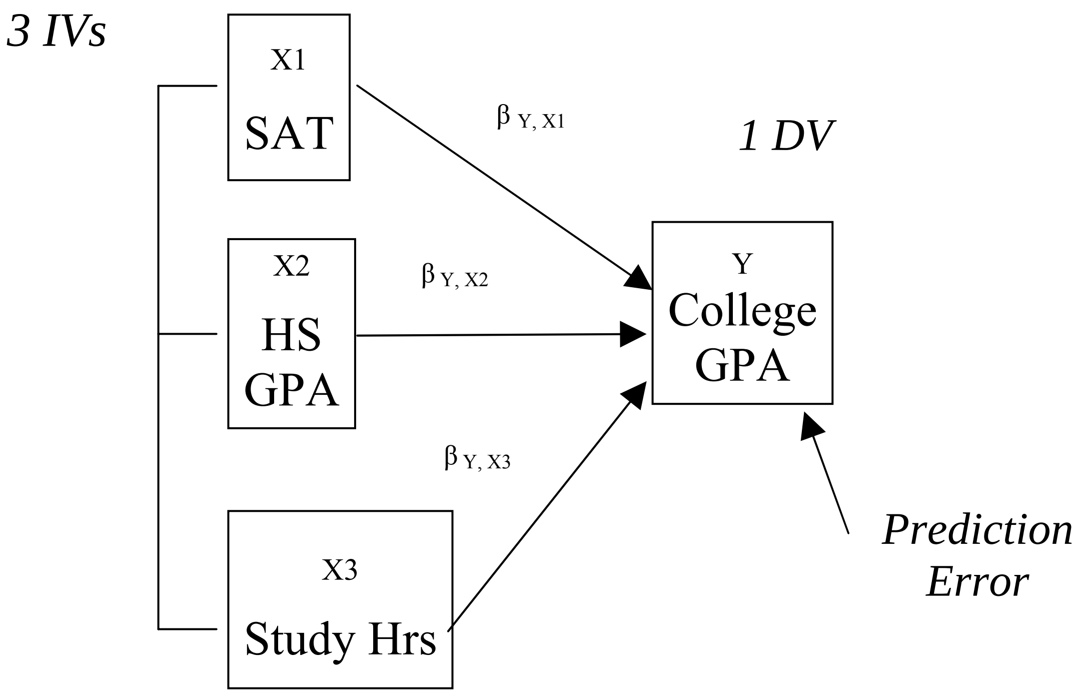

Figure 2: Depiction of a Multiple Regression Application with 3 IV-Predictors and 1 DV plus Prediction Error, Focusing on the Standardized Regression Weights, β, between IVs and DV

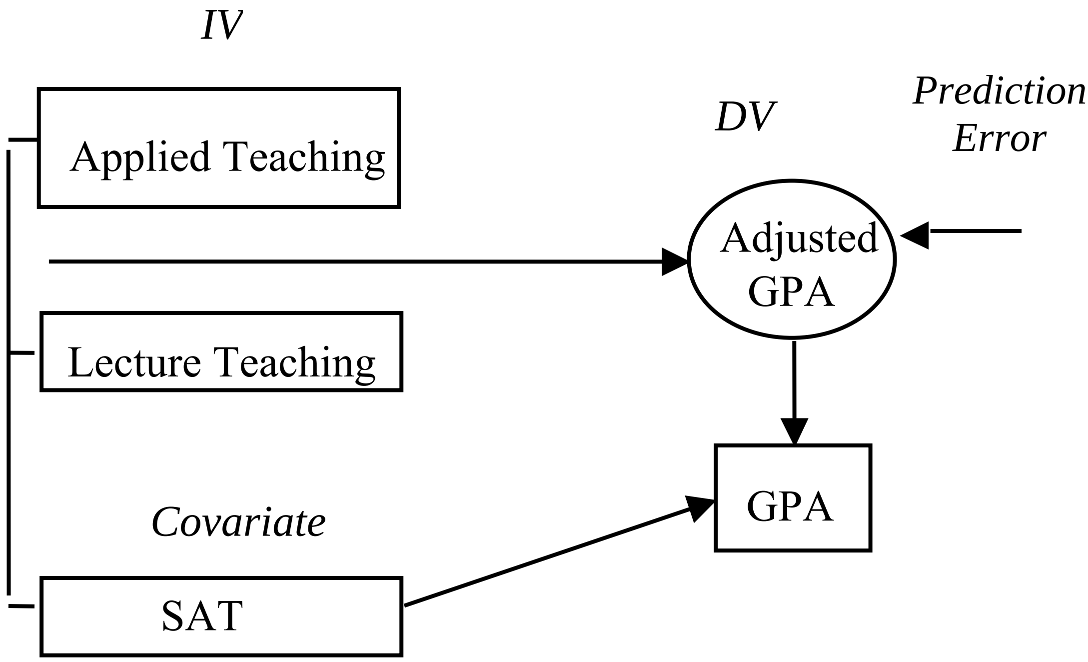

Figure 3: Depiction of an ANCOVA Application with 1 Categorical Independent Variable (IV), Teaching Method (Applied vs. Lecture), 1 Continuous (IV) Covariate (Standardized Achievement Test (SAT) Score), a Dependent Variable: DV (Grade Point Average (GPA)), and a DV that is Adjusted for the Effect of the Covariate. Focus is on the Difference between Means for Applied vs. Lecture Teaching Groups on the Adjusted GPA Score, after taking into account SAT scores.

CONCLUSION

I presented an overview of 7 main questions that can be used to highlight the common themes and concepts across several different kinds of multivariate methods (i.e., Correlational, Prediction, and Group Difference). Recognizing that there are other excellent multivariate methods that may not be easily described with the themes given above, the multivariate methods presented here can be described as focusing on either the correlation among variables, the prediction of a DV from a set of IVs, or mean differences between 2 or more groups. I argue that statistical methods can be seen as more reliable and generalizable to the extent that they can encompass multiple considerations (e.g., theories, hypotheses, measures, samples, time points, and controls). In this regard, structural equation modeling probably addresses more multiplicity considerations than the other multivariate methods presented here, allowing for more rigorous and accurate conclusions. Further, many multivariate methods can be described by either a correlational, linear combination prediction, or group difference model, usually analyzing some ratio of variances and/or covariances between and within the variables or groups. Additionally, most multivariate methods can be assessed at a macro level by a significance test and an effect size showing whether the magnitude of the shared variance between independent and dependent variables is significantly different from chance. At a micro level, most methods can assess specific variables or groups using significance tests and/or standardized weights or standardized differences between groups. Micro level assessment informs researchers as to what aspects of an analysis is the effect occurring. Several applications were briefly depicted to highlight an example of a correlational, prediction and group difference method, respectively.

After conducting multivariate statistical methods, several next steps could include replicating the results using different samples and methods, noticing patterns and themes that emerge in the data, and above all, enjoying multivariate thinking and application.

REFERENCES

Anastasi, A. and Urbina, S. (1997). *Psychological Testing.* Upper Saddle River, NJ: Prentice Hall.

Cohen, J. (1988). *Statistical Power Analysis for the Behavioral Sciences*. San Diego, CA: Academic Press.

Cohen, J., Cohen, P., West, S. G., and Aiken, L. (2003). *Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences* (3rd edition). Mahwah, NJ: Lawrence Erlbaum Associates.

Grimm, L. G. and Yarnold, P. R. (1995). *Reading and Understanding Multivariate Statistics.* Washington, DC: American Psychological Association.

Grimm, L. G. and Yarnold, P. R. (2000). *Reading and Understanding More Multivariate Statistics.* Washington, DC: American Psychological Association.

Hair, J. F., Black, B., Babin, B., Anderson, R. E., and Tatham, R. L. (2005). *Multivariate Data Analysis* (6th edition). Upper Saddle River, NJ: Prentice Hall.

Harlow, L. L. (2005). *The Essence of Multivariate Thinking: Basic Themes and Methods*. Mahwah, NJ: Lawrence Erlbaum Associates.

Johnson, R. A. and Wichern, D. W. (2002). *Applied Multivariate Statistical Analysis* (5th edition). Upper Saddle River, NJ: Prentice Hall.

Kirk, R. E. (1996). Practical significance: A concept whose time has come. *Educational and Psychological Measurement,* 56, 746-759.

Kline, R. B. (2004). *Beyond Significance Testing: Reforming Data Analysis Methods in Behavioral Research.* Washington DC: American Psychological Association.

Pedhazur, E. J. and Schmelkin, L. P. (1991). *Measurement, Design And Analysis: An Integrated Approach.* Mahwah, NJ: Lawrence Erlbaum Associates.

Simon, H. A. (1969). *The Sciences of the Artificial*. Cambridge, MA: The M.I.T. Press.

Tabachnick, B. G. and Fidell, L. S. (2001). *Using Multivariate Statistics* (4th edition). Boston, MA: Allyn and Bacon.

Wheatley, M. J. (1994). *Leadership And The New Science: Learning About Organization From An Orderly Universe*. San Francisco, CA: Berrett-Koehler Publishers, Inc.