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TREES AND GENERALIZED VENN DIAGRAMMES : A RICH DUALITY.

Résumé :
Nous avons déjà présenté les **diagrammes de Venn généralisés** dans différents livres et articles (bibliographie 1-2). Voici maintenant l'étude de la **dualité** au sens de la théorie des graphes, entre les **arbres** et les **diagrammes généralisés, mis sous forme canonique**.
Nous retrouvons alors la définition d'une hiérarchie indicée au sens de Bourbaki.
Tout ceci nous donne un **grand ensemble d'images axiomatiques** et nous permet de réunir les présentation non seulement du calcul des probabilités mais aussi de la logique, de la théorie, de la décision, de l'inférence statistique et de la théorie des jeux. **Réunir des visions scientifiques différentes** est un progrès selon Descartes. Mais le but principal de notre étude est de **faciliter l'enseignement de ces disciplines**, permettant d'allier le confort de l'image à la sécurité dans l'utilisation des axiomes classiques.

We have already presented « GENERALIZED VENN DIAGRAMMES» in different books and papers (bibliography 1-2-3-5), with their syntax . Let us present now the DUALITY, regarding graph theory, between TREES and DIAGRAMMES, within their canonic form. Then, we reach the BOURBAKI INDICED HIERARCHIES.
All this gives us an important set of AXIOMATIC IMAGES and allows us to agglomerate the presentation of STATISTICS, PROBALITY, LOGICS, DECISION THEORY an GAMES, which is a progress in itself.
It also allows us to facilitate the corresponding teaching, giving the comfort of images, with the security of using the right axioms .

I - DEFINITIONS.

1 *Let us first remind the definition of a hierarchy : (bibliography 4))
Let E be a finite set. let H be a set of parts of E
H is a hierarchy if it respects the following axioms :
i $(\forall x \in E)(\{x\} \in H)$
ii $E \in H$
iii $(\forall A \in H)(\forall B \in H)[(A \cap B) \neq \phi \Rightarrow ((A \subset B) \vee (B \subset A))]$

2 * Let us add the definition of the set of successors of an element of H.
S is an application of H into P (H) such as the following axiom is respected :
$(\forall A \in H)[(X \in S(A)) \Leftrightarrow ((X \subset A) \wedge (\nexists Y / ((Y \in H) \wedge (X \subsetneq Y \subsetneq A)))]$
If S (X) = ϕ, then X is a singleton of E.

3 * We also define a COHERENT-PSEUDO-INDICE (C.P.I.) :
f, application of H into R is a C.P.I. if the following axioms are respected :
i $(\forall A \in H)\ (f(A) = \sum_{X \in S(A)} f(X))$
ii $f(E) = 1$
(if f is defined on the singletons-set, then it is totally defined)

4 * And we add the definition of SUCCESSING-PSEUDO-INDICES (S.P.I.) g_A, when we know a C.P.I. : f.

Let A be an element of H. The C.P.I. g_A is an application of the set of successors of (S (A) into R, such as :

$(\forall A \in H) (\forall B \in S(A)) (g_A(B) = f(B) / f(A))$

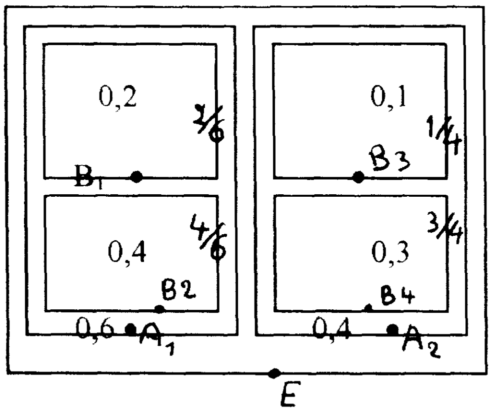
II - REPRESENTATION AND DUALITY.

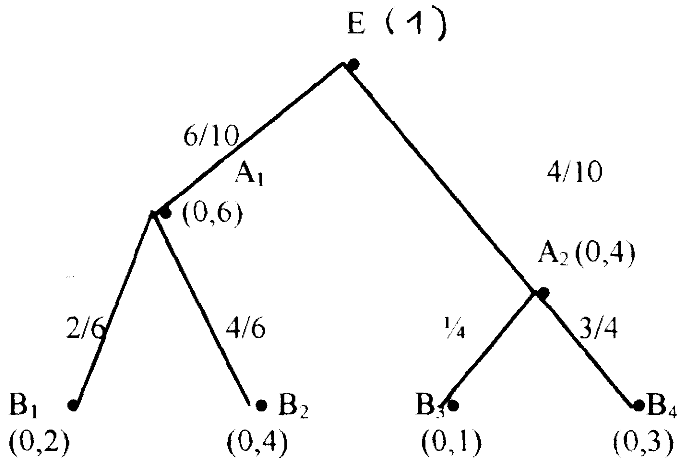
All this can be represented in two ways :

1 * as a tree with coherent-pseudo-indices (C.P.I.)on its summits ans succession-pseudo-indices (S.P.I.) on its branches.

2 * as a Generalized-Venn-Diagrammes within its cannonic form with C.P.I. on its singleton faces, and S.P.I. on its vertices.

Exemple :





III - CONCLUSION

Both of these drawing are special graphs in wich Euler's formula can be applied. They are in DUALITY regarding the classical graphs-definition (exchange between faces and summits, vertices becoming « cutting » vertices) . Both represent an indiced hierarchy with E = (B1,B2,B3,B4) and H=(B1,B2,B3,B4, A1,A2,E) with its respective indices-value (0.2,0.4,0.1,0.3,0.6,0.4,1)+).

Thus, we have shown THE TRIPLE EQUIVALENCE between INDICED HIERARCHIES, C.P.I. TREES and CANNONIC COMPLETE DIAGRAMMES.

THIS SHOULD ALLOW A SIMPLE UNIFIED PRESENTATION OF PROBABILITY, STATISTICS, DECISION THEORY, TEST THEORY AND GAMES.

+)Allowing singleton-indiced values to be different from 0.

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