Statistical thinking: How can we develop it?

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1. The Framework

According to Snee (1999) the development of statistical thinking is the next step in the evolution of the statistics discipline. The impetus for this evolution started in the early 1990s with particular people in the American Statistical Association (e.g. Moore, 1990) and the quality management field. Statistics education research on statistical thinking is only just starting to emerge (Garfield & Ben-Zvi, in press). Wild & Pfannkuch (1999) attempted to characterise statistical thinking in empirical enquiry in a four-dimensional framework. This framework was built upon historical, quality management, psychology, epidemiology, and statistics education literature, as well as interviews with statisticians and statistics undergraduate students.

Wild & Pfannkuch (1999) stated that statistical thinking comprises four dimensions: an investigative cycle, types of thinking, an interrogative cycle, and dispositions. The four dimensions contain generic and specific statistical thinking habits and are operative within the thinker simultaneously. The five types of thinking that were identified as fundamental elements in statistical thinking were: recognition of the need for data, transnumeration, consideration of variation, reasoning with statistical models, and integrating the statistical with the contextual.

When considering the framework and these types of thinking many questions arise for learning, teaching, and the curriculum such as: How are these types of thinking manifested in beginning students? Are there particular ways of teaching that can elicit such thinking? How does the teacher draw students' attention to notice and to attend to this thinking? How is such a habit of thinking communicated in a curriculum document? The purpose of the framework was to characterise statistical thinking rather than define students' growth in statistical thinking and was not primarily intended to address teaching.

The framework is not developmental or hierarchical. If the framework is to be used for teaching then extensive work may need to be done on ascertaining levels of thinking for each element that can be expected from students across the school years. The basis of the levels could be the SOLO taxonomy as used by Watson (2001), and Jones et al.(2000). Much research has already been conducted on a number of elements of the framework such as variation, and reasoning with models and this implies that other models could be used to inform and delineate the framework into one that was more applicable to teaching. On the other hand, the framework in its present form may be sufficient to communicate the habits of thinking that should be fostered in students. It may not be necessary to convey in detail every nuance of statistical thinking. In fact Snee (1999) criticised the framework on the grounds that it was too complex and that for effective communication a model containing four to seven key elements would be sufficient. A model can be used for communication with others and as a thinking tool. It can be used to critically reflect upon and to describe experiences. Whether the framework needs to be amplified into a developmental framework for teaching and the curriculum, or reduced, or used in conjunction with other frameworks, or is

sufficient as an overall guide will perhaps depend on how it is chosen to be used. Thus the next stage in our research is to employ the framework as a guide to develop our knowledge on how to foster students' statistical thinking in the school setting.

2. A Preliminary Case Study

At the end of last year preliminary steps were undertaken in a school to implement a statistical thinking focus into a Year 11 (15–16 year olds) teaching programme. The constraints on such an implementation are: the statistics unit of the mathematics course is limited to a four-week teaching block including assessment; students only have access to basic calculators; and the unit is an internally assessed component towards a national qualification. The research project is operated on the principle that the development of the unit will be a collaborative process between teachers and the researcher. The development of statistical thinking will be about making changes and transformations in the classroom based on the notion that there should be an evolutionary development of a living system. Teachers can visualise how the current situation might be changed, identify problematic situations, and explore alternatives to create an imagined situation (Skovsmose & Borba, 2000). The imagined situation can eventually reside in a teaching unit. To move from the current situation the teachers must implement the teaching unit, the arranged situation, upon which, through observations and experiences, critical reflections are produced on the teaching and learning process.

The negotiation process with the teachers in the school determined that: the researcher could evaluate the current statistics unit taught by having access to the resource and assessment material used; the teachers would participate in a workshop that would endeavour to communicate through teaching activities what statistical thinking was; two teachers would be interviewed about the current situation; and a new teaching unit would be developed collaboratively.

The documentation (D) of the statistics unit and its assessment provided the researcher with insights into specific content that was taught and how it was taught. Armed with this knowledge and conversations with the teacher in charge, a workshop was developed that addressed and challenged some of the broad tenets of their curriculum design, but with the focus on enhancing statistical thinking. Research informed all of the activities given and a data set which they had all used in class with their students was included. Observations (O) of what had occurred in the three hour workshop were written up. The one hour interviews with the two teachers, an experienced (ET) and a beginning teacher (BT), were transcribed. From this documentation the main current barriers to the development of statistical thinking were identified. These will now be discussed in terms of some of the elements of the framework.

Dispositions. Promoting a disposition in students towards statistical thinking was identified by the teachers as a key area of concern. They referred to students' lack of engagement in the class activities, and of a propensity not to seek a deeper understanding of the data. They also recognised a lack of success in igniting and fostering students' imagination.

> [there is] the whole issue of engaging with the data I mean do they really care about the muscles of the shells you know, it's that context that is lacking so it's really hard for them to understand why they might look at shell lengths (ET)
>
> A lot of the stuff we did wasn't really meaningful for the students … sometimes if the data is not that interesting, not relevant, I get a little put off … there is so much data out there that we could do … something that they could connect with. (BT)

Apart from indicating that students were only superficially interacting with the data, they realised that by coaching students for a particular assessment that they were limiting students' opportunities to explore data in different ways.

Empirical enquiry cycle. Moreover, the assessment required students to undergo all but the beginning stages of the statistical enquiry process, from posing a question of given data, analysing the data-set, to drawing conclusions, albeit with question prompts throughout the process. The students tended to do each question in isolation and failed to link the stages of the enquiry cycle together.

> The biggest failing maybe was that they would forget to go back to the very beginning of the question and remember what they were doing. They just kind of looked at the data and just talked about the data instead of going back to the question. (BT)
>
> You know each of the component parts seems to be quite straight forward but then when you try and paint a big picture or get them to do the whole process it's hard so they can't. (ET)

They attributed this situation partly to the way in which they taught.

> Well I taught each part lesson by lesson. I think if we had an ongoing investigation that might actually show them that it is all connected, every bit is connected with the other bit rather than just fragments … (BT)
>
> If it's always taught in isolation then it's no wonder the students never link it to making decisions about data and the question that you are trying to answer … We did chunk it [the unit] up into little bite size pieces rather than really thinking about the whole overall process. (ET)

Integrating the contextual with the statistical. The focus on addressing skills associated with statistical investigations resulted in data being given to practice drawing graphs or posing questions. Lack of attention to contextual knowledge about the data and to the 'stories' contained in the data-sets limited opportunities for the students to interact with data, interpret and draw conclusions from the data, and question the data. When teaching students to pose questions from given data the teachers drew students' attention to the data without regard to the contextual aspects.

> When I gave them the mussel data (an activity they had used with their classes) they said they never considered the context - went straight to the data. Did not even draw what a mussel looked like on the board! (O).

Transnumeration. When students were given data teachers focussed on posing narrowly framed statistical questions according to a template (D) and choosing one correct graph to analyse the data.

> This idea of posing a question, it is hard … we have a particular understanding of the word question like we know when we are doing stats topics and we are talking about a particular type of question maybe students haven't yet formed that understanding of what a question is, how you can ask a question in a set of data. (ET)

Drawing multiple graphs was not a feature of their teaching. When questioned about an activity where multiple plots were given connections between the plots were not made.

> I think I did a really limited job of that especially looking at these graphs now because we didn't explore the different types of graphs. (ET)

Reasoning with models. The teachers felt that their students were able to read information from graphs. They identified that students found it difficult to argue with data and justify statements they made based on the data. The students could think of contextual aspects to argue a specific position but this was largely unrelated to the data-set. On further investigation of one particular assessment item the teachers were referring to, it seemed that the students were not looking within the data-set and analysis to work out how the data had been manipulated. Again the teachers reflected that in their teaching they had not placed much emphasis on this aspect, rather they had spent time on the mechanics of graphing rather than the interpretation. Furthermore, they tended to use only univariate or bivariate datasets rather than multivariate datasets.

Variation. The messiness and variability of data led to students, according to both teachers, being frustrated that there were no rules to follow. This was exemplified in the problems students were having in choosing scales for their axes as this depended upon the nature of the data-set, and in deciding whether to use the mean or median, or how to group data. A misconception identified by the experienced teacher was that some students interpreted box and whisker plots as if they were histograms.

> Say they are comparing two results, maths and English and the English results were very very spread, they will be saying things like oh English got much higher marks just because of the fact that the box is really big and the box is spread but they are trying to apply the reasoning that they would use for a histogram or bar graph onto a box and whisker. (ET)

The teacher felt she had addressed this misconception in class but "it's not really understood, they've not learnt relationally". Scatterplots resulted in queries by students about whether to join the points or not, and the nature of the trend and ascertainment of outliers were also problematic (D). In the class activities where multiple graphs or data-sets were given, students' attention was not drawn to ideas about comparing variability between data-sets nor seeing variation represented in a variety of ways.

Recognition of the need for data. Students had difficulty in arguing with data and tended to argue from their own experience or own contextual knowledge base. Statistical thinking, however, requires students to acknowledge that conclusions must be drawn from carefully analysed data. This was another area that needed to be addressed.

Other. After the workshop the teachers were all generally worried about their own statistical thinking and that they either were having trouble thinking that way or needed to improve their thinking. There was a feeling amongst the interviewees that computer technology needed to be introduced into the statistics unit.

3. Practical Implications of the Framework

There are three main practical implications for this framework. First, the teachers and the researcher need to come to a common consensus of what they mean by the term statistical thinking and thus be able to communicate. Second, the teachers need to reflect critically on their current teaching and identify areas which are acting as barriers to the development of their students' statistical thinking. Third, the constraints that are imposed externally on teaching need to be recognised and acknowledged. The identification of the current main barriers to statistical thinking led to the two teachers and the researcher designing a new statistics teaching unit. New activities were chosen on the basis that they would be interesting to the students and would allow sufficient exploration of the data within an empirical enquiry process. The next part of the project will investigate how one teacher facilitates statistical thinking in her classroom and how her students develop their statistical thinking. The research project is envisaged as a three year action-research process which will gradually enhance and promote statistical thinking.

Starting with a theoretical framework on statistical thinking has led to a promising overview of the current situation in a school. The test, however, will be the implementation in the classroom and whether the teacher can start to enculturate her students into a statistical way of thinking.

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RÉSUMÉ

On présente un aperçu du schéma à quatre dimensions de Wild et Pfannkuch (1999) pour la réflexion statistique en recherche empirique, ainsi que les implications théoriques de ce modèle. On aborde également une étude de cas préliminaire s'inspirant du schéma. Cette étude de cas a été menée dans une école qui compte mettre en pratique la réflexion statistique dans son programme d'enseignement de 11^{e} année.