Asymmetric Fairness and Unfairness: Reinventing Distribution with a Computer Game

Efi Paparistodemou
University of Cyprus
Department of Education
5 Ellados street
2003 Nicosia
Cyprus
e.paparistodemou@ucy.ac.cy

Richard Noss
Institute of Education, University of London
Department of Mathematics, Science and Technology
20 Bedford Way
London WC1H OAL
r.noss@ioe.ac.uk

Abstract

This paper focuses on how young children constructed the idea of distribution in a computer game. The game offered children the opportunity to manipulate sample space and distribution in order to solve problematic situations of fairness and unfairness. The paper describes children's expressions and constructions of asymmetric fairness and unfairness. We present some case studies, which illustrate how the game mediated children's probabilistic understandings.

Introduction

In the field of learning probability, there appears to be a gap between intuitions and the formal knowledge of the concept of probability. The research of understanding probability has many studies on conceptions of probability, most of them focusing on misconceptions (for example, Kahneman & Tversky, 1982). However, it seems that little attention has been paid to the tools that learners have available for expressing themselves. Our starting point is that by designing tools that are specially used for expressing randomness and chance, learners may do things that cannot be predicted by misconceptions or by stages of thinking (see Piaget & Inhelder, 1975). Expressiveness in our work means the way one can express ideas in a concrete form while actions are carried out by interacting with a tool (diSessa, 2000). The process cannot be separated from the product, and that brings us to the idea of webbing (see Noss & Hoyles, 1996), connecting knowledge to become familiar with more abstract ideas.

The aim of this paper is to describe how the game mediated children to construct fair and unfair situations. It illustrates the idea of asymmetric fairness and unfairness, two categories that children employed for their constructions to reinvent the idea of distribution.

The game

The game was designed to afford children the opportunity to talk and think about probability and was built to allow a connection between both local and global events. Children could make changes in a 'lottery machine' in which a small white ball bounces and collides continually with a set of blue and red ball (blue balls appear in the figures as the light grey balls, see for example

Figure 1). Children could change and manipulate a number of aspects to construct their own lottery machine: the number, the size, and/or the position of the balls. This lottery machine controlled the movement of a space kid, which represented the global events in the game. A more detailed description of the game can be found in Paparistodemou and Noss (2003). The case studies presented in the paper are part of a broader study, which is describing children's expressions of randomness. The children were interviewed during their interaction with the game. Each children worked with the game individually for a duration of between 2 to 3 hours.

Reinventing distribution

The children expressed ideas of distribution while they were facing with problematic situations, of constructing a fair or an unfair probabilistic environment. Their manipulations in the lottery machine let them change the probability of an event in it.

For example, Simon (7 10/12 year-old boy) described his construction as having 'a sabotage'. This factor made Simon and other children to try to construct a fair environment with different number or/and size of balls as is described in the next category. These examples showed how children made changes on distribution in order to get the same probability for the two events. Simon made a fair environment by placing the balls around and thinking about which balls were sabotaging to the others.

Simon: I think we are going to get equal numbers. Let's see… (see Figure 1)

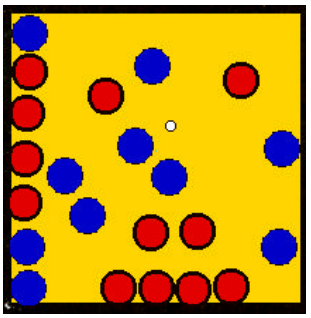

Figure 1: Simon's mixed up fair construction

He starts the game.

S: Oh….look they got equal numbers…

Researcher: I don't understand why. The reds are more than the blues.

S: **It doesn't matter how many they are**. What it does matter is the shape. One ball might sabotage another one.

Because of the representation of sample space Simon developed a strategy where the number and size of the balls was not a strict characteristic of a fair environment, but the place of the balls played the major role. So, he constructed a sample space with 12 reds and 10 blues, with all balls having the same size, and placed them in a way that made him expect that the two colours would get equal points.

Concerning unfairness children also made constructions where they changed the probability of each ball to be hit in their lottery machine in order to make the one or the other team to win. The space that each ball occupied in the sample space was a criterion for children to judge an unfair environment. Brian (6 6/12 year-old boy) explained in the following arrangement:

Brian: The ball will get more points because it's bigger. It will get more points. It has much bigger place and this is very small and if it goes like this it will touch it and like this it won't touch it. It can get more points faster than the red one, because it's bigger. If it goes like this and like this it will get more points (see Figure 2).

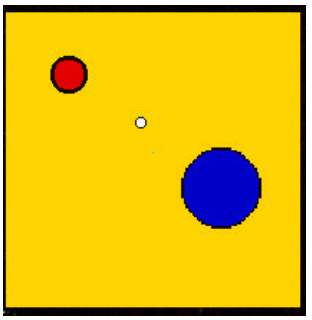

Figure 2: Brian's unfair construction

So, in an unfair environment the winning ball is bigger than the other, as it takes more space and it gets the points faster than the other ball. Anne (6 10/12 year-old girl) also described the importance of having bigger balls inside the sample space in the following snapshot.

Anne: The blue balls are bigger. (She laughs…)

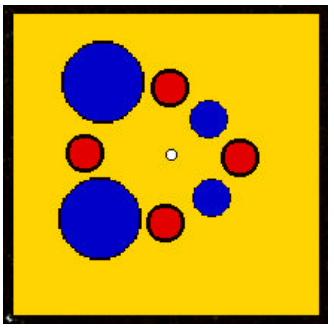

Figure 3: Anne's unfair construction

Researcher: Is it going to be fair now?
A: I don't think so…It's not!
R: Why? I have four blue balls and four red (see Figure 3).
A: Because these two blue balls are bigger and they take much more space. The ball can hit them more because they have more space.
R: What do you mean by 'more space'?
A: It means is more full. So, the ball will touch mostly those ones.
R: What does it mean if you have more space in the lottery machine?
A: It is like cheating…

In Anne's situation there were an equal number of blue and red balls, but the two blue ones take more space around the circle. This made it very obvious for Anne to describe this sample space as unfair and that the blue balls are going to win. As she said, when the balls take more room in the sample space the result is like cheating. She also described the unfair environment as a 'cheating' one when she made a spatial arrangement of the same size and number of balls. Although Anne placed the same number and size of the balls, she believed that the way she arranged the balls inside the sample space was an unfair representation. She seemed to realise that when you unbalance the distribution, you unbalance the outcome as well.

Discussion

The construction of fairness and unfairness in the game made children build for themselves representations where they had to engage with the idea of distribution. By constructing fairness children reinvented the idea of equiprobable distribution and by constructing unfairness children

realised that by unbalancing the probability of each event you can increase or decrease the likelihood of an event to occur. The lottery machine of the game encouraged some children to base their construction of unfairness, and changing the probability of an event, not only by changing the quantity of that event in the sample space, but also by changing the distribution. The children implicitly used the idea of distribution in order to change the likelihood of an event occurring. It can be argued that in children's construction of unfairness there is a situated abstraction of distribution (as situated abstractions are defined by Noss, Healy and Hoyles, 1997).

These expressions diverge from Piaget and Inhelder's (1975) argument, which finds that at this age there is an absence of the concept of distribution of the whole. As they claim the problem of random mixture brings up the problem of the forms of distributions and that the final positions of the elements in the mix necessarily take on certain distributive forms of the whole. On the contrary this study shows that children in the game had a need to express distribution in their constructions. They seem to have achieved this because they had a tool that allowed them to do so.

REFERENCES:

diSessa, A. (2000). *Changing Minds: Computers, learning and literacy* M.I.T. Press

Kahneman, D., Slovic, P. & Tversky, A. (1982). *Judgement Under Uncertainty: Heuristics and Biases* Cambridge: Cambridge University Press

Noss, R. & Hoyles, C. (1996). *Windows on Mathematical Meanings: Learning Cultures and Computers* Dordrecht: Kluwer

Paparistodemou, E. & Noss, R. (2003) 'Designing a task for expressing randomness' *6th International Conference on Computer Based Learning in Science* (in press)

Piaget, J. & Inhelder, B. (1975). *The Origin of the Idea of Chance in Children.* London: Routledge & Kegan Paul.