Teaching Probability via its History: Reflections on a Case Study

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Abstract

More than 20 years ago, I was moved to switch the teaching of my third year probability course (bits and pieces more or less Feller Vol 1 level) onto a historical basis. The experiment continued for three years and was then discontinued.In this talk I want to reflect on this experience. Why was it started? Why was it stopped? Can one draw any morals from the experience? It remains in my mind as one of the more genuine innovations that I attempted as a teacher, but ultimately not one that I wanted to continue.

1 Introduction

I should be embarrassed to give this talk, for I am an arm-chair historian, with no pretensions to doing original work in that field, and no great pretensions as a teacher either. However, over a long period teaching at all levels at a small university, this is one course that I do remember with pleasure, and reflect on with interest, even though it remained in existence only for a short period. It helped me to try out some of the things that I had come to feel about teaching probability, it helped me to understand what some of the early writers on probability had actually done and how they did it, and it helped me to collect material that later found a place in the introductory chapter of the text on point processes that Daryl Daley and I started working on soon afterwards. So it was a valuable experience for me at least, and one that I am happy to share with this session if it fits the session topic. In the end, however, it leaves me with no clear moral to offer, no strong convictions about the role of history in teaching probability and statistics, more questions than solutions.

I propose to first outline the course, then examine some of the issues surrounding its inception and demise.

2 Course summary

Full year course, 24 weeks, 2 lectures + tutorial per week.

1st trimester (Probability).

- Early history: up to Huyghens
- James Bernoulli and LLN
- de Moivre and CLT
- Bayes and his theorem
- Role of the normal distribution:Laplace, Gauss, Ellis
- geometrical probabilities

2nd trimester (Stochastic processes)

- Machinery for discrete distributions; Poisson, compound Poison, negative binomial
- Poisson process (discovery and properties)
- Branching processes (Galton, Watson and others)

- Markov chains (matrix case)
- Renewal process
- Life insurance and collective risk

3rd trimester *either*

- Elementary time series, *or*
- Introduction to measure and probability

This course was given in the late 1970's, under the old regime, where courses lasted the full academic year, about 24 weeks of real teaching time. It was one of two third year courses in probability and statistics that ran in parallel, at two lectures a week and one tutorial each. It was still then a Department of Mathematics, and the students taking the course were majoring in mathematics. At that time a good third year student might take from four to six courses of a similar length, generally including one on applied analysis, and another on applied algebra. Student numbers ranged from 12 to 20 or so (but are now much less).

3 Motivation

First I should emphasize that this was not a course on the history of probability, but the regular third year course on probability, envisaged broadly as covering selected material from Feller Vol 1. It was preceded by a second year introduction to probability and statistics, and ran in parallel to a similar-length course on statistics. The courses were part of the general smorgasbord laid out in front of what we hoped might be future mathematicians (in some weak sense). Several other staff at that time had an interest the history of mathematics, but none of us contemplated a special paper on that subject. The small staff and student numbers, and the obligation that we felt to introduce students to a full range of topics in modern mathematics, meant that we saw our primary task as being to make accessible some of the key concepts in the chosen field of the course. In statistics, our task was to expose the mathematical ideas behind the statistical techniques, not so much to teach practical competency in the techniques themselves. Or at least that is how I saw our role at the time.

From this point of view, the historical approach seemed to offer a number of advantages. First of all, I had discovered by then that it is sometimes much easier to understand a mathematical idea by seeing how it first struggled into existence historically, than by following a systematic, logical development. So my scheme was to break the Feller material into topic headings, outline for each the beginning steps in its history, and then sketch in its later development, trying to illuminate its role

or significance. In this way I hoped that I could convey both the essence of the ideas, and some inkling of the role they played, with a minimum of formal development.

A second advantage was the opportunity that I felt it might give of introducing students to some texts that they might actually enjoy. Modern undergraduate texts, designed for efficient learning through mnemonic aids such as displayed summaries, coloured print, or the like, may succeed in their purpose, but with a few notable exceptions they are little fun to read. I had been beguiled by the elegance of style of the writings of some of the famous mathematicians of the immediate past (Hardy, Khinchin, Feller, Titchmarsh) as well as by autobiographical or biographical fragments from some of the older figures. One of my favourites was the account reproduced in the Dover edition of 'Doctrine of Chances' of de Moivre's all too human frustration at being required by his tutoring work (all he could obtain) to spend his time
"...teaching my pupils and walking. But since the city (i.e. London) is very large, a considerable part of my time is employed solely in walking. That is what reduces the amount of profit I can make, and cuts into my leave for study.."
[What he would have made of traffic jams, one hates to think. My unsympathetic view is that the walking strengthened his constitution and gave him excellent opportunities for reflection on mathematical themes!] In the same way, many of the historical writers, beguiled by their subjects, write about them entertainingly and with enthusiasm. Better material to give on-coming mathematicians, I felt, at least for once, than the next expository text, however efficiently written.

Thirdly, I was aware of the criticism directed not so uncommonly at mathematical graduates that, whatever skills they might have in doing mathematics, their skills at expressing themselves in English were barely rudimentary, a trait we were sometimes accused of unwittingly encouraging by being too lenient in the final examinations, kindly trying to drag up an extra mark or two from a script so illegible and difficult to comprehend that just conceivably a glimmer of mathematical understanding might lie hidden beneath the inadequacies of the exposition. The historical approach called for essay-writing. However unwelcome to the students, at least they would have one chance to show literary skill if they had it, or to tackle the basic problem of saying something in words if they did not.

Finally, I have to confess, I was keen to try out a new approach and see what came of it.

4 Results

The first year that I embarked upon this project, as I recall, it was only for the first term of the course (around 16 lectures), which was sufficient to deal only with historical developments to around the time of Laplace and Gauss. But this was not a serious limitation, for many of the great themes of statistical thinking had emerged

by the 1830's: the law of large numbers and its elusive inverse; Bayesian arguments; the central limit theorem and the normal distribution.

In preparing the lectures, I found that I had underestimated the amount of time it would take to sort out and order the material that I wanted to include, including glimpses into future developments. The result was that when the time for the lecture came, I found that I lectured with enthusiasm but often transgressed the schedule that I had set myself. In particular, I recall serious delays in developing the weekly assignment sheets, which tended to reflect real issues, requiring more thought and research than the students were able to give it, so that in the tutorials we would still be working on topic A when we had already advanced in the lectures to topic C.

Nevertheless, I was encouraged by comments from one or two older students, and staff who were sitting in on the course, who told me that they enjoyed it, and was not too discouraged at the lack of imagination shown by the many students who chose to model their essays around the biographical accounts of famous mathematicians in certain well-recognized encyclopaedias.

In the second year I expanded the programme to bring in the beginnings of stochastic processes (elusive, because the beginnings are surprisingly early: many problems in straight probability developed out of attempts to model processes), streamlined the timing and organisation of the course, and settled into a more consistent assessment format: weekly problem sheets differing not greatly in content from those offered when the course was being taught more conventionally; examination questions which for the most part touched on standard theorems; and the inevitable essay question, usually centred around the life and contributions of a particular scholar, occasionally around the development of a particular idea. The teaching ran more smoothly, many but not all of the earlier bumps and mistimings disappeared, and students went through, but less memorably it seems.

In the third year I was away in the first half of the year, and the course was taken up by a volunteer replacement; by the time I came back it was hard to rekindle my earlier enthusiasm, new teaching requirements had arisen, and the year following I had different committments. The result was that the course never reappeared in historical garb.

Despite its short period of existence, however, ingredients from this course coloured my teaching ever afterwards. The knowledge that I had been forced to acquire about the contributions of the early statisticians, and the historical development of the subject, proved an enormously valuable resource. It increased the confidence I felt in my grasp of the subject, empowering me to drop in a comment about a favourite incident, such as de Moivre's concern at the time he was forced to spend walking the streets of London, whenever an opportune momented presented itself. In developing new topics also - for example actuarial mathematics - it became a pleasure to try and dig up their sources and see how they fitted into other strands of the historical picture. In the teaching programme I continued to inflict an essay

on the students who ventured into my course, finding that more of the students than I had expected seemed willing enough, at least for once, to try their hand at an essay in exchange for yet another set of examples or computing exercises.

5 Termination

I have often tried to pinpoint the reasons why I did not continue or return to this historical style of teaching, despite a satisfactory outcome and even some requests to do so. One reason undoubtedly was the pressure of conformity; if other courses had been taught in a similar style, perhaps I would have continued also. But I was not confident that the approach was successful, or that I was doing it well, and felt uncomfortable at continuing with such a divergent approach in a small department.

A related reason was the unease I began to feel for the very subjective selection and organization of material. The choices of content and emphasis were arbitrary decisions, based on my personal views, unsanctioned by any shared view of syllabus content. If I chose to dwell on some characteristic feature of the normal distribution, because of my perception of its historical relevance, students were stuck with that; they missed out on a more systematic account. It might be quite fun at the time, but it might not be so useful later. Instead of giving them a rounded view of the development of the subject, which had been my goal, I was giving them an arbitrary, idiosyncratic selection of topics, possibly at the expense of material generally considered basic for the subject.

At the same time I began to be aware that the approach was more demanding on students than I has initially imagined. In order to appreciate the historical account, the student really had to know something about the topic to start with. If the topics were encountered for the first time, and with an exam at the end, the historical element became just one more dimension that had to be understood. The student was required to master, not just the concept itself, but a whole lot of material about the historical context and the evolution of mathematical concepts. He or she might have neither the mathematical background nor the historical knowledge to benefit from the historical viewpoint.

In this respect, the situation of a regular undergraduate student was very different from that of a staff member or graduate student who sat in on the course for enrichment or pleasure. For the latter it was truly enrichment material, that the individual could take or leave as they wished, enjoying some parts but discarding what they did not find interesting or relevant. But for the former, particularly for a weaker student, the history might appear as no more than an additional obstacle to understanding the key ideas of the subject. The course provided little opportunity for practice in handling particular methods or procedures, or in reinforcing their understanding of the principal logical arguments. I attempted to redress this deficiency through example sheets, but this depended for its success on the students'

ability and willingness to work on the examples. Again it was a task more suited to the handful of the most able students than the students at the middle or lower end of the class. Finally, the essay question posed a severe and perhaps unfair handicap on the Asian students who were often leaders in the mathematical work. I was reluctant to accept this argument as a reason for changing the assessment, but it was an element in the overall perception of the course.

Such reservations came together at their strongest in trying to put together an examination for the course. On the historical side, one could at best expect the students to regurgitate the limited material they had been told. On the mathematical side, one was equally restricted to the limited, non-systematic material that had been broached as part of the historical exposition, giving that material an apparent emphasis that it might not deserve.

6 Conclusions

I find it difficult to draw any clear conclusions form this experience. At bottom, I feel that there should be value in undertaking such a historical course, even when it has to form part of the main programme. The lecturer will undoubtedly benefit from the exercise, and colleagues are also likely to find it stimulating, at least on an occasional basis. What the students will make of it is likely to be more varied. Graduate students and the best undergraduate students may gain something hard to find in more conventional courses, but for weaker students it may even prove counterproductive, depriving them of the familiar crutch of well-ordered theorems and standard examples, and, should they continue, serving them worse than a more conventional course directed more specifically towards their needs.

As a continuing element of the undergraduate programme, a course of this kind is always likely to be difficult to justify on pragmatic grounds; it may be too costly in time and effort to justify a permanent place on the syllabus. But in a department which offers two or three courses at three or four levels, one would like to think that there was enough room for occasional experiments of this kind, to leaven the teaching loaf, and to encourage both staff and students to maintain a feeling for the historical picture.

At the the heart of my uncertainty lies a question of values. The history of a subject surely has a clear place in the eyes of anyone who values the subject in its own right, as a creative, scholarly or even professional discipline. Entering into the history of the subject, trying to understand the motivations and achievements of its outstanding representatives, is one of the surest ways of revealing a field's underlying culture. But if the subject is seen as having little intrinsic value, then any detailed study of its history becomes a labour without purpose. It takes courage and commitment, more than I possessed at the time, to maintain a course centred on what are essentially scholarly matters, in an environment which seems ambiguous about

the values to which it is committed, and uncertain in particular to its commitment to scholarship.

So, as I review the issue at the present time, the difficulties seem less a question of course content or teaching technique than of the underlying goals of the instruction. Statistics runs the danger of being seen largely as a service subject, by which I mean here a subject valued only for the immediate usefulness of the techniques it imparts. If the courses are valued only in this way, or even if they reflect only the immediate goals and preoccupations of the majority of their students, then it may be a profitless task to expend time on cultural and historical aspects of the subject. My own belief, however, is that the goals should be broader, and that giving students some feeling for the subject's interior culture and values can be just as important to their overall grasp of the subject as the mastery of any particular technique. If this view is accepted, then it is surely worth expending some teaching time on historical aspects, and addressing more detailed issues such as how best to introduce historical aspects within a department's overall offering.