Symbolic Computing Systems in Teaching Statistics in the Czech Republic and Poland

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1. Introduction

In the last decade, a possibility of teaching with the use of computers has increased at both elementary and secondary schools in the Czech Republic very rapidly. It obviously concerns also bases of statistics. Unfortunately, statistics and probability are not involved in the obligatory content in mathematics at secondary schools. In spite of it, some activities to teach students of secondary schools probability using computers have been born; see [1].

There exist three main reasons for teaching in more interesting way by modern methods at secondary schools, i.e.:

- better understanding principles of the discipline by students,
- finding the talented students, and
- support of talented students.

The aim is to make the discipline interesting for larger number of students. It gives possibility to discover hidden talents. This does not concern only statistics and probability. This problem is similar in other specializations, too. The support of talented students in chemistry in the Czech Republic can be mentioned as an example. This support gives talented students assumptions to be successful during their studies at universities, including in PhD programs being a good base for finding jobs and further career.

2. Teaching probability with MuPAD

Considering limited financial funds which can be invested to the improvement of teaching, mainly cheap software systems are used at secondary schools. In the textbook [8], teaching probability with the use of the MuPAD system is presented. MuPAD is a symbolic computing system which can be used also for numeric computing. In the textbook, mainly numeric computing is shown. However, students use own programs for these purposes. By creating a program, they express their requirements in a symbolic way. They learn programming language close to Pascal and probability together. The most examples use random number generation. Students obtain figures (or graphs) as results. However, each student has own generated values and own result and has to interpret his (her) own task. In some cases, the number of trials is a parameter and so a student has a tool for solving different problems obtained by changing this parameter.

The textbook has eight parts. They consist of the introduction to MuPAD, simulations, probability, calculations with probabilities, independent trials, conditional probability, random variables and random walks. In the first chapter, the principles of programming are explained. However, the book is mainly a textbook on probability. It includes theory needed for learning probability, a lot of examples and their solutions. The MuPAD system is applied for the illustration of some parts. For example, students can compare results based on a generation of a lot of random values and results obtained by calculations according to the formulas. Relative frequency of the occurrence of the certain event can be compared with the theoretical value of corresponding probability.

The book covers only discrete distributions (binomial, geometric) and characterization of them by a mean and variance; continuous distributions are not included. It is a textbook for such secondary schools in

which students learn the basics of the probability theory. It is a suitable tool for better understanding its principles. It can be also used for motivation of talented student because examples from Mathematical Olympiads are included. Since the probability theory is not involved to the obligatory content in mathematics at secondary school in the Czech Republic (and it cannot be involved to Mathematical Olympiads), there are examples from Mathematical Olympiads from Czechoslovakia, Hungary, Russia and Poland.

However, for talented students which attend mathematically oriented secondary school, a range of the theory and illustrative examples could be broadened by continuous distributions. In these specialized schools, students learn to differentiate and integrate. So, computations needed in the case of continuous distributions can be included in teaching of the probability theory. In this case, further possibilities of the symbolic computing system could be used. The advantage of such software systems is a possibility of combining symbolic and numeric approaches.

The probability theory is taught both in some secondary schools and also at universities, usually as a part of courses of statistics. So, a proposed way of teaching of probability theory can be applied also at universities.

3. The use of MuPAD in teaching statistics in Poland

According to the Internet resources it seems that the MuPAD system is expanding in Poland. Since 2002, MuPAD workshops and MathPAD conferences on using MuPAD have been held in Poland; see [5, 6, 7]. In [5], the MuPAD system is used for describing the data file both by figures and graphs. The examples on computing median, modus, mean, variance and standard deviation, and plotting frequency graph and boxplot are included. It means that numeric calculations and plotting are involved but computation of mean and standard deviation is explained by means of programming, i.e. in a symbolic way.

In [7], plotting graphs is explained. It includes bar and pie charts, histogram and boxplot. As concerns of histogram, input data are generated by simulation of rolling three dices. Each value represents the sum of number of pips on these three dices. Moreover in [6], graphical representation of time series is described.

4. Teaching statistics with Derive

Beside of MuPAD, the Derive system, see [12], is used in the teaching in the Czech Republic. This software system is recommended for teaching at elementary and secondary schools by the Ministry of Education, Youth and Sports of the Czech Republic. It is also used at some universities. For example, a lot of tools for teaching with this system have been prepared at the Faculty of Management in Jindřichův Hradec (University of Economics), e.g. [2, 3, 4]. Derive is used in teaching of economy, mathematics, econometrics, and, obviously, in probability and statistics.

In [4], following examples are solved in the part on statistics: computation of the parameter value in the probability density function (integral of this function equals one), calculations of probability density function from the cumulative density function and vice versa, computation of modus, mean, variance and standard deviation of the continuous random variable, and the example of probability computation. These examples could be used in teaching at secondary schools, too. In some examples, numeric computation is a main aim. However, for its achievement, it is necessary to transform expressions, integrate and differentiate, and so symbolic computing is applied as well.

At other universities and at elementary and secondary schools, Derive is used mainly in teaching mathematics; see [9, 10, 11]. Concerning the probability theory, the parts of textbooks concerning combinatory analysis can be used. This problem is included, e.g., in [9].

In the same way, other symbolic computing systems, as Maple or Mathematica, are available to students at some universities in the Czech Republic, can be used in teaching of probability theory. In fact, they are used mainly in mathematics.

In Slovakia, the system Derive is also promoted. In 2004, the workshop Mathematics with Derive 6 was held as a part of the INFOVEK 2004 conference on techniques in teaching at elementary and secondary

schools. The Ministry of Education of the Slovak Republic was a sponsor of this conference. In the annotation of the workshop, the topics of probability and statistics are mentioned, see [14].

5. Conclusion

In [8], teaching probability at secondary schools using MuPAD is proposed. However, only discrete distributions are involved in this textbook. It would be suitable to broaden teaching of talented students of secondary schools by introducing continuous distributions as well. On the other hand, to teach students at universities by means of symbolic computing systems is also important. In some universities, statistics is scheduled for the first year of studies. Teaching probability theory in more interesting way using computers helps to find the talented students and to support them further.

Some algorithms described in the textbook [8] on MuPAD can be realized by means of tools designed for web pages management. In 2000 and 2001 we created the interactive textbook of statistics – IASTAT, see [13]. As concerns probability theory, for selected discrete distributions (binomial, Poisson, hypergeometric), probabilities, mean, variance and standard deviation can be computed on the basis of specified parameters. Rolling a die is also applied but it is involved in chi-square goodness-of-fit test.

In the Czech Republic and Poland, several papers, textbooks and web pages concerning using symbolic computing system were prepared. On the Internet, documents on Derive (in the Czech Republic) and MuPAD (in Poland) are available.

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