STATISTICAL EDUCATION AT SCHOOL LEVEL

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TEACHING STATISTICS AT SCHOOL LEVEL IN SOME EUROPEAN COUNTRIES

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1. Introduction

In 1977, the International Statistical Institute set up a task force on the teaching of statistics at the school level (TOTSAS) under the chairmanship of Professor Vic Barnett. Amongst other things, this committee published a book entitled "Teaching Statistics in Schools throughout the World". The section on Europe included reports on England and Wales, Scotland, the Federal Republic of Germany, France, Hungary, Italy and Sweden. This paper reports on some of the changes in all of these countries except Hungary, and also includes a reference to the current position in Spain.

The original survey showed a wide range of approaches to the teaching of statistics between the countries. There was disagreement on the amount of statistics, the nature of the appropriate material, the age at which it should be introduced, and the balance between probability and statistics.

Barnett reported on the situation in England and Wales. In the primary schools there were no national guidelines and, although some good work on data collection and pictorial presentation could be found in some schools, in others there was very little. For pupils aged 11-16 there was little statistics in the standard mathematics course but there were optional courses in probability and statistics that had quite full syllabuses. He reported on a major curriculum development project, the Schools Council Project on Statistical Education, which produced teaching material with the aim of introducing statistics for citizenship, stressing the practical relevance of statistics by introducing concepts and techniques almost entirely through the medium of detailed real-life problems, chosen from a wide range of application areas. Barnett also reported on the setting up of a Centre for Statistical Education at the University of Sheffield.

Dinges, from Germany, reported that there was no uniform picture. He noted that the views of what stochastics should be differed between states, and that states varied from those with centrally controlled written examinations to those where individual schools had more freedom. Stochastics was acknowledged in both basic and more intense courses within mathematics. Although probability could vary from the intuitive

to Kolmogorov's axioms, the tendency was to formalise probability to events in the sample space and formal manipulation rules. He expressed a desire to see a change to the position where applications should not merely be seen as illustrations to enhance motivation.

From France, Hennequin reported that there was no work on probability for pupils younger than 14. He noted that teaching statistics remained formal because of lack of time and detected an over-emphasis on formal probability, involving questions on balls in urns. No reference was made to any statistical work in primary schools.

In Italy, Zuliani noted that the syllabus for 11-13 year olds included *ordering and correlating data* and *drawing attention to the differences between certain and probable.* Of six theses in the mathematics syllabus, one concerned statistics. There was a more complex picture for the upper secondary schools (age 14+, courses up to 5 years long). Vocational schools included some stochastics, mostly probability theory and description. There was some simple descriptive statistics, calculating simple statistical measures. Again no reference was made to any work in primary schools.

Ekenstam reported that in Sweden courses for grades 4-6 (age 10-13) included the collection of statistical data, tables and graphs. The work for grades 7-9 included frequency tables, graphs, mean, median, some measure of spread and probability. He found directions that suggested grade 1-3 children should gather and report information in pictorial form.

That was the position at the end of the 1970's. How does it differ today?

2. Why do we teach statistics?

It is clear that there are two different traditions meeting here. What we choose to teach will depend on our philosophy and definition of statistics.

One tradition has a very broad view of statistics. It sees statistics as part of society, including uses made of data in all walks of life. Since, on this definition, it is a numerate subject, it is up to mathematics teachers to be responsible for this teaching - but they will not be the only teachers to be so involved. Statistics will also occur in the teaching of science, geography, economics, etc. This tradition is summed up in the following quotation from the British Cockcroft Report from (Cockcroft, 1981).

> §781. Statistical numeracy requires a feel for numbers, an appreciation of appropriate levels of accuracy, the making of sensible estimates, a common-sense approach to the use of data in supporting an argument, the awareness of the variety of interpretation of figures, and a judicious understanding of widely used

concepts such as means and percentages. All these are part of everyday living. Good statistics can encourage pupils to think in these ways.

This philosophy of statistics will mean that at least lip service is paid to the place of statistics in other areas of the curriculum. Again from the Cockcroft report:

> §776. Statistics is essentially a practical subject and its study should be based on the collection of data, wherever possible by the students themselves. It should consider the kinds of data it is appropriate to collect, the reasons for collecting the data and the problems of doing so, the ways in which the data may legitimately be manipulated and the kinds of inference which may be drawn. Work in subjects such as biological science, geography and economics can therefore contribute to the learning and understanding of statistics.

The non-statutory guidelines of the National Curriculum in Mathematics for England and Wales (Department of Education and Science, 1989) refer to having a mathematics policy for the whole school and using data from other subjects. In Spain, the Andalucian expansion of the Ministry of Science and Education syllabus refers to the place of probability and statistics in our world culture. In Italy, the *programma didattico* for *scuola elementare* stresses the use of mathematics in subjects to help scientific understanding of problems and a logical aproach to solutions. It also stresses the practical uses of mathematics - which by implication includes the sections on probability and statistics. The scheme for the upper secondary schools in Italy refers to the importance of statistics in science and real life in general.

The second tradition that can be detected behind the proposed syllabuses is that of mathematical statistics or stochastics. In Germany *stochastics* also incorporates applied probability, including risk analysis and decision making under uncertainty. In this tradition, statistics is seen as a subset of mathematics (including applied mathematics) and the main concern is with modelling uncertainty and developing theoretical models. There is a much greater emphasis on probability and distributions, and much less on data and data collection. This attitude can be seen behind goals such as those described by Nemetz (in Barnett, 1981, p. 97) for Hungary:

> a. Make the pupils acquainted with the notion of randomness. Make them able to view stochastically the notion of random events.
>
> d. Get the children acquainted with basic laws of probability theory and the main ideas of mathematical statistics.

e. Show them how to construct stochastic models in real life problems.
(goal *b* was on evaluating statistical data and the limitations of stochastic arguments, and goal *c* was on the stability of relative frequencies).

The ASU group from France (Ambroise *et al.*, 1991) refer to this tradition when they criticise the positioning of the phrase *organization et gestion des données* immediately next to *fonctions*. They also refer to textbooks where statistics is only an excuse for probability. Morin (1991) writes of the strong influence of mathematics and pure mathematicians.

At its worst this tradition can lead to sterile theoretical models and much work in pure mathematics, with no real thought about applications. At its best it can make pupils think deeply about the structure of a real life problem and see how to work to its solution using probability theory. At its best it can also sharpen up concepts in statistical inference.

Fortunately, there does seem to be some coming together of these two strands in the countries studied, although the actual balance reflects society and the history of mathematics teaching in each country. As an example, two successive aims in the Swedish syllabus are:

to develop some knowledge and skills in arithmetic, geometry and statistics and insights in the concepts of probability and functions, and

to develop basic statistical concepts and methods for collecting and handling data and using tables and diagrams to describe and compare important properties of statistical information.

Even so, fine statements such as these need to be matched against what occurs in the classroom. The reality can be very patchy, particularly where teachers are insecure in their own statistical knowledge and teaching ability. They may avoid teaching statistics if they can and, even where they do teach it, they may find it difficult to meet such high aims.

3. What statistics should we teach?

It is interesting to note that in many countries the word *statistics* did not convey what the course designers wanted it to include. In some countries it was associated with too narrow a view (as might be implied by the word *stochastics*). In others, it was associated too much with official statistics and the amassing of numerical information with no analysis or theoretical inference.

In the United Kingdom, in an attempt to solve this problem, the

content is put under the heading of *Handling Data* while French syllabuses refer to *Data Organisation.* In Germany, you may find some of this work under *Sachrechnun* (Calculating in a Context). Such phrases appear to have been chosen to emphasise an active approach to data and the idea of using real data to obtain meaningful insights and draw meaningful inferences. They are a good counterbalance to the tendency to teach techniques for their own sake, based on fictitious data.

There is a common core of techniques and concepts at the heart of the syllabuses from all the countries considered. These include summarising the data in pictorial form such as bar charts or histograms; using and calculating measures of central position such as the mean and the median; having some idea of variability, which may be measured by interquartile range or even standard deviation; and probability, as a measure of uncertainty and the basic rules of manipulating probabilities (at least for independent and for mutually exclusive events).

There is not a common pattern with reference to the inclusion of EDA techniques such as stem and leaf or box plots, nor to the idea of exploratory data analysis as such. Neither is there a common reference to sampling and the fundamental statistical idea of using a sample to draw inferences about a population. There is no common reference to the distinction between a survey and an experiment, nor to the fundamental idea of randomisation in experimental design. There is no common attempt to try to get pupils to appreciate the importance of statistics in society nor to think statistically when making decisions. Although not one of these things is common, elements of each occur in some of the syllabuses. Other courses include reference to the use of computers, databases, spreadsheets and statistical packages.

The reports from France seem to be rather pessimistic. The ASU, after noting that primary pupils could collect data, put them into tables, and give graphical representations, write that on the face of it the situation seems good but that the reality is different. Apparently, most teachers do not teach that part of the programme. Morin (1991) reports that teachers are supposed to deal with data organisation in the last year of primary school, but whether this is actually done depends on the individual teacher's attitude to statistics. The BEPC does include a course in basic statistics for all pupils aged 11 to 16, but it is estimated that only one third of teachers teach statistics inside the mathematics classes. In a personal communication, Morin reports that the BEPC test in *Geography* is full of statistics. Students must draw histograms, comment on statistical tables and handle conditional distributions. In short, in geography the students handle more statistics than in mathematics. Reports from Germany also indicate a growing use of computer supported data analysis

in subjects other than mathematics.

The approach to teaching probability varies immensely. To some extent it depends on the age at which the ideas are first introduced. In England and Wales the approach is through events that may or may not happen, with the idea of *more* or *less likely* being hardened up to a rank order that can be expressed on a 0 to 1 scale. *Equally likely outcomes* are encountered as a special case, and *relative frequencies* are introduced after equally likely events. The Italian primary school syllabus recommends that the probability should be introduced through real life and through games involving judgements about *perhaps, certain* or *possibly*. Later it recommends using games to learn probability. In other countries, the equally likely approach predominates and is much more closely linked with combinatorics. In Spain, the Andalucian expansion of the Ministry of Science and Education syllabus includes combinatorics and sample spaces leading to conditional probability.

4. When should we teach probability and statistics?

On this point, there is a wide range of views. Most states in Germany include some work in probability and statistics in mathematics before the age of 16 (although there are some exceptions where none is included). All states include stochastics for grades 11 to 13 (the next age goup). The most common practice is for some such work to be included at least in the syllabuses for pupils aged 10+. For some countries the start is made much earlier.

In England and Wales (Department of Education and Science, 1991), work on sorting (leading to data representation) starts at the age of 5. Subjective use of probability words such as *uncertain, possibly* etc. leading to ranking probabilities begins soon afterwards. In Scotland (Scottish Office Education Department, 1991), data representation begins in the same way, but there is no work on probability until after the age of 14. Although France allows such work in primary schools, apparently not as much work happens as our French colleagues would like. A more positive start is made at the secondary level. In Spain (Godino, 1992), primary school pupils are taught to use elementary techniques of data recognition to obtain information, and to represent it in graphical and numerical form. Some elementary probability is also introduced. States in Germany tend to start at the secondary level. Sweden starts in the early years of the primary school, so that at the end of five year's schooling Swedish pupils should be able to interpret data in tables or diagrams and to use the concepts of median and mean. Italy has a detailed programme for

primary schools including probability and statistics. These pupils are expected to be able to recognise data sufficiency and see contradictions in data.

Wherever recommendations are made about how statistics should be taught they are always along the lines of doing practical work, using real data, and teaching in real life contexts.

5. Other considerations

5a. *Central Direction*

There is a wide range in the amount of central direction of the teaching content. At one extreme is Germany where the decisions are left to the individual states, and even then there may be only outlines which schools or individual teachers have to interpret. At the other extreme is the United Kingdom. From being one of the most diverse systems at the time Barnett was writing in 1982, it has now become the most centralised of all the countries being considered here. The National Curriculum in Mathematics for England and Wales (there is a separate but similar document for Scotland) applies in detail to all the schools in the country. It gives a highly detailed syllabus, including work in probability and statistics, split into ten levels that are to be covered in the age range 5 to 16. There are national tests at ages 7, 11, 14 and 16.

Spain is much more typical. There, a general syllabus is laid down nationally and local areas put in their own detail. Italy and Sweden have similar systems

5b. *Testing*

It has not been possible to obtain examples of the different ways used to test students' understanding of probability and statistics. This, though, is a most important aspect of teaching. The nature of the test is probably the single biggest factor in determining how the subject is taught. To borrow, more or less, a phrase from word-processing "What You Test Is What You Get".

If the teacher sees that the assessment emphasises something then (s)he will rightly teach the students to be able to pass this assessment. If the assessment only tests the trivial then that is what will be taught. More fundamental understanding will then tend to be ignored. This is a matter for concern in the United Kingdom.

The ASU group in France put the matter well when they reported that there was a lack of statistics questions in examinations; questions test calculating ability and memory, but not a critical spirit.

6. Conclusion

There does seem to be a coming together in Europe concerning what should be taught in probability and statistics at the school level when compared with 10 to 15 years ago. There is, though, a lot that we can still learn from each other and there are many ways in which we could be of mutual help.

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Syllabuses in mathematics were obtained from the following countries:

England and Wales
Germany - several of the federal states
Italy - national syllabus and the expansion used in Umbria
Scotland
Spain - the national syllabus and the Andalucian expansion
Sweden
Finland

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