# SIMULATION AND THE STUDENT ENGINEER

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## 1. Introduction

The aim of this paper is to stress the importance of the simulation of random situations in today's engineering world and that for this reason simulation should be treated in all basic stochastics courses and especially so in such courses for engineering students. R. W. Hamming mentions in Hamming (1993) that in the 1950s at Bell Lab one out of ten experiments was made on the computer and nine out of ten in the laboratories but that later the situation was reversed with nine out of ten experiments made on the computer, that is, by simulation.

For some engineering students, for example, students of computer science engineering or industrial engineering and management, it might be appropriate to have a specific course on simulation.

Simulation can also be used as a didactical tool to give the students concrete experience and background for abstract concepts and specific methods in stochastics.

Simulation is, according to Ralston and Reilly (1983), "the representation of certain features of the behaviour of a physical or abstract system by the behaviour of another system".

The following is, according to Law and Kelton (1991), a list of some particular problems for which simulation has been found to be a useful and powerful tool.

1) Designing and analyzing manufacturing systems.
2) Evaluating hardware and software requirements for a computer system.
3) Evaluating a new military weapons system or tactic.
4) Determining ordering policies for an inventory system.
5) Designing communication systems and message protocols for them.
6) Designing and operating transportation facilities such as freeways, airports, subways, or ports.
7) Evaluating designs for service organizations such as hospitals, post offices, or fast-food restaurants.

8) Analyzing financial and economics systems.

Some of the reasons why simulation is used in these and other areas of applications are the following.

1) An analytical solution is not possible.
2) An analytical solution is too complicated.
3) Simulation can be used to estimate unknown probabilities, distributions, expectations and so on.
4) Simulation can be used to give a concrete illustration of what random variations and stability can be expected in a given system.
5) Simulation can be used to check an analytical solution.
6) Simulation can be used as a didactical tool.

Note that even if it is possible to find an analytical solution to a problem, for example in queueing theory, it might still be worth while to simulate the situation to give a concrete picture of what random variations or stability can be expected in the system.

Monte Carlo methods are closely related to simulation. By Monte Carlo methods are usually meant methods of solving deterministic mathematical problems by experiments with random numbers. For example, complicated multiple integrals can be estimated by simulation.

Simulation of large complex systems is not an easy task and requires both a good understanding of the simulated system and a good working knowledge of simulation techniques. This is illustrated by the following citation from *Newsweek*, March 18, 1991.

> In simulations before the attack, Air Force Computers projected that as many as 150 planes would be lost the first night. When the real thing came, all planes returned safely.

Fig. 1 gives a scheme for the simulation of a real life situation. Note that a simulation requires a mathematical simulation model. So simulation is not, as some might believe, a way to avoid using mathematics. Validation means to check if the simulation really is relevant for the practical situation which is to be simulated. This can be done, for example, by running the simulation program for a simple case of the practical situation for which an analytical solution is known.

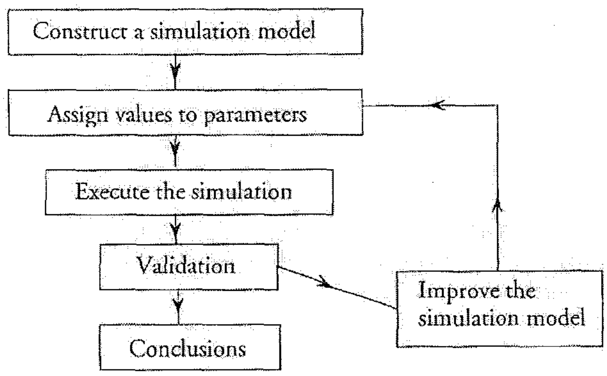


Figure 1. *Flow chart for a simulation*

## 2. Teaching simulation

Elements of simulation can easily be covered in all basic stochastics courses. For example, when discussing various basic probability distributions, we should show not only their mathematical properties and areas of applications, but also how they can be simulated. Here the most effective simulation algorithms need not be used but methods which will give the students understanding and insight. Some examples are the following.

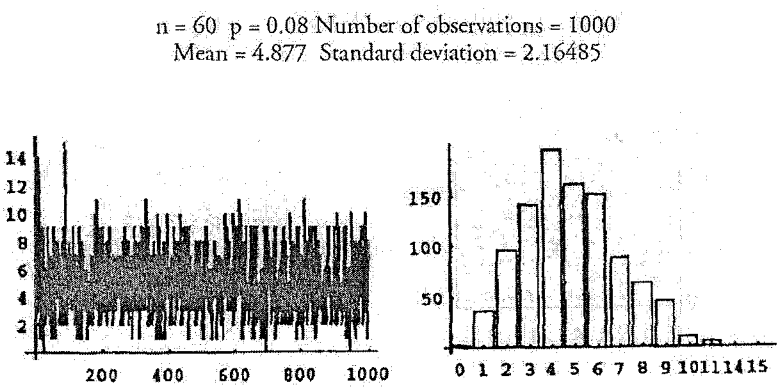


Figure 2. *Simulation of a binomial distribution*

1) The binomial distribution should be simulated by adding binary random digits. This will help the students see that a variable with a binomial distribution can be considered to be the sum of simple random variables, which can be used to calculate moments and to understand why a binomial distribution can be approximated by a normal distribution.

2) The Poisson distribution should be simulated through the simulation of the Poisson process by counting the number of events in an interval of fixed length. The Poisson process can be simulated by adding simulated observations of a random variable with an exponential distribution. This will help the students understand the important relationships between the exponential distribution, the Poisson process and the Poisson distribution.

3) The normal distribution should be simulated by adding a number of random numbers. This will give the students practical experience of the central limit theorem. A good exercise is to show why the addition of 12 random numbers is a good choice.

When using statistical computer software to do statistical data analysis, it is very appropriate to use simulated data produced by the computer.

Fig. 2 shows an example of results from such a simulation. In this case 1,000 observations on a binomially distributed random variable with $n = 60$ and $p = 0.08$ have been simulated. The data can be interpreted as the observed numbers of defective units in samples of 60 from a production process with the failure probability $p = 0.08$.

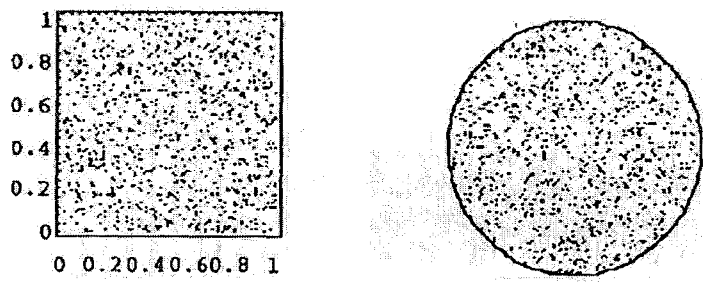


**Figure 3.** *Random points in square and circle*

It can also be instructive to have the students perform simulations of some simple random experiments. Examples of such experiments are:

1) Simulation of exponential or Weibull time to failure for some manufactured products.
2) Simulation of time to failure for series, parallel or other technical systems of components with random life times.
3) Generation of random points in a square and a circle.
4) Simulation study of the distance between two points chosen at random in a square.

For some engineering students simulation can be predicted to be such an important activity in their future profession that it is highly desirable to give them a specific course on simulation. Such a course should include at least one large practical simulation project. The following is a suggestion for the content of such a course.

1) Random numbers and random number generators.
2) Test of random number generators.
3) Simulation of simple random experiments.
4) Simulation of exponential, gamma, Weibull, normal, binomial, Poisson and hypergeometric distributions.
5) Simulation of the Poisson process.
6) General simulation by inversion, acceptance-rejection and the table methods.
7) Statistical analysis of simulated data.
8) How many observations?
9) Use of computer software.

It is not possible to discuss here details of how to treat these topics. It should, however, be pointed out that even if most statistics computer software has pre-programmed routines for the simulation of various standard distributions, it is important to discuss at least some algorithms for such simulations.

Textbooks on simulation often discusses at length several different methods for variance reduction. However, today the interest in such methods is not so great as it is usually very cheap to generate random numbers. It is probably enough that the students are informed, via illustrative examples, how to use control and antithetic variables for variance reduction.

## 3. Use of software

It is essential that the students when learning about simulation have access to computers and appropriate software. Most statistical computer program packages have commands for simulation. For example, Fig. 4 shows the menu for simulation in the Statgraphics system. Basic simulation exercises can also be made with general mathematical program packages like Mathematica. The illustrations in this paper have been made with this program. Simulation programs can also be written in general-purpose languages like FORTRAN, C, Pascal and BASIC.

There exist also special software for simulation and also for simulation of specific applications. This kind of simulation software can be used by the students for project work. In Law and Kelton (1991), where such simulation packages are discussed at length, the following programs are described:

GPSS (General-Purpose Simulation System)
SIMAN (SIMulation ANalysis)
SIMSCRIPT II.5
SLAM II (Simulation Language for Alternative Modeling)

Random Number Generation

Distributions available:

| | | | | | |
|---|---|---|---|---|---|
| (1) | Bernoulli | (7) | Beta | (13) | Lognormal |
| (2) | Binomial | (8) | Chi-square | (14) | Normal |
| (3) | Discrete uniform | (9) | Erlang | (15) | Student's t |
| (4) | Geometric | (10) | Exponential | (16) | Triangular |
| (5) | Negative binomial | (11) | F | (17) | Uniform |
| (6) | Poisson | (12) | Gamma | (18) | Weibull |
| | | | | (19) | Multivariate normal |

Distribution number: 10

**Figure 4.** *Menu for simulation in the Statgraphics system*

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