From **Brunelli, Lina & Cicchitelli, Giuseppe (editors). Proceedings of the First Scientific Meeting (of the IASE).** Università di Perugia (Italy), 1994. Pages 395-400. This pdf file is from the IASE website at http://www.stat.auckland.nz/~iase/publications/proc1993. Copies of the complete Proceedings are available for 10 Euros from the ISI (International Statistical Institute). See http://isi.cbs.nl/sale-iase.htm for details.

STATISTICAL SIMULATIONS

Mary Rouncefield
Chester College of Higher Education
Cheyney Road, Chester, CH1 4BJ, UK

1. What is a statistical simulation?

A statistical simulation is a model of a real situation in which chance or random factors are involved. In a queuing situation, for example, a simulation can be used to help answer questions such as:

- How many pumps are needed at a petrol station?
- How many pay booths are required at a toll bridge?
- Is a central queuing system more efficient than queues at separate counters in a bank or post office?
- Will the introduction of an appointments system mean that a doctor's patients will have to wait shorter lengths of time for consultation?

In order to answer these questions the simulation will need to model both the arrivals of cars, or customers or patients, and the time taken to "process" each one. Chance elements come into play at both ends of the queue, arrivals and departures, and these chance factors must be built into the model. This is a fairly advanced statistical simulation and is explained in Rouncefield and Holmes (1989).

One way street simulation

The simplest traffic simulation can be based on a "one way" street. Here the problem is described in Fig. 1.

Pupils can use the data provided in the text or they can collect their own data on a one way street near their school. First of all, the results can be examined to find the longest time a pedestrian would have to wait to cross the road. In order to get a fuller picture, more data is needed, and this will take a considerable time to collect, we can generate further results in a simulation. In order to do this a frequency table can be drawn up and relative frequencies calculated. These relative frequencies can be used as probabilities and further results generated using random numbers or a spinner as in Fig. 2.

The class can then discuss the original problem: "Is a pedestrian crossing needed"? A simulation can then be run to see what would happen

if a pedestrian crossing was introduced:

- How long will pedestrians have to wait? (on average).
- How long will vehicles have to wait? (on average).

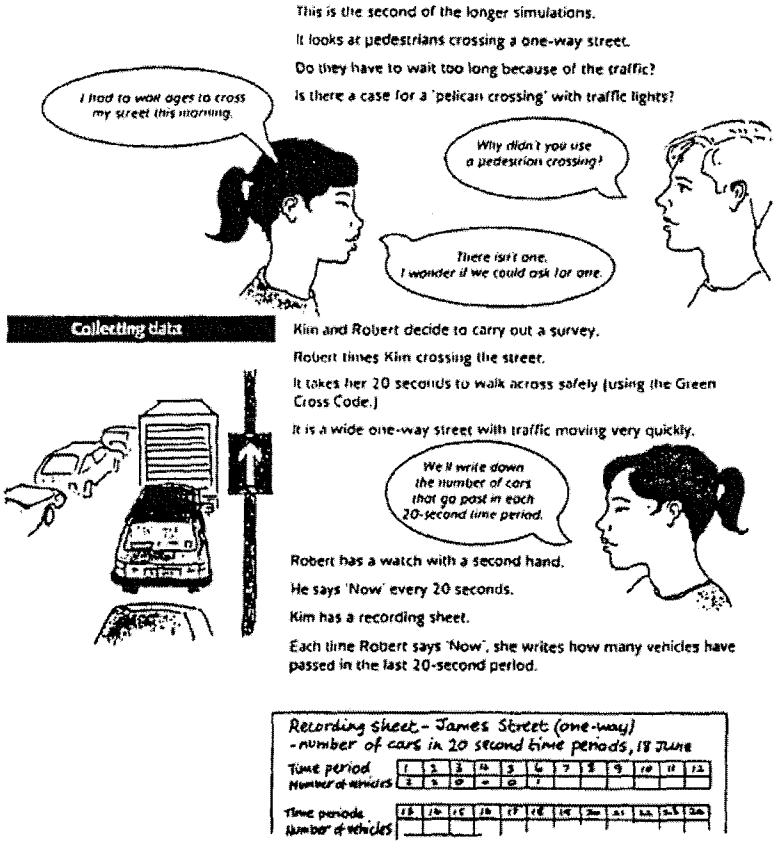

Figure 1. *One-way street*

Relative frequencies

We can use relative frequencies to estimate the chance or probability of each result.

Number of vehicles	*Frequency*	*Relative frequency*
0	29	0.5
1	14	
2	12	
3	5	
Total	60	

1 Copy the table and fill in the other relative frequencies.

Give your answers correct to 1 decimal place.

2 Now make a 'One-way street' spinner to do the simulation experiment.

There is a worksheet for this spinner if you need help.

Try to explain why it is designed this way.

You could use Logo to generate random numbers between 0 and 9, instead of using a spinner.

Y 7/8 HANDLING DATA – E

Figure 2. *Relative frequencies*

Simulation and reality.

This "One Way Street" simulation has some important features:

1. Real data are used.
2. Relative frequency is used as an estimate for probability.
3. Statistical and probability ideas and skills are linked and used in conjunction. In many cases, pupils work either with probability or

statistics, rarely with both. In the minds of many pupils (and teachers) statistics and probability are separate entities and links between them rarely made. (One exception being the tabulation and analysis of a probability experiment).

Two way street simulation

Using the skills and ideas acquired in the One Way Street simulation, pupils can progress to a more complex simulation; a two way street in which:

- Traffic approaches from two directions. Two sets of observations are required to generate probabilities so that a two-way flow can be simulated.
- Pedestrians arrive at both sides of the street, wishing to cross over. Again two sets of data are needed to simulate this two-way flow.

Altogether 4 flows are involved in the simulation. A team of pupils can work together using a map and toy cars and figures as shown in Fig. 3.

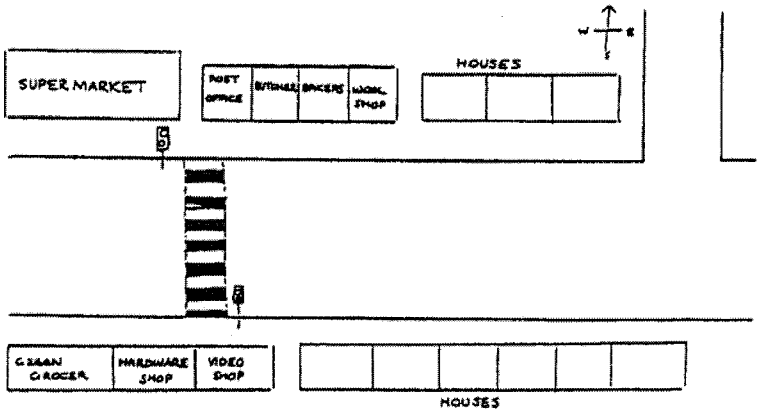

Figure 3. *Two way street*

The simulation can be run both without and then with the pedestrian crossing.

A more enjoyable, although possibly more chaotic version of this simulation, can be run by drawing a roadway on the playground in chalk. Pupils themselves can take the roles of pedestrians and cars.

2. Population models

Many pupils are interested in ecology and conservation issues. They can use library books to research data on the breeding and mortality of endangered species. As ornithologists are such enthusiastic observers and collectors of information, some of the best data available relate to bird populations.

Fig. 4 shows a flow chart showing the processes involved in modelling a 2 year breeding cycle for a pair of Royal albatross.

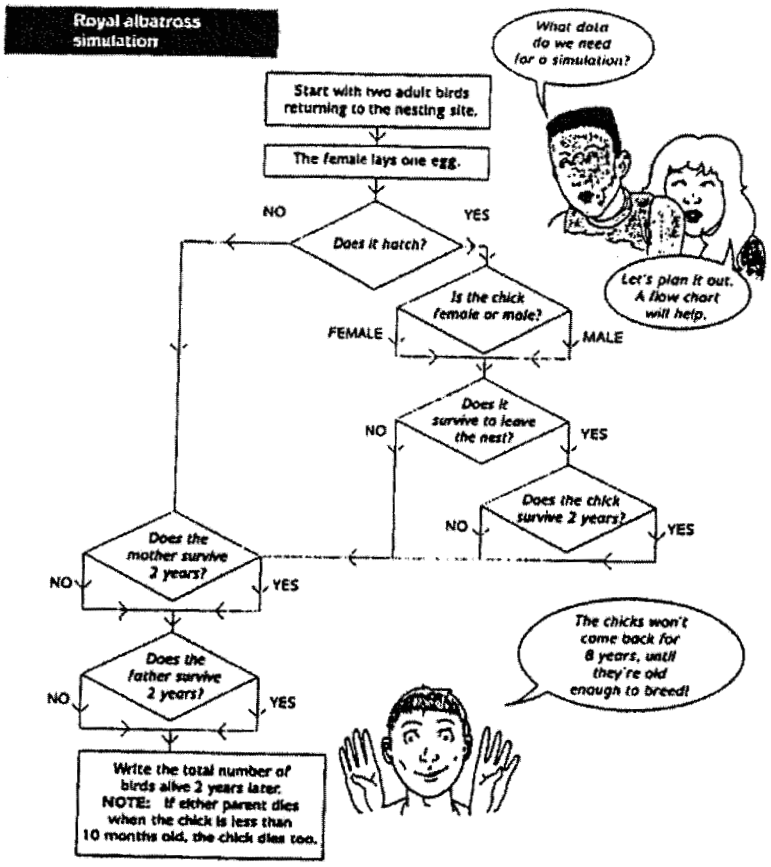

Figure 4. *Royal albatross simulation*

Each decision box in the flow chart requires randomised results based on probabilities calculated from real data.

3. The use of statistical simulations in the classroom

In these statistical simulations, probability and statistics appear as interwoven strands of the same theme rather than as disjoint areas of mathematics. Real situations are studied and real data are used.

Mathematics is applied in real context of interest to the pupil.

Statistical simulations provide the opportunity for cross-curricular projects. Situations can be modelled from numerous subject areas including: Ecology, Biology, Population Studies, Business Applications, Physics and Mechanics.

Students are also motivated to write simple programs for graphical calculators or micro computers.

Acknowledgements

Figures 1, 2, and 3 are taken from the *Handling Data Extension Year 7/8* book, part of the *Century Maths* materials published by Stanley Thornes. Figure 4 has been reproduced from the *Handling Data Extension Year 9* book in the same *Century Maths* series.

Bibliography

Hudson B. and Rouncefield M. (1991), *Handling Data Extension Year 7/8*, from the Century Maths series, Cheltenham, Stanley Thornes, U.K.

Hudson B. and Rouncefield M. (1992), *Handling Data Extension Year 9*, from the Century Maths Series, Cheltenham, Stanley Thornes, U.K.

Rouncefield M. and Holmes P. (1989), *Practical Statistics*, Macmillan Education, London.