STATISTICS TEACHING IN GERMAN SECONDARY HIGH SCOOLS

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This report focuses on a research project that combines two aspects of a statistics curriculum related to teachers' classroom practice and their students' statistical knowledge. Data were collected with questionnaires. The development of the questionnaires derived from results of a qualitative research project will be sketched. Afterwards, some results will be discussed.

INTRODUCTION

In recent years, statistical reasoning (SR), statistical literacy (SL) and statistical thinking (ST) have been declared as the three overarching goals of modern statistics teaching (see for example, Ben-Zvi & Garfield, 2004). In many countries, e.g., in Germany, these goals involve a considerable change from teaching probability to teaching statistics. Hence, Garfield (2002) mentions that teaching statistics in regard to SR, SL or ST requires a change of statistics teachers' instructional practice. However, the classroom practice of "conventional" statistics teachers is often unknown (Shaughnessy, 2007). In this report, the focus is on a research project involving a quantitative survey concerning the classroom practice of German statistics teachers based on the following questions:

1. What is the content of the curriculum of conventional statistics teachers? (In Germany, teachers are able to some degree to select content for their own choice.)
2. What are the objectives of the curriculum of these statistics teachers?
3. What knowledge and beliefs do students attain in statistics courses?
4. What is the impact of the teachers' statistics curriculum on the students' knowledge and beliefs?

The research project is based on two results of the research on teachers' beliefs. First, Chapman (1999) states that mathematics teachers' thinking is the key factor in any movement towards changing mathematics teaching. Secondly, there is strong evidence that the knowledge and beliefs students attain are determined by their teachers' beliefs and their teachers' instructional practice (Calderhead, 1996).

THEORETICAL FRAMEWORK AND METHODOLOGY

The research project discussed in this report is part of a larger research project involving a qualitative designed investigation of statistics teachers' classroom practices and the impact of the latter on students' learning (Eichler, 2007a; Eichler, 2007b). The results of this qualitative research frame the design of the quantitative survey. The qualitative research (Eichler, 2007a) yields five aspects that shape the teachers' planning of statistics instruction (Table 1):

Table 1. Five aspects of statistics teachers' planning

The content of statistics instruction	(aspect 1)
The teachers' objectives of statistics instruction	(aspect 2)
The teachers' objectives of mathematics instruction	(aspect 3)
The teachers' beliefs about the students' benefits of statistics instruction	(aspect 4)
The teachers' beliefs about effective teaching of mathematics	(aspect 5)

The qualitative research provides four types of (individual) statistics curricula that are similar concerning content (see Table 1, aspect 1) but differ considerably with regard to the teachers' objectives or beliefs (see Table 1, aspects 2-5). The distinction between the four types is characterised by differences of the teachers concerning two dimensions. The first dimension can be described with dichotomous pairs of a static versus a dynamic view of mathematics. The second dimension can be described with the orientation towards formal mathematics versus

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mathematical applications. The four types of statistics teachers (Figure 1) were characterised with respect to their main objectives as follows (Eichler, 2007a).

(dimension 2)	static view of mathematics	dynamic view of mathematics
application	*Application preparers*: Their central goal is to have students grasp the interplay between theory and applications. Students first must learn statistical theory in order to cope with mathematical applications later.	*Every-day-life preparers*: Their central goal is to develop statistical methods in a process, the result of which will be both the ability to cope with real stochastic problems and the ability to criticise.
formal mathematics	*Traditionalists*: Their central goal is to establish a theoretical basis for statistics. This involves algorithmic skills and insights into the abstract structure of mathematics, but it does not involve applications.	*Structuralists*: Their central goal is to encourage students' understanding of the abstract system of mathematics (or statistics) in a process of abstraction that begins with mathematical applications.
	(dimension 1)	

Figure 1. Four types of statistics teachers

To investigate the characterisation of the four types, a questionnaire was developed with four parts. The first part concerns the instructional contents of statistics courses (Table 1, aspect 1). The teachers have to mark the content given in a list that they teach in statistics courses. Further, there is room for the teachers to add content to the given list. The other three parts of the questionnaire concern: teachers' objectives for statistics and mathematics instruction (part 2: Table 1, aspects 2 and 3), teachers' beliefs about the students' benefit of statistics or mathematics instruction (part 3: Table 1, aspect 4), and teachers' beliefs about effective teaching of mathematics (part 4: Table 1, aspect 5). In each of the latter three parts of the questionnaire (parts 2-4), the teachers were asked to rate typical statements made by teachers who represent one of the four types (from full agreement to no agreement, coded with 1 to 5). In each of these three parts two statements of every type have to be rated. In addition, the teachers were given space to formulate their own main objectives.

In the qualitative research project (Eichler, 2007b) the construct of statistical knowledge, which Broers (2006) describes as the core of SL, SR and ST, was used. Furthermore Broers' distinction of declarative knowledge, procedural knowledge, and conceptual knowledge was used following the description of these three aspects of knowledge proposed by Hiebert and Carpenter (1992). Results of the qualitative research suggest that students differ in their knowledge, especially concerning declarative knowledge and conceptual knowledge. The differences exist between the students of one teacher and between sets of students of different teachers. The questionnaire for the students involves items concerning declarative knowledge and conceptual knowledge. Concerning their declarative knowledge, students were asked to rate a list of 28 statistical concepts and whether they:

- are not able to remember the statistical concept (coded with 0),
- are able to remember the statistical concept (coded with 1),
- are able to approximately explain the statistical concept (coded with 2),
- are able to exactly explain the statistical concept (coded with 3).

Concerning conceptual knowledge, students were asked to indicate interconnections among the consecutively numbered concepts. Furthermore the students were asked to indicate examples of statistical applications along with the related statistical concept.

In addition to knowledge, students have beliefs concerning statistics or mathematics (Broers, 2006). The qualitative research showed students' beliefs about statistics and mathematics and their relevance for society and for the own life (Eichler, 2007b). For this reason, the questionnaire involves several items (open items and closed items) in which the students were asked to indicate examples that give evidence of the relevance of statistics. Other items on the questionnaires for both students and teachers will be briefly described.

A random sample of 240 German secondary high schools was selected. These schools were asked to conduct the survey, and 166 of the high schools agreed. From each school two teachers and, for each of the teachers, three students with different statistical performance were asked to fill out the questionnaire (January to July 2007). The completed questionnaires from 110 teachers and 323 students were analysed. Due to the fact that Germany has a low tradition in statistics teaching, it must be assumed that the survey includes those teachers who have taught statistics as well as teachers who have never taught statistics.

RESULTS

The statistics curriculum is dominated by a so-called classical block of probability. Except for hypothesis testing other (pure) statistical concepts are rarely taught (Table 2).

Table 2. Percentage of teachers including different instructional content in their teaching

Block	Topics and percent of teachers teaching the topic (n=110)
Classical block of probability	Frequencies (98%), Laplacean probability (97%), statistical probability (72%), probability tree (100%), Bernoulli experiment (99%), binomial distribution (100%), expected value (95%), standard deviation (95%)
Inferential statistics	Hypothesis testing (89%), confidence intervals (51%), Bayesian statistics (27%)
Conditional probability	Conditional probability (81%), (in)dependence (80%), theorem of Bayes (74%)
Distributions	Normal distribution (79%), hypergeometrical distribution (49%) Poisson distribution (49%)
Descriptive statistics	Frequencies (98%), mean (87%), spread (74%), median (52%), regression and correlation (16%)

Factor analysis concerning the objectives of the teachers' statistics curricula in response to the questionnaires yielded three interpretable factors (Table 3), which include 14 of the 24 items referring to the objectives of the statistics curriculum. For each factor the table shows the number of items and the Cronbach's Alpha.

Table 3. Number of items, Cronbach's Alpha and interpretation of factors concerning the objectives of the statistics curriculum

Factor	Factor 1 (5 items, $\alpha = 0.689$)	Factor 2 (5 items, $\alpha = 0.707$)	Factor 3 (4 items, $\alpha = 0.779$)
Interpretation	Traditional curriculum, little reference to real data	Curriculum with high reference to real data	Curriculum with high reference to process

Table 4 lists the items included in Factor 1. The items were constructed from statements of the teachers.

Table 4. Items included in Factor 1 in the questionnaire given to the teachers

Item statement	Type of teacher*
The objective of teaching statistics is to establish a theoretical foundation of statistics.	T
Students must learn to deal successfully with abstract and formal systems	S
Algorithmic skills constitute the basis of learning statistics or mathematics	T
Students must be well prepared concerning final exams and studies	T
Students must learn a precision in reasoning in order to deal successfully with abstract and formal mathematics	S

*T stands for traditionalist teacher, S for structuralist

In addition to each item the table presents the type of teacher giving such a statement (traditionalists: T; application-preparers: AP; every-day-life preparers: EP; structuralists: S). About 43% of the teachers predominately agreed with the objectives listed concerning Factor 1, about 44% concerning the objectives listed concerning Factor 2, and 26% concerning the objectives listed for Factor 3. In regard to the analysis of teachers representing traditionalists (Eichler, 2007a), these results give evidence that 43% of the surveyed teachers focus on probability but pay little attention to real data.

Table 5 shows the mean of students' self estimated ability to explain different statistical concepts (the students' declarative knowledge) and the rank of each mean concerning the means for all the concepts. For instance, "probability tree (2.89; 1)" indicates that the mean of the self estimated declarative knowledge of all students in respect to this concept is 2,89. The second number (1) describes the rank of the given mean (2.89) in respect to all means. The self estimated declarative knowledge of the students correlates with percentages of teachers teaching this instructional content (see Table 2).

Table 5. Average student's self estimated ability concerning statistical concepts and rank

	Topics with average of student's self estimated ability and rank (n=323)
Classical block of probability	probability tree (2.89; 1), Bernoulli experiment (2.35; 3), expected value (2.31; 4), Laplacean probability (2.22; 5), binomial distribution (2.15; 6)
Inferential statistics	hypothesis testing (1.55; 14), confidence intervals (0.44; 23)
Conditional probability	dependence (1.86; 10), independence (1.71; 12), Bayesian theorem (1.23; 17)
Distributions	normal distribution (1.81; 11), hypergeometrical distribution (0.69; 21)
Descriptive statistics	median (0.74; 20), regression (0.45; 22), correlation (0.36;24)

Although the students estimated their declarative knowledge by themselves, these estimations give evidence of the students' factual knowledge. The students were asked to give examples concerning connections between statistical concepts (con1), statistical applications (AW), the use of statistics connected to reality (NU), and connections between statistical concepts and statistical applications (con2). The variables con1, AW, NU, and con2 represent the number of the examples the students indicate. In addition the variables AW, NU, and con2 were divided into examples given by the students concerning games of chance (g, including classical random generators, e.g. urns, cards, dice, etc.) and those that do not concern games of chance (a). The correlations between the students' declarative knowledge (d, c.) and the value of the variables described above are shown in Table 6.

Table 6. Pearson's correlations between students' declarative knowledge and 10 other variables

n=323	con1	AW	AW_a	AW_g	NU	NU_a	NU_g	con2	con_a	con2_g
d, c.	0.44**	-0.064	0.17**	-0.23**	0.11**	0.28**	-0.18**	0.31**	0.29**	0.18**

The correlations are predominately weak, although they are significant different from zero. However, the correlations as a whole give evidence that the students' self estimated declarative knowledge (d, c.) measure in some sense the students' flexibility of dealing with statistical concepts. Further, there is evidence that the higher rating for the students' flexibility in dealing with statistical concepts, the higher their reference to realistic statistical applications is, and the lower the reference to games of chance is.

To address conceptual knowledge, the students were asked to indicate connections among statistical concepts given in the questionnaire (see above). The indicated connections are shown in Figure 2. The numbers in the boxes as well as the numbers in the lines measure the mean of the number of connections between statistical concepts the students have indicated. For example, the number 67 in the box concerning conditional probability means that the students have indicated the six possible connections among the four concepts about 400 times (which yields a mean of about 67; each of the six possible connections could be indicated by each of the 323 students. Thus, the potential maximum was 323). This overview in regard to the students' conceptual knowledge suggests that the students remember statistical concepts in some respects as clusters like chapters in the textbooks. In Figure 1 only the means of indicated connections that are greater than 10 were included. If the mean of the connections between two concepts was greater than 40, a cluster was defined. The concepts concerning procedures (italics), e.g., probability tree were not clustered. It is striking that there are no or few connections between statistical concepts, e.g., median, regression, or confidence intervals to other statistical concepts. Further, the learning divides the statistical concepts in two parts. On the one side, there is the classical block of probability with some connections to hypothesis testing, and the subject matter concerning conditional probability on the other.

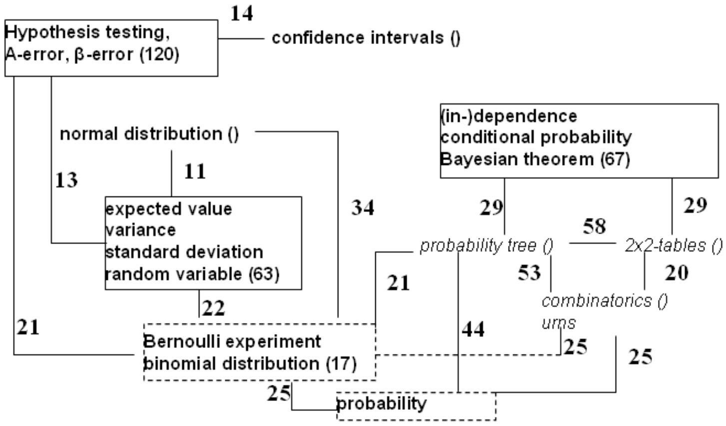

Figure 2. Students' conceptual knowledge

The evaluation of connections between the curricula shaped by the teachers' objectives and the students' learning is difficult because there are only a few completed questionnaires concerning a set of one teacher and his students. Sometimes the teachers returned the questionnaire, while their students have not. Sometimes the students returned the questionnaire, while their teachers have not. For this reason the following results must be interpreted carefully. The teachers were clustered in two groups with high or low acceptance of the objectives related to one of the three factors (Table 2). Hence, two sets of students were constituted. The means of these two groups of students concerning the variables presented in Table 5 (con1, AW, NU, and con2) were compared using a t-test.

These results produced some evidence that the higher the teachers' acceptance of a curriculum with reference to mathematics as process, the higher the declarative knowledge of the students and the higher the students' ability to indicate examples concerning the relevance of statistics. However, there is little or no evidence that a teacher's orientation towards a traditional curriculum (Factor 1) or a curriculum that includes real data (Factor 2) promotes (or impedes) students' learning in reference to the students' declarative knowledge, the students' conceptual knowledge, and the students' beliefs concerning the relevance of statistics. In contrast, the low interrelations between the teachers' objectives concerning the statistics curriculum and their students' knowledge provide stronger evidence that the amount of content has an impact on the students' knowledge. So, the greater the number of statistical concepts taught by the teachers, the lower the declarative knowledge of the students seems to be (Pearson's correlation coefficient $r = -0.29^{**}$).

CONCLUSION

The results of the quantitative survey concerning the curriculum of statistics teachers and the learning of students give evidence that:

- "The traditional way of teaching statistics, with its heavy emphasis on formal probability" (Broers, 2006, p.4) is still existent in German secondary high schools;
- the teachers' instructional contents are similar but the teachers' objectives differ

considerably;

- the quality of students' declarative knowledge affects their conceptual knowledge and their beliefs concerning the relevance of statistics;
- the students predominately have low conceptual knowledge that Broers (2006) declared as prerequisite for a meaningful grasp of statistics;
- the teachers' orientation towards a process oriented curriculum affects the students knowledge;
- the amount of statistical content the teachers teach affects the students' knowledge.

To change a traditional statistics curriculum to the teaching of statistics with regard to SR, SL, ST requires a change in statistics teachers' thinking (Garfield, 2002). For this reason, it seems to be self-evident that we need to increase research related to teachers' thinking using qualitative and quantitative approaches.

Statistics teachers' thinking is determined by their fundamental orientation, i.e. the teachers' system of objectives concerning statistics teaching or the teachers' central beliefs concerning statistics teaching. Pajares (1992) stated that it could be difficult to change the teachers' central beliefs. One approach to change these central beliefs may start from the teachers' conviction that a changed curriculum actually will promote students' statistical knowledge. For this reason it would be desirable to have more research into the statistics teachers' curricula, the students' statistical knowledge and beliefs, and, in particular, the relations between statistics teachers' curricula and the students' statistical knowledge or beliefs.

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