CONCEPTIONS OF FOUR PRE-SERVICE TEACHERS ON GRAPHICAL REPRESENTATION

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In this study we analyze the conceptions of future secondary school mathematics teachers on the teaching of statistics and their influence in classifying the problems in which graphical statistics play a role. For this purpose we present a case study of four students taking the course 'Introduction to the Teaching of Mathematics', who responded to different data collection instruments and were interviewed afterwards.

INTRODUCTION

There is scarce research in the field of training statistics teachers regarding their knowledge and conceptions (Shaughnessy, 1992; Batanero, Garfield, Ottaviani & Truran, 2000). This is also confirmed by Batanero, Garfield, Ottaviani & Truran (2001), who recognized the need for statistics teachers to develop specific knowledge on teaching emerging from pedagogical content knowledge (PCK) (Shulman, 1986). This has been a construct little used in the field of research on teaching statistics.

Researchers and teacher trainers are paying increasing attention towards analysing how future teachers learn to teach and what implications their conceptions and pedagogical knowledge have on the teaching of statistics, in order to use this knowledge to design and implement strategies for training teachers and improving the teaching and learning of statistics in any school system. Among the studies that address this issue, Chadjipadelis (1999) developed a program to improve the training of elementary teachers in six phases in which the teacher must design, implement, collect data and evaluate a teaching project on statistical topics. Nicholson and Darnton (2003) suggested using classroom activities that involved real contexts to motivate discussion and achieve a deeper understanding of the concept for the students as well as developing the knowledge of conceptions, mistakes and difficulties of their pre-service teachers. Lopes (2006) implemented a project that included a community of practice as a strategy for training and developing professional knowledge for elementary statistics teachers. Pfannkuch (2006) conducted a study to improve teaching and learning of box plot distributions based on Wild and Pfannkuch's (1999) model for informal inferential reasoning.

Graphical representation is an object of study and interest at all levels of statistics education. A statistical graph is a "construct which was developed in specific cultural contexts to mediate interpretation of data… an activity which is related to a complex range of elements and processes" (Monteiro & Ainley, 2006, p.1), and the graph is considered as a reasoning tool to learn something new about the context it represents, gain new information or learn from the data (Pfannkuch, 2006). For this reason, pre-service teachers should develop pedagogical content knowledge that will help them design activities to increase the conceptual understanding of statistical graphs in their students, based on the relationships between the main components of the graph and the necessary process of its interpretation (Friel, Curcio & Bright, 2001). However, there have been several studies on statistics education (Burgues, 2002; Monteiro & Ainley, 2006; Espinel, 2007) that address the problems that pre-service teachers have in interpreting graphs. Therefore, researchers agree that future teachers' training must take into account their statistics conceptions and knowledge. In this short exploratory study we will try to get closer to the teachers' cognition, i.e., what teachers know, what they do and why they do it. Our main objective is to understand the forms of knowledge and conceptions of four pre-service high school teachers regarding the teaching of graphical representation of statistical data and to relate them to their conceptions of mathematics in general and statistics in particular.

METHOD

This is a qualitative research study that was carried out with four students: Ana, Carmen, Carlos and Diego (two male and two female), aged between 20 and 24, and enrolled at

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the University of Salamanca (Spain) in the free elective subject 'Introducción a la didáctica de la matemática' (Introduction to the Teaching of Mathematics) in the academic year 2006-2007 and who participated voluntarily. Two of them were finishing a degree in mathematics (Ana and Carmen) and the other two in physics (Carlos and Diego), and they all chose this course because their goal, in the future, was to become high school teachers of mathematics. This free elective subject deals with the content of teaching secondary education mathematics.

This study offers a description of a brief exploration based on the methodological approach by Llinares (2000) and Sánchez and Llinares (2003) in the study of PCK, and it also uses the cognitive levels of graphical representation of Curcio (1987) and Friel, Curcio and Bright (2001) in the analysis of statistical content. The main objectives are: (a) to analyze the criterion used by future mathematics teachers in classifying the problems related to statistical graphs and (b) to compare the conceptions and knowledge prospective teachers have regarding teaching mathematics and teaching statistics, and, more specifically, statistical graphs. The research carried out was organized in three well-differentiated stages:

Stage 1: Classification of graphical representation problems

The pre-service teachers had to classify 20 school problems (with their respective parts or items) and indicate the criterion for classification by giving a justification to obtain information about the characteristics of the problems they perceived and emphasized. These problems were chosen from different textbooks used by high school students, taking into account the following dimensions:

a) The inclusion of different statistical graphs: histogram (9 items), sector diagram (7 items), bar diagram (14 items), stem and leaf diagram (5 items) and frequency polygon (16 items).
b) The involvement of different levels of statistical thinking (Curcio, 1987, 1989): 24 items were identified as reading data, 18 items as reading between data and 9 items as reading beyond the data.

The activity involved in the problems corresponds to a translation between different representation systems (Janvier, 1987); in some problems there was a question referring to the construction of the graph (four problems) and in others the interpretation of the graph (10 problems) or both (five problems). In all the problems, the activity consisted of making a graphical/numerical translation (in one direction or another), but in some problems a graphical/graphical translation had to be made as well (three problems).

Stage 2: Task analysis

Five problems were selected from the list of problems used in the first stage, taking into account that all types of graphs and levels of thinking were included. For each of these problems the students had to respond to the following requests or questions:

- Describe the problem in your own words.
- Do you think this type of task should be included in the teaching of statistical graphs at secondary school? Why?
- What mathematical content can be learned by doing this problem?
- What are the aimed objectives?

The idea was to identify the reasons the pre-service teachers had for including each particular problem in their teaching and how they thought their students would solve it.

Stage 3: Individual semi-structured interview

The interview carried out with each pre-service teacher comprised several parts. A first general part contained questions common to all four students and was aimed at obtaining information about the student's mathematics training and some data on his/her conception of mathematics and its teaching. The second part comprised questions that varied according to the

answers given to the first instrument by each of the students and was thus adapted to each of them. In the third part, the students were asked about the answers given to the second instrument.

RESULTS

The use of different instruments ensured triangulation. The interviews were taped and transcribed completely for subsequent analysis. From the analysis, the arguments used were initially identified by the pre-service teachers and the information obtained was organized in three dimensions according to the aims showed in the method: conceptions about mathematics education; knowledge of statistics and in particular of statistical graphs; and conceptions concerning statistics education. We will describe the results obtained in each of the three dimensions and the criteria used for classifying the problems.

Conceptions about Mathematics Education

Carlos and Carmen valued the traditional role of the teacher, i.e., they view the teacher as the person who directs and organizes knowledge and activities in the classroom; they thought that the teacher's main function is to transmit knowledge, explain, communicate and let students know what mathematical concepts are for, attaching a high value on group work:

> The thing is that he has the ability to summarize, to transmit information,… he explains it once and nearly everyone gets it and that is great [Carlos, E82, E84-85];
> They know how to explain well… they know how to give you the rule and apply it to anything [Carmen, E59, E69-70].

They all considered that the ideal teacher is the one who teaches the students to solve problems, has a method for solving problems, acts like an expert in solving problems, and the students can imitate him/her and then be able to apply what they have seen in order to solve other problems:

> Doing exercises is essential, or else you will not develop non-logical, but almost intuitive instruments [Carlos, E116-117].
> He/she solves exercises from A to Z and does not leave out anything, that is to say, he/she solves it very well [Diego, E77-78].

Ana confronted the theoretical aspects in a practical way by pointing out that if you do not understand the concepts, learning by rote occurs, whereas applications help one to know the reason for things. Diego stressed an added value, motivation of the student.

Knowledge about Statistics and Statistical Graphs.

The pre-service teachers considered that statistics is the easiest part of mathematics, but they were aware that their training in statistics during high school was scant or null:

> You very seldom saw anything. You did see some of the probabilities, but very superficially [Carmen, E120-121].

Consequently, the knowledge they recalled from compulsory education and, more specifically, related to statistical graphs was scant. For example, Carmen was not familiar with frequency polygons, and none of the students had worked with stem and leaf diagrams; hence, they indicated that they would not include these graphs when they had to teach statistics. Despite this, they valued the presence of statistical graphs in secondary education, the possibility of working with different types of graphs and knowing how to choose the right one for each moment:

> …because you can summarize much more information in a graph, many times more than in a chart [Carlos, E162-163].

> The representation will determine the properties that can be extracted from the data, but a good representation adapted to the data you have will be able to give you information that otherwise you are not going to find [Carlos, E163-166].

Some students had difficulties with direct work based on statistical graphs, for example, needlessly turning the data from a graph into a table (Carlos). In general, they all found it easier to work with numerical data than from a graph.

Conception Concerning Statistics Education

All the students taking part in the research agreed on the importance of teaching statistics in high school and argued in favour of the following: a) its usefulness in life, b) the presence of information and statistical graphs in the media, and c) the instrumental nature it may have in their future higher training or in their profession.

> You use it for a lot of things [Carmen, E124].
> Because they give you a lot of data and you can learn a lot from them, …you're reading a paper or looking at a survey on TV and they use graphs [Ana, E80-82].
> There are many more cases applied to the specific area they are studying for them to see that it is not really something totally disconnected from what they are going to study or what they are going to do, but it is actually an instrument they can find useful [Carlos, E153-158].

Thus, they perceived that the teaching of statistics must be something eminently practical, valuing the problems that applied issues pose, not only as a source of motivation but also as learning in relation to daily matters (e.g., electoral results, mortgages):

> I would give more examples and would try to do so in a much more practical way, because I think that they would understand it better, rather than giving just a formula, a table, or distributions [Ana, E72-76].
> I would bring things closer to them… because if you don't know about the topic, you're lost …So, if they put things that motivate you more, music, cinema, things like that, I think this would motivate them more and they would do it better [Ana, E353-354, E359-360].

Despite the fact that they all agreed on the applied aspect, they took specific positions on some matters, for example, unfamiliar topics were in themselves an added difficulty (Ana); topics such as elections and urban waste can cause a conflict of interests with the students (Carmen and Diego); topics that in their opinion stray a little from the mathematical content (Diego). They repeatedly insisted that, until now, the teaching of statistics has been done in a very normative way, which leads to learning by rote, almost without meaning, and does not teach how to reason and think, and work things out for oneself.

> They give it to you mechanically, they don't explain things, they put the graph, find the mean, find the median. They don't teach you how to interpret it, no, they don't teach you that. I mean, sometimes they give you an exercise, but they don't teach you [Ana, E91-94].

Criteria for Classifying the Problems

To classify the problems Ana used as criterion the type of activity the students had to do: prepare graphs, interpret graphs and make calculations. She and Carmen both thought that the first thing the pupils have to learn is to make graphs, then be able to interpret them, later on to be taught concepts, and finally to learn to make calculations and handle formulae. Besides this, Carmen pointed out that making graphs is not statistical knowledge. In order to classify problems, Diego looked at their usefulness for teaching based on the wording and considered the learning of measurements from the graph. Carlos referred to the type of representation (verbal, graphical, table) used in stating the problem and the transformation the student is asked

to make in order to answer the question.

Although Ana used interpretation as one of the criteria, she did not distinguish between different forms of interpretation and considered that an interpretation involving the reading of data from a graph was at the same level as a reading that involves reading between data or one that refers to reading beyond the data. She simply maintained that interpretation is an obstacle for the students merely because they are used to making calculations. This way of teaching makes the student feel comfortable; she considered problems with numerical data where it is merely necessary to apply a formula as easier than those in which a graph has to be interpreted. Diego and Carlos established a distinction, although at a very superficial level, when it is a matter of reading data. Diego established five categories when classifying the problems: not at all appropriate, not very appropriate, appropriate, very appropriate and excellent; they were based on the clarity of the wording, their adequacy for high school, the interest of the topic, the familiarization with the graph and the relations between mathematics and society. Carlos did so with a table, indicating:

> to me the easiest problems are those that merely had to be answered from the data, directly from the graphs [Carlos, E229-231].

Despite our insistence in some questions or problems on explaining how the students could answer them or what they needed to know, none of the prospective teachers at any time posed anything related to "reading beyond the data"; neither did they distinguish between the different types of graphs in the problems.

DISCUSSION AND CONCLUSIONS

From a general point of view, mathematics teaching was perceived by these preservice teachers in the way expressed by Sánchez & Llinares (2002) as the transmission of contents, the handling of useful instruments and as communication, so that although learning requires a personal effort, it is also linked to good transmission on the part of the teacher. These preservice teachers had a broad knowledge of mathematics and perceived its formative aspect in higher education. They valued the usefulness of statistics (aim b), its contextual nature (Nicholson & Darnton, 2003) and the need for education focused on solving practical problems in daily life (Batanero et al, 2000; Batanero et al, 2001) that justifies its inclusion in the curriculum and its instrumental and utilitarian aspects. In spite of this, they considered statistics as a simple part of mathematics, easy to learn, and they did not perceive the difficulties associated with its learning.

These pre-service teachers had a precarious knowledge of statistics, together with a scant or limited knowledge and treatment of graphical representation, no training in matters relating to the curriculum and the processes of learning and teaching; specifically they knew nothing about stem and leaf graphs. They did not perceive the different cognitive levels associated with graphs (Curcio, 1987) or the various components and processes linked to their interpretation (Monteiro & Ainley, 2006). Their conceptions about graphs came from the criteria used to classify the problems (aim a). They were centred in the procedimental aspect of graphs, i.e., the activity the student has to carry out, distinguishing between more interpretive activities and more mechanical ones or simply considering the acquisition of skills. The teaching, according to their conception, focused on the construction of graphs, the analysis of concepts and the application of algorithms and formulae (Batanero, Godino & Navas, 1997). They knew nothing about the process of learning statistical graphs and the difficulties (Burgues, 2002; Monteiro & Ainley, 2006; Espinel, 2007) that students have around this topic.

The implications from this study are that pre-service teachers need a specific training on different representations and their role, features, difficulties and value in the process of teaching and learning. In this context, the pre-service teachers must design (Chadjipadelis, 1999), discuss (Lopes, 2006), implement and evaluate activities related with different representations, and from all these, their PCK could emerge (Shulman, 1986).

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