NAVIGATING LAYERS OF UNCERTAINTY IN TEACHING STATISTICS THROUGH CASE DISCUSSION

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The dynamics of an online case discussion among a group of fifteen prospective secondary mathematics teachers are described. During the discussion, participants offered and debated conjectures about general pedagogy, statistical content, and content-specific pedagogy. Their collective discourse showed that cases can help catalyze online conversations in which prospective teachers challenge one another's claims and interpretations. It also suggested that discussion moderators may need to help participants consider factors in addition to teacher explanations when analyzing the path of students' statistical learning. The paper closes by suggesting that a carefully-sequenced case-based curriculum may have the potential to build prospective teachers' statistical knowledge and challenge persistent misconceptions.

BACKGROUND

Teachers of statistics must deal with two layers of uncertainty in their daily work. The first layer relates to disciplinary knowledge, which is one of the vital aspects of the knowledge base for teaching statistics (Batanero, Godino, & Roa, 2004). Teachers need to be able to help students appreciate the discipline of statistics as being fundamentally concerned with "the study of uncertainty" (Lindley, 1984). Statistical ideas such as hypothesis tests and confidence intervals, along with informal inferential reasoning techniques (Pfannkuch, 2006), are not be taught in a procedural, deterministic manner but rather should be portrayed as tools that provide a basis for making reasoned conjectures about actions to take in uncertain situations. Portraying uncertainty as a central object of statistical study can be particularly difficult for teachers trained mostly in mathematics because mathematical proofs and solutions to problems are often given in absolute terms (Gattuso & Pannone, 2002).

The second layer of uncertainty relates to knowledge of students and pedagogical practices, which are also essential elements of the knowledge base needed for teaching statistics (Batanero, Godino, & Roa, 2004). Uncertainty is ubiquitous in teaching because of the unique and dynamic interactions among teacher, students, and subject matter in any given classroom (McDonald, 1992). Using both qualitative and quantitative observations, teachers make numerous decisions each day about issues such as how to organize the classroom, how to meet individual student needs, how to deal with students' responses to questions, and how to discipline students (Fennema & Franke, 1992). In most cases, the teacher has no precise algorithms to follow to resolve these situations and eliminate uncertainty. Hence, teachers must understand and navigate the uncertainty inherent to both statistics and the classroom simultaneously in order to function effectively.

Practice-based artifacts provide a means for helping prepare teachers to engage with both layers of uncertainty simultaneously. Artifacts like student work samples, teacher reflections, and descriptions of classroom events contain information about the interplay among teachers, students, and content from actual teaching episodes. Lampert and Ball (1998) suggested that artifacts of practice be used to catalyze discourse in which prospective teachers form reasoned conjectures about teaching situations. An important goal for such discourse is for prospective teachers share their conjectures about the artifacts so that they may "probe and even disagree with one another's claims" (p. 116). Through such critical conversations, prospective teachers can begin to hone classroom decision-making heuristics at the same time as they unpack the statistical content embedded in the artifacts.

PURPOSE OF THE STUDY

The purpose of the study was to investigate the nature of the conjectures made by prospective teachers during a discussion about an artifact of practice from a statistics classroom. In particular, their interactions and conjectures surrounding a written case of

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classroom events are described. The different types of conjectures and interactions that emerged are then analyzed in order to draw possible implications for the practice of statistics teacher education.

METHOD

The present study describes the conversation occurring among 15 prospective secondary mathematics teachers as they discussed a case entitled, "It's Time for a Tail" (Merseth, 2003). The case described classroom events transpiring when a teacher (given the pseudonym "Ms. Brady") asked students to make predictions about situations involving uncertainty. In one task, for example, the students were told that a coin had been tossed and had come up "heads" five times. Students were asked to predict the likelihood of obtaining "heads" on the next toss. In another task, students were told that a coach was trying to decide which of two basketball players, Dennis or Michael, should be called upon to take the winning shot in a basketball game. They were told that Dennis made 5 of every 10 shots, on average, but that he has made every one of his last five shots. Michael, on the other hand, made 7 out of every 10 shots, but he had missed his last three shots. Ms. Brady and her students struggled to come to a conclusion about the task, reflecting cognitive difficulties that have been documented by researchers studying other individuals' conceptions of the "hot hand" in basketball (Gilovich, Vallone, & Tversky, 1985). The events of the case were documented through student work samples, excerpts from classroom discourse, and Ms. Brady's reflections on the lesson. At the conclusion of the case, several questions were given to readers to spark discussion and conjecture formation, such as one question asking what the teacher should do next with the class, and another asking for a judgment about the existence of "streak shooting" in basketball.

The discussion of the case took place in an asynchronous (time-independent) online discussion board setting. The online discussion took place during the second semester of a two-semester teaching methods course sequence. The course was taught by the researcher, and ordinarily met face-to-face rather than online. The online discussion was given as an outside-of-class assignment, and participants had seven days to complete it. They were told to post at least four different messages on four different days. Messages were to be directed toward analyzing the case and conjecturing about its implications for teaching practice. There were no face-to-face class meetings held during the time period participants completed the assignment. The group had completed similar online discussion assignments in the past (Groth, 2007).

As the conversation unfolded, all messages were captured on the software program used to host the online discussion (*LiveText*, 2006). The researcher functioned as a discussion moderator by resolving questions that arose, commenting on some posts, and at other times withholding comments to allow the conversation to flow freely (Simonsen & Banfield, 2006). At the conclusion of the discussion, the full transcript was loaded into Atlas.ti (Muhr, 2004) to facilitate coding. Conjectures that participants made related to the educational issues associated with the case were tagged along with the supporting evidence, if any, offered for the conjecture. Conjectures with similar characteristics were then clustered into categories with one another (Miles & Huberman, 1994). Sub-categories were also formed when further distinctions within a category were discernible (for example, a cluster dealing with general pedagogical issues was further broken down into two sub-clusters: classroom management and student affect). The categories of conjectures that emerged are described in detail in the next section.

RESULTS

During the process of data analysis, three broad categories of participant conjecture were discerned. Conjectures dealt with: general pedagogical issues, content-specific pedagogical issues, and content-specific issues (see Figure 1). Sample comments from each of the three categories will be reported in this section. All participants have been assigned pseudonyms.

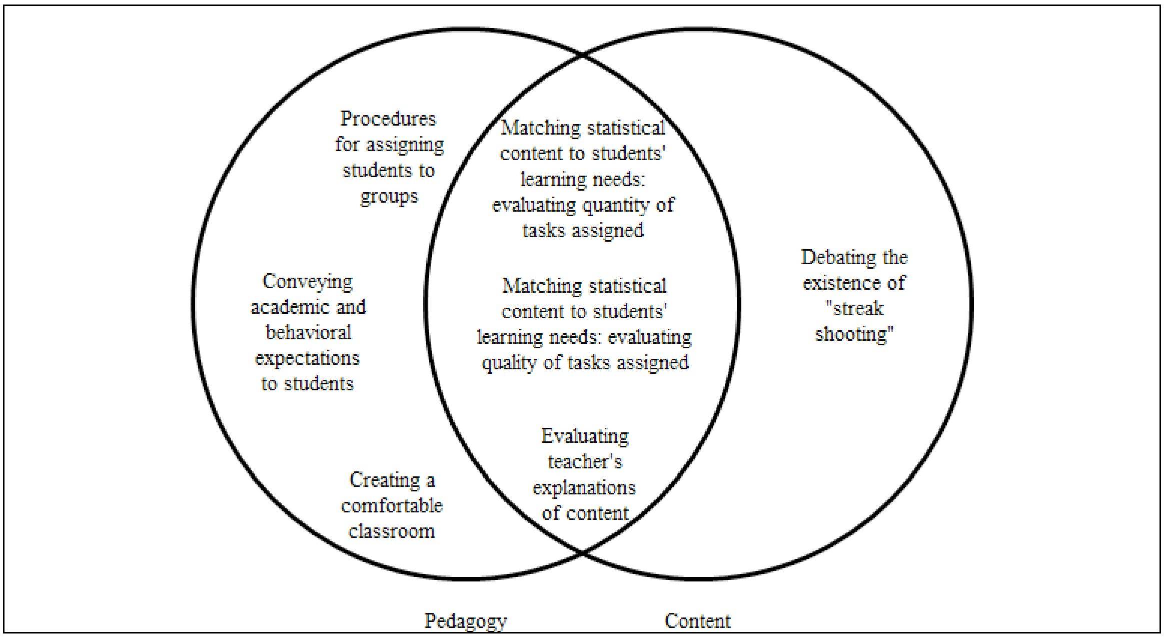

Figure 1. Conversational themes occurring during the case discussion.

Twenty of the conjectures posted to the discussion board dealt with general pedagogical issues. Several participants conjectured that Ms. Brady, the teacher described in the case, needed to change aspects of her classroom management practices. For example, at one point in the case, Ms. Brady allowed her students to select their own group partners. Some participants felt that assigning students to cooperative groups rather than letting them choose their own would minimize disruptions. Others conjectured that the teacher should have set clearer expectations for behavior, although the validity of that conjecture was disputed by some participants. In another strand of general pedagogical discussion, participants conjectured that affective issues were the root cause of problems with the lesson. Harold, for example, conjectured that students needed to feel more comfortable expressing confusion and that they need to feel that teachers have high expectations of them. Others, like Olivia, conjectured that Ms. Brady needed to convey more enthusiasm.

There were 34 conjectures made during the conversation that dealt with issues lying at the intersection of pedagogy and content. One strand of content-specific pedagogical conjecture dealt with the quantity and quality of the problems Ms. Brady selected for the lesson. Six participants conjectured that Ms. Brady simply asked students to do too many problems in a single class period. Some of this strand of conjecture was precipitated by Ivan, when he stated,

> The examples were good for understanding probability, but maybe she used too many, too fast. The teacher didn't spend enough time with each example/experiment and when a question wasn't answered from one example, the students brought their misconceptions to the next example.

Several others felt that it was not the quantity, but the quality of the tasks that were to blame for students' confusion. Eight participants conjectured that the basketball task (described earlier in the method section of this paper) was too complex for students, and that a simpler problem should have been chosen. Kelly, for example, stated,

> This [the basketball task] is a good example to get kids thinking about statistics in the real world, but it may just confuse them more than build their understanding. I think that when you are first introducing certain concepts you should stick to ideas that are pretty much cut and dry. Perhaps you should make sure that the students completely

> understand the concepts of probability before you challenge them on a higher level of thinking. It may do more harm than good.

After the researcher made a post asking if “cut and dry” statistical situations run the risk of being perceived as artificial and contrived, Claire and Laura conjectured that the basketball task actually was suitable for the lesson in the case. Claire, for example, supported the use of the task by stating,

> I don’t think that there is anything wrong with her examples. It is important to see many examples to fully understand a concept and apply what you have learned to other examples. If you can only use probability with coins but not with dice or lotteries, etc. then you aren’t really learning the concepts.

Claire went on to state that the difficulties documented with the lesson were due to the quantity of problems that the teacher tried to accomplish in the lesson.

A second strand of content-specific pedagogical conjecture dealt with the quality of Ms. Brady’s statistical explanations. Elaine took issue with the way Ms. Brady introduced the problems, conjecturing that the definition of “probability” she provided before having students work on problems was misleading:

> I was looking at the situation that arose with the student Benjamin. The class was discussing a homework problem in which there were 3 favorable outcomes out of 5 possibilities – so the probability of the given event was 3/5. But Benjamin was convinced there was only one winner. Do you think the problem could have been in wording that the teacher used? She described probability to be the number of winners divided by the number of possibilities. Under this wording, I think Benjamin was correct, because there can be only one winner per trial. I think a more favorable choice of words may be the # of favorable outcomes divided by the total # of outcomes.

Olivia, Harold, and Diana all made conjectures that resonated with this analysis. Greg and Nancy, however, disagreed with the idea that Ms. Brady’s conjecture caused the students’ confusion. Nancy attributed students’ difficulties to another source:

> I think that the teacher did do a good job of explaining how many winners there are. She talked them through the problem so that they could see that there were 3 possible winners. Although the problem could have been worded better I think that the real problem is how she stopped trying to explain the problem because some of the class was having their own discussions. Just when Ben was beginning to make some ground she moved on, almost giving the impression that she didn't care whether or not he understood the material.

Kelly, Harold, Diana, and Fran all conjectured that students’ understanding could have been built by having them present solution strategies to one another. Kelly explained, “By having the students that do understand explain in their own words, you are likely to hear lots of different explanations of the same topic. The students that don’t understand are bound to understand one of those explanations.”

Finally, there were 14 conjectures posted about the content-related issue of streak shooting. Several participants offered an analysis of the basketball problem embedded in the case. Andrew, Kelly, and Ivan all conjectured that Dennis should take the final shot because he had made his previous five shots. Andrew explained,

> I think confidence is definitely an issue that isn’t involved in the calculations of who should take the last shot…Theoretically, as Kelly said, we would like to pick the one who had the better average during the season. But experimentally, today is not that same person’s day and we should stick with the “hotter-hand” today.

Elaine, Greg, and Olivia disagreed with this analysis, and conjectured that Michael, who had a higher overall shooting percentage, would be more likely to make the last shot. When Olivia shared an article she found online that refuted the idea of streak shooting, Elaine expressed agreement with the ideas in the article, stating,

> I think "streak shooting" happens in the sense that an athlete will have moments of doing well. But…this streak does not mean the player is changing their average shot rate. Just as in a coin flip, you may flip a series of tails, but later on you may also flip a series of heads. In the long run, a player's average shot rate will not necessarily be affected by streak shooting.

Claire, Harold, Fran, and Brittany conjectured that it didn't matter who took the last shot. After giving a lengthy discussion of the factors involved in making the decision about who would shoot, Harold offered a conjecture that summed up this line of thinking:

> I do not think there is a correct answer in this case. It is a judgment call on the part of the coach. Another coach may decide to go with his hottest player. Either way if you pick the hottest player or the one who delivers the most often, that player you pick will either make the basket or will not make the basket.

This final group of four students simply discounted quantitative information as a source for helping the coach make his "judgment call" about the last shot of the game.

CONCLUSION

The case discussion described in this paper provided a venue for prospective teachers to deal with uncertainty arising in both statistics and teaching. They offered, and at times debated, claims about general pedagogical issues, content-specific pedagogy, and statistical content itself. This type of critical dialogue, where colleagues challenge one another's interpretations, is essential to fostering reflective teaching practice (Lampert & Ball, 1998). It is also significant that asynchronous online conversation of a case supported this type of dialogue, because online conversations among statistics teachers have not been uniformly successful in getting them to interact and critically question one another's claims (Peck & Gould, 2005).

Despite the instances of critical dialogue, the participants' comments also suggest problematic dynamics in online case discussion which might need attention from moderators. One dynamic relates to prospective teachers' ability to appreciate the uncertainty inherent in classroom situations. At times, participants proposed solutions to classroom dilemmas that resonated with naïve transmission-oriented approaches to instruction. For instance, at one point during the case discussion, conversation was around the idea that the key to improving the lesson was to enhance the quality of teacher explanations. Although teacher explanations may influence students' learning, the complexity of the learning process is lost if the focus remains solely on teacher talk. To help the participants avoid coming to premature closure on such an issue, discussion moderators may need to draw attention to other factors that influence learning, such as the preconceptions students tend to bring to the classroom. Students' statistical preconceptions were particularly pertinent to the case used to catalyze discourse in the present study, as researchers have documented numerous prevalent probabilistic notions that interfere with statistical learning (Garfield & Ahlgren, 1988).

Just as students' preconceptions are pertinent to the interpretation of a case, prospective teachers' statistical preconceptions influence the ways in which they analyze the events presented in a case. In the present study, for instance, some participants seemed to believe that it didn't matter who took the last shot in the hypothetical basketball game outlined in the case, because the player selected would "either make the basket or not make the basket." Such a position seems to reflect an "equiprobability bias" (Lecoutre, 1992), in that both events, since they are random, are perceived as being equally likely. The specific misconceptions that emerge during a case discussion are likely to be related to the statistical problems included in the case. The emergence of such misconceptions presents an opportunity for the discussion moderator to challenge participants' reasoning and hence deepen their content knowledge. Therefore, it seems that a promising possible future direction for statistics teacher education would be to sequence case conversations in a manner that systematically attends to the content teachers are responsible for teaching so that misconceptions they have about it may be challenged. As teachers build content knowledge and overcome their own statistical and probabilisitic misconceptions, their capacity to help students understand and appreciate statistics as the "study of uncertainty" is likely to increase.

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