STATISTICAL CONTENTS AND LESSONS IN THE JAPANESE CURRICULUM OF MATHEMATICS FOR LOWER SECONDARY SCHOOLS

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In the new Japanese curriculum of mathematics for lower secondary schools which will be implemented in 2021, statistics education has been strongly emphasized toward this knowledge-based society. The new curriculum of mathematics in lower secondary level contains descriptive statistics (histogram, box-plot, etc.) and inferential statistics (sample survey). And one of the big targets of new statistics education in Japan is using the Statistical Inquiry Process and Critical Thinking in usual lessons of lower secondary level. In this paper, we analyze the pilot lessons of the box-plot and its evaluation to clarify the actual condition of teachers and students at junior high school in Japan.

BACKGROUND

After the Second World War, the Japanese curriculum has been revised every decade. Statistics are arranged in subject Mathematics. Compared to Numbers and Algebraic Expressions, Geometrical Figures, and Functions, the content of statistics changes greatly in each revision. In the 1999 curriculum revision, each subject had decreased due to the introduction of five-day weekly system and new subject of comprehensive learning. In mathematics, statistical content was greatly reduced. In the 2008 curriculum revision, the mathematics time had returned to almost the same as the 1989 curriculum revision under the influence of the PISA shock and so on. The contents of the statistics almost returned to original. In the 2008 curriculum revision, box-plot had been introduced to mathematics of high school for the first time in Japan.

In the 2017 curriculum revision, the contents of statistics are put more emphasis and the box-plot is to be taught in the grade 8 (Table 1). Also, as stated as "grasping and analyzing data according to purpose, reading the distribution tendency of the data, critically thinking and judging" (grade 7), statistical inquiry Cycle and critical thinking are emphasized.

Therefore, this study focuses on the teaching materials and student's understanding in the pilot lessons of box-plot for students in grade 7 in Japan. We use evaluation tool which made by the teacher of the lesson to capture students' understanding of the box-plot. Based on the above, we will examine the characteristics of the teaching method of Japanese teachers and derive some implications for statistical lessons.

Table 1
Teaching Contents of Statistics of Junior High School in Japan

Grade	Course of study 2008	Course of study 2017
Grade 7	Descriptive statistics (Mean, Median, Mode, Histogram, Relative frequency)	Descriptive statistics (Histogram, Relative frequency, Cumulative frequency) Statistical probability
Grade 8	Probability（Statistical probability, Mathematical probability）	Descriptive statistics (Box-plot), Mathematical probability
Grade 9	Inferential statistics (Sample survey)	Inferential statistics (Sample survey)

Note. 1: In the 2017 curriculum revision, representative value (Mean, Median, Mode) is taught at Grade 6 of elementary school.
Note. 2: About the 2017 curriculum revision, it will be implemented in 2021.

In M. A. Sorto, A. White, & L. Guyot (Eds.), *Looking back, looking forward. Proceedings of the Tenth International Conference on Teaching Statistics (ICOTS10, July, 2018), Kyoto, Japan.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org

OUTLINE OF PILOT LESSONS

This chapter illustrates one advanced practices of statistics focused on the Statistical Inquiry Process in lower secondary level. This is the pilot lessons based on the new course of study (Ishiwata, 2016).

Description of the pilot lessons （Students, Period and Class Hours）

The pilot lessons were conducted about 3 lesson hours (one lesson hour is 50 minutes) given to the 7th grade students at a public Lower Secondary School in Setagaya-ku, Tokyo, in March 2016.

Unit title: Box-plot

The teacher in charge taught guidance on the box-plot for the students of the 7th grade who had studied the histogram and the frequency distribution polygon. As a preliminary step of introducing box-plot, dot plot was used. Dot plot is handled in grade 6 of elementary school.

1st lesson：Introduction of a box-plot based on dot plot
2nd lesson：Comparison of box-plot and histogram
3rd lesson：Reading parallel box-plots

At the 1st and 3rd lessons, the students actually gathered data and created a box-plot. Also, at the 2nd lesson, they used the data collected at the 1st lesson.

Aim of the Unit

* Students can write box-plot and read it.
* Students can visually grasp the distribution of data.
*Students can examine and judge trends based on data.

THE PILOT LESSONS

Peripheral vision training （1st lesson, 2nd lesson）

17	45	48	3	26	8	67	62	50	1
80	87	90	25	89	98	29	10	16	94
13	4	15	78	22	86	28	2	55	100
74	30	61	9	38	5	82	69	12	20
57	42	79	32	73	77	11	47	34	33
39	46	99	14	51	93	81	56	53	97
19	96	64	7	65	40	76	24	6	31
44	35	18	68	49	27	21	58	66	37
23	41	70	83	59	75	63	43	52	84
60	95	54	88	36	91	85	71	72	92

Figure 1. Peripheral vision training

Teaching material. Viewing field of human is 180 degrees to 200 degrees. The range in which the color and shape can be clearly recognized is about 20 degrees centered of viewing field. This view is referred to as the central vision. The other parts are called peripheral vision. There is peripheral vision training to train peripheral vision. It is said that it is effective for restoring vision by searching the numbers from 1 in order from which is randomly placed. The teacher prepared a table in which 1 to 100 numbers are randomly arranged in a hundred squares grid (*Figure 1*). In the class, the students took a record by examining how many checks were possible within the time limit.

1st lesson: Introduction of box-plot based on dot plot. The teacher explained peripheral vision training and students tried peripheral vision training collected their data which is how many numbers they could check in three minutes (*Figure 2*) .The results are shown in the dot plot (*Figure 3*).

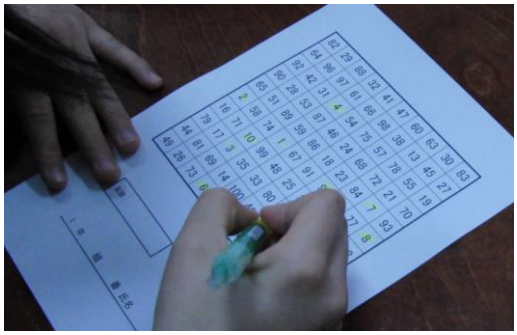


Figure 2. Student's peripheral vision training

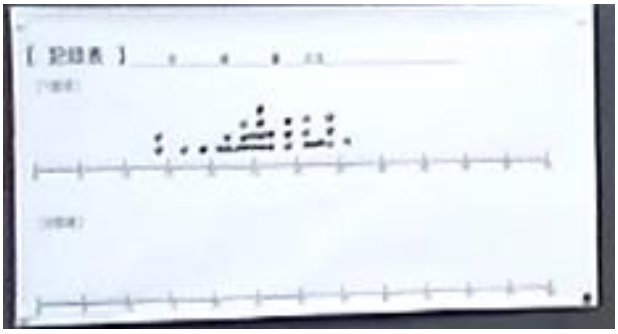


Figure 3. Dot plot of data of peripheral vision training

Next, the teacher took the time to think about ‘I notice, I wonder’ from the dot plot, and the teacher made the students speak their thoughts.

S1 "It is gathered around 20."
S2 "Many students are from 20 to 30."
S3 "The range is 22."
S4 "The maximum is 36.
S5 "The fewest is 14."
S6 "The middle is 26."
S7 "There are 30 or more people who are a quarter of classmates."

The students of S1 to S3 capture the distribution with focusing on cluster and variation of data. The students in S4 to S6 express the distribution using the statistical terms they have learned. Students of S7 had a thought related to quartile. Also, many students were thinking about where their own records were located in the distribution. In order to grasp the relative position of student's record, the teacher showed that data could be divided into 4 parts, and it leads to the idea of box-plot (*Figure 4*).

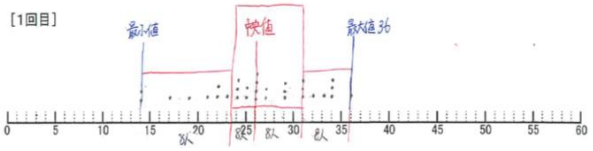


Figure 4. The idea of box-plot diagram

After drawing a box-plot, the teacher taught the terms such as the first quartile, the third quartile, etc., and also taught that the whisker and the part of box contained data of 25% each. After that, students tried to peripheral vision training again, and the first and second records were compared in the box-plot (*Figure 5*). Students focused on the position of the boxes, maximum value, minimum value, median value and so on, and could read that the second record were better than first.

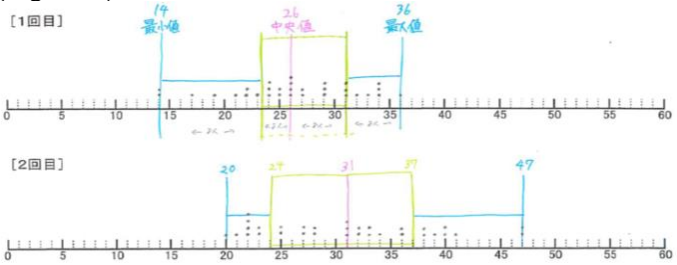


Figure 5. Comparison of two box-plot diagrams

2nd lesson: Comparison of box-plot and histogram. Based on the record of the peripheral vision training carried out in the 1st lesson, the students again drew a box-plot, checked the terms such as the quartile number, and looked back on the features of a box-plot.

Next, the students compared histogram with box-plot of same their data (*Figure 6*). As a merit of the histogram, the students got such information as "the mode can be read immediately", "the frequency of each class is immediately known" and "the number of data is understood". As a merit of the box-plot, students got such information as "it is easy to draw", "It is easy to compare the results of the two data sets", "The density is known every 25% ", " it is easy to understand " and so on. Also, the teacher asked the students a comparison of box-plot and histogram.

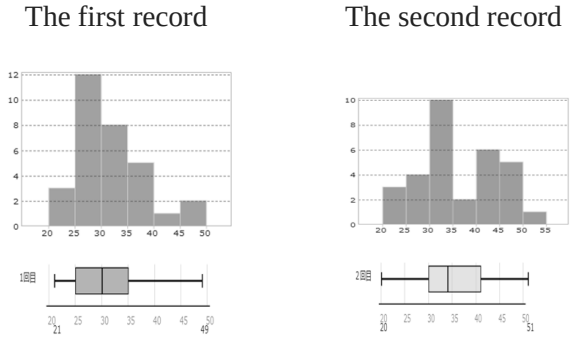


Figure 6. box-plot diagram and histogram

S1: Although it takes time and effort to draw histogram, it is very easy to see a certain range. Some information are left out in box-plot, but it’s easy to draw and easily understood.
S2: It’s easy to read the mode from histogram and the median from box-plot.

S3: Each two figures are sometimes unreadable to read. But, by looking at the two figures, I can see how the data looks very much.

By comparing the two graphs side by side, students were able to understand the characteristics of each graph more deeply. Also, as students' words of "easy to see" and "easy to read ", it is possible for students to be aware of the importance of capturing data information from the figure and reflecting data information in the figure.

One second sense (3rd lesson)

Teaching material. Using a stopwatch (measurable up to 1/100), students measure one second without looking at the display screen. The student closest to one second has a good sense. Each student made 30 trials and examined the trend of the data using a box-plot. Based on the results, the students discussed with group (5 to 6 people) and selected one student with one second sense form each group.

3rd lesson: Reading parallel box-plots. The teacher conducted a lesson of "one second sense" that selects students who will give values close to one second without seeing the screen of the stopwatch. The task of the lesson is to select a representative student from each group who is likely to give results closest to one second.First of all, students discussed throughout the class how to select representatives from a group.

S1: I will try it several times.
S2: Decide on one trail!
S3: I will choose a person who can get a perfect match for 1 second.
S4: I will choose a person whose average is close to 1 second.

Students discussed "how many times to try" and "what is the criteria for us to choose ". About the number of trials, they decided "30 times". Next, the students discussed the criteria to choose. A lot of opinions to push "average" were presented, but based on an opinion from other student, that is, the average from the data of 0.1 second and 1.9 seconds becomes 1 second. It turned out that the average was not necessarily suitable as a representative value in some cases. As a result of the discussion, it was decided to select "a person who gives more results close to one second" as a representative, looking at the scattered data.

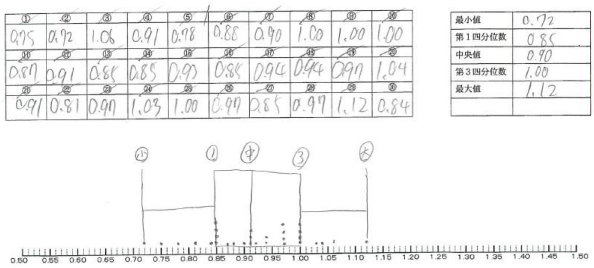


Figure 7. Box-plot diagrams of 30 trials

Each student performed 30 trials using a stopwatch and represented the data in a box-plot (*Figure 7*). Next, in order to select representative student from each group, the data of the members were compared in a parallel box-plot (*Figure 8*). At the group of Fig. 8, they were discussing whether one of the first and second box-plots students would be chosen as representative. They also seemed to compare the number of times of one second and around one second. Finally, this group focused on the difference in maximums of two students' data and chose the first student as representative. Other groups could also select representatives focusing on the shape of the box-plot and median.

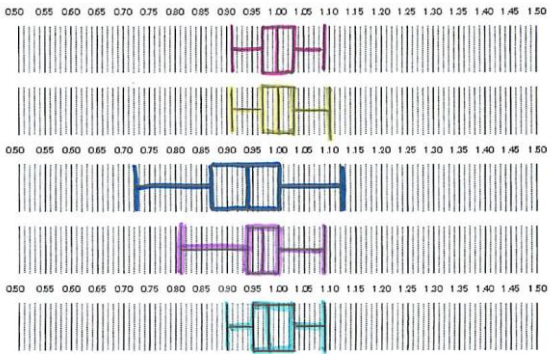


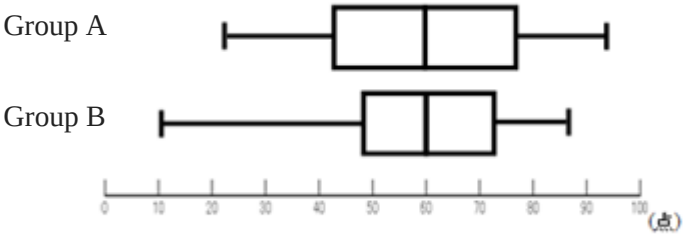
Figure 8. Parallel box-plot diagrams

ASSESSMENT TEST

This teacher conducted an assessment test on 98 students who completed 3 hours of lessons (*Figure 9*). Question 1 of this evaluation test adopts Oguchi (2011). Question 2 adopts the University Entrance Examination of Japan (2017). The results are shown in Fig. 10.

Question 1

The distribution of the scores of mathematical tests of Group A and Group B is shown in box-plots. The number of students in group A and group B are 36. Look at the box-plots and answer the question.



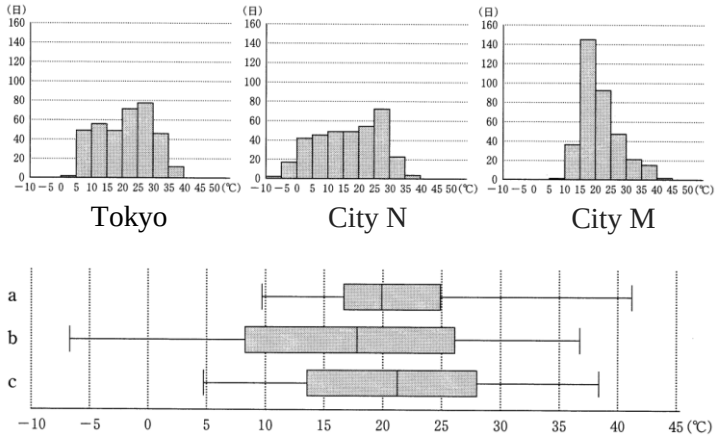
(1) Which group did the students who took more than 60 points? Choose one of the following X, Y, Z, W.

(2) Which group did the students who had less than 30 points? Choose one of the following X, Y, Z, W.

X: Group A
Y: Group B
Z: Both classes are almost equal
W: I can not say either

Question 2

The figure on the right is a histogram and a box-plot showing the record of the highest temperature of each day of a certain year in the three cities (Tokyo, City N, City M). Choose the correct one of the following P, Q, R, S, T, U as the combination of city name and box-plot.



P: Tokyo－a、City N－b, City M－c

Q: Tokyo－a、City N－c, City M－b

R: Tokyo－b、City N－a, City M－c

S: Tokyo－b、City N－c, City M－a

T: Tokyo－c、City N－a, City M－b

U: Tokyo－c、City N－b, City M－a

Figure 9. The assessment test

From the results of Q 1, some students believe that if box-plot are long, there are many data contained therein. Oguchi (2011) pointed out that "Learners are hard to catch the ratio of data, even though they hold their knowledge about box-plots." Junior high school students are also hard to capture the ratio of the data in the box-plot (A.Bakker, R.Biehler, C.Konold, 2004). On the other hand, as shown in the result of Question 2, the correct answer rate was high in the judgment of the correspondence relationship between the histogram and the box-plot. It can be said that in the class practice, it is the result of incorporating a comparison between the histogram and the box-plot.

Table 2
The Result of The Assessment test

Question 1 (1)		Question 1 (2)		Question 2	
X	68.4%	X	11.2%	P	4.1%
Y	5.1%	Y	49.0%	Q	3.1%
Z	17.3%	Z	1.0%	R	2.0%
W	9.2%	W	38.8%	S	10.2%
total	100%	total	100%	T	6.1%
				U	74.5%
				total	100%

Note. N=98.

DISCUSSION

Evaluation on teaching of box-plot in Japan

In teaching a box-plot by Japanese teachers, teachers have introduced quartiles based on the fact that the students divide the data equally, using the dot plots that were already learned. Students draw box-plot from dot plot, so they could return to the original data. Japanese teachers teach students to draw out concepts. In addition, Japanese teachers taught students awareness of the goodness of each graph through comparing histogram and box-plot. The characteristics of the two materials were to use the student's own data, and the student could learn positively.

Evaluation of student's activity

Through a three-hour lesson, students became able to understand the outline of box-plot and compare multiple data to read trends. In addition, comparison of the histogram and box-plot of the evaluation test, and reading of the parallel box-plots in class practice were able to make judgments based on the positions of the box-plot, the maximum value, the minimum value, the median value, etc. From these facts, it is sufficiently possible to instruct the box-plot in middle school. Students were able to have criteria to judge data more diversely. On the other hand, as shown in the results of the evaluation test, in this practice, understanding of the relationship between the length of box-plot and the number of data contained therein is insufficient. It is necessary to pay sufficient attention to this point when teaching.

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