APPLYING PROBABILITY RULES TO GAMES OF CHANCE THAT INVOLVE RISK: A DIDACTICAL STRATEGY

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This paper explores the effects of an innovative didactical strategy (called The Probability Festival, PF) to help university students to signify the concept of probability. This strategy has been implemented four times (one semester each) by three different college teachers. The PF consisted in dividing the group into teams. Each team had to create games of chance involving risk and to explain how concepts related to probability were involved in the game. A worksheet assessment, which was modified throughout the Probability Festivals, was used to evaluate students' explanations. The results we analyze come from the 42 students who participated in the third PF. Data interpretation shows that students improve their understanding of concepts once they have to explain how their games are in their favor.

INTRODUCTION

The interest of several researchers to teach statistics (Delmas, Garfield & Ann, 2003; Batanero, 2001; Tauber, Sánchez & Batanero, 2004; Barragués & Guisasola, 2007) is due to the fact that students have come across difficulties to understand procedures and formal concepts related, mainly, to chance (Batanero, 2005; Jones, Langrall & Money, 2007). To overcome these difficulties, it has been advocated to transform the teaching of statistics through the modification of contents, didactic strategies and the use of technology in the classroom (Antoch & Cihák, 2007, Batanero, 2005; Barragués & Guisasola, 2007; Friz and Figueroa, 2011). In the specific case of issues related to random events, elaborating efficient didactic proposals to help improve the understanding of the concept of probability represents a challenge for statisticians (Jones, Langrall & Money, 2007). A guide that helps on the development of these proposals are the suggestions of Barragués & Guisasola (2007), who consider that teaching probability theory consists of defining the specific contexts where students wish to apply this concept. For example, organized teaching proposals in sequences of tasks that show that students achieve greater capacity for reasoning than in conventional education.

This article will demonstrate the effects that the implementation of a didactic strategy called Probability Fair (PF) had on the learning of the concept of probability. The PF has been organized for four years by the professors of a university course on Probability and Statistics. The results shown in this article correspond to those of the fourth fair.

The probability festival as a didactical strategy

Five years ago, two professors of probability and statistics taught undergraduate students who were majoring in business administration. These professors organized several informal meetings in which they proposed possible activities that could help reduce the rate of failing said course. As part of these proposals, the idea of carrying out a Probability Fair arose. The objective of this was to organize a team of students (4-5 students) that would propose games of chance in which the calculation of probability was involved. The students would take the knowledge acquired in the classroom to make a Probability Scheme in the game, which would allow them to put into practice the different concepts previously seen.

Each team designed a probability game. One of the main requisite was to be creative. The completed games were presented to an evaluating jury. The members of the jury were previously selected. The criteria for members of the jury was to have knowledge in the area of probability.
The jury took as reference an assessment worksheet designed by the professors. At the last probability fair held (PF-4) where new variables were introduced that help improve the learning process. The classes were recorded throughout the course, corresponding to probability (unit 3). In total, three assessments worksheets have been modified for the evaluation of student learning. The modifications have been based on the previous experience of the professors (Viramontes &

In M. A. Sorto, A. White, & L. Guyot (Eds.), *Looking back, looking forward. Proceedings of the Tenth International Conference on Teaching Statistics (ICOTS10, July, 2018), Kyoto, Japan.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org

Rodríguez, 2016). Next, the evaluation form will be described taking into account the last assessment worksheets.

It has two main sections: probability game package (with a value of 75%) and presentation (with a value of 25%). In the first section, the name of the game and printed instructions, the design and the entertaining aspect have a percentage of 5%, 20% and 10% respectively. The creative and didactic aspect is 15%, and the clear explanation of the probability scheme is 25%. The professors specify these facets in three points:

First. Explain the probability approach used. There are three approaches to probability: classical probability, empirical probability, and subjective probability. It is important that the student knows how to differentiate between these three types of approaches and the application of their game is a way to identify these concepts in the process.

Second. Explain the probability rule used for the calculation of probabilities if this is the case.

Third. Explain if you used any tool for calculating probabilities such as Bayes theorem, Venn diagrams, the tree diagrams or combinatorial analysis. In the event that students do not apply probability rules, they can use some of the tools seen in class to calculate probabilities in their games of chance. All the tools are explained in the classroom and even modified to be applied to probabilities in games of chance.

In the case of the presentation of the game that the team is organized in terms of their attire, punctuality and organization in the presentation of their game a percentage points of 15% is taken, and the clear explanation and refreshment for the evaluators and the judges, of 10% percentage points.

A checklist of specifications is included, such as defining a name for the team and, in addition, the team that collects the most income receives a 5% more rating, assuming that the probability in favor of the house is calculated. The professors will evaluate the calculation. The price of the game will be defined by each team, with a maximum of five mexican pesos. The professors will consider that the game is to not be expensive and be accessible to all players and students must design a poster alluding to the event for the promotion of the fair and a design contest is to be held where the first place will be awarded.

METHODOLOGY

Two groups of university students participated in this study, which is the last fair that has taken place. The professors of both groups taught the same course: Probability and Statistics. The professors agreed to evaluate the learning of the concepts of Unit III of their course by requesting their respective groups to elaborate games of chance that implied risk. The groups were divided into teams of 4 to 5 members. In this article we describe and show some proposals of the presented games and we present the probability fair as a didactic strategy. The concepts involved in each game should be the following concepts, shown in the textbook of Lind, Marchal and Wathen (2008): special rule and general rule of addition, special rule and general rule of multiplication, general rule for events not independent. The professors explained these rules in class and, in addition, they studied the meaning of the combinatorial analysis in the application of problems related to games of chance. In this way, what was intended was for each team to explain how these concepts could be identified in the proposed game of chance.

DATA ANALYSIS

In total, 12 games are played on this occasion. Some of them are described below and the way in which the student applies the concepts of probability seen in class. All the games are described in a summary table at the end of this section.

The first team, 1, uses a classical probability approach and its game consists in throwing two dice at the same time. The participant can choose between drawing a sum of 10, 8 or 6 (Event A) or an odd or even number in the two dice (Event B). The participant must choose only between two different events. To make the calculation of probabilities students make a Venn diagram. If the participant wins with his combination, he is given a prize of $5 pesos. For example, as event A you can choose a sum of 10: P (A) = Sum of 10 and as event B you want an even number in the two dice: P (B) = Pair in both dice.

The students do the calculation in the following way:

P (A or B) = 3/36 + 9/36 - 2/36 = 10/36

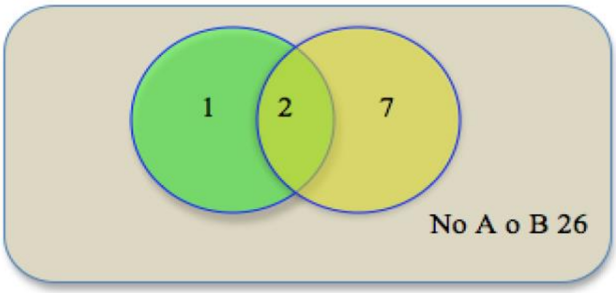

Figure 1. Venn diagram drawn up by students

The team's proposal allows the use of the rules of addition for both mutually exclusive and not mutually exclusive events. For example, selecting the two events shown in the Venn diagram shows that the events are not mutually exclusive since there are two sums of 10 that are also even numbers in the two dice (4,6) and (6,4) joint probability that must be subtracted from the sum; otherwise, the probability would be doubled.

The team argues that "if the participant chooses the combination a sum of ten or odd number increases his odds of winning because there is only one sum of 10 odd (5.5)." This information in the calculation of probabilities is the student's understanding of the rules of the sum. The participant simply chooses his two events and does not perform calculations like in a casino. However, the student does do it and applies the concepts seen in class.

Team 2 developed a contingency table (see Figure 2) to calculate the odds of winning in their game. The game consists in asking the participant to select, without seeing, one of three figures (a triangle, a circle and a square) and, later, guess the color of this figure (three possible colors were: red, blue or orange). For example, once the participant extracted a square, he must guess its color. The calculation of the probability is: P (B1/A3) = 3/9 P(B2/A3) = 1/ 9 , and P(B3/A3) = 2/9

Figuras/Colores	Rojo B1	Azul B2	Naranja B3	Suma
A1	2	3	1	6
A2	4	2	3	9
A3	3	4	2	9
Suma	9	9	6	24

Figure 2. Table of contingencies prepared by students

As can be seen, the highest probability of winning is obtained if the blue color is chosen. The subject of probability calculation with the use of contingency tables was difficult for the members of Team 2 to understand; however, the fact of asking them to elaborate a game of chance in whose explanation this topic is involved, they were forced to study it in greater detail. The creation of a game of chance helped the members of Team 2 understand the rules of multiplication for students. Launching the creativity of the student and calculating probabilities in his own new game helps the learning of probability.

Team 3 proposed a game that consisted of showing the participant three cards. In each card, previously, a grid had been drawn whose numbers had been written in squares from 1 to 1. The participant had to choose three numbers from each card. Next, one of the members of Team C removed, without seeing, one of 15 numbered papers that had been placed in an urn. The said member repeated this withdrawal three times. If the trio obtained by the team member coincided with one of the shortlists chosen by the participant (no matter the order of the numbers extracted), he won a prize.

The novelty of this game is in the fact that the team members involved counting rules to calculate the probability that the participant had to win. This is relevant, since the professors of probability explain the combination only as a rule to count, but not as a tool to calculate probabilities. Even though in the lottery the combination is used to calculate probabilities, the members of this team mentioned that, before the probability class, they believed that the probability of each triple was of P(A) = 1/15, without taking into account the combinations that greatly diminish the possibility of winning. With the creation of this game, now the members know that the probability of each triple being the winner is of P(A) = 1/455 , and that the probability of winning is of P(A) = 3/455.

The following table shows a summary of the probability approach used, rules for computing probabilities and probabilistic tools used by the students.

Table 1.
Summary table of the games

Team number	Approaches to probability	Rules for computing probabilities	Probabilistic tools
1,5	Classical	General rule of addition	Venn Diagram
2, 12	Classical	General rule of multiplication	Table of contingencies
3, 6, 7, 8, 10	Classical	Special rule of multiplication	Counting rules
4, 11	Classical	Special rule of multiplication	Tree Diagram
9	Empirical	Special rule of addition	None

Most teams use the classic probability approach. The Venn diagram, the contingency table and the tree diagram are probabilistic tools on which the students have relied to calculate the probabilities. On the other hand, no one team has used Bayes' theorem because it seems more complex to transfer this concept to a game of chance. Summarizing, students prefer to use the multiplication rule because they can apply rules for counting or other probabilistic tools. However, the rule of addition helps them to understand better some concepts as joint probability or the rule of the complement.

CONCLUSION

In this article, three applications of probability rules are exposed to games of chance that are proposed by students. The main result is that this teaching strategy causes a change in the way of understanding the concept of probability, as well as being a more active way of learning it. The explanations that their games should provide the students, at the same time provoke the development of their creativity, strengthening their ability to solve problems and ask questions arising from their new proposals. Developing efficient didactic proposals that help improve the understanding of the concept of probability has been a challenge as Jones, Langrall and Money (2007) mentioned. Even more, derived from this didactic proposal, some advantages are observed related to the teaching not only of the concept of probability, but also of statistics. One of them, for example, is to produce a pedagogical practice in a non-traditional context outside the classroom that motivates the student to reflect on the importance of studying the concepts involved in games with attention to detail.

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