A SURVEY ON THE USE OF CONTEXT FOR TEACHER TRAINING IN PROBABILITY AND STATISTICS: FOCUSING ON THE TUESDAY BIRTHDAY PROBLEM

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Context plays a significant role in teaching probability and statistics; although its importance has been contended in previous studies, there is not sufficient research on the role of context. Therefore, this study aimed to clarify the actual use of context in problem solving through pre-service mathematics teachers. We focussed on the Tuesday Birthday Problem, which requires awareness of context. Data were obtained from surveys involving 98 teachers and analysed by classifying answers based on the answer types related to context. Results revealed that there are three hurdles related to context in this problem: 'generation of the idea of conditional probability,' 'identification of the necessary and sufficient context for the condition,' and 'correct use of the context.'

INTRODUCTION

In today's age of volatility, uncertainty, complexity, and ambiguity (VUCA), addressing social problems such as those arising from the 2019 coronavirus disease (COVID-19) pandemic has become imperative. To confront these problems, students need statistical and probability literacy to deal with uncertainty. Specifically, probability literacy is one of the competencies that must be developed as part of mathematical literacy to survive in today's society. Gal (2005) characterises the knowledge elements of probability literacy as knowledge of big ideas such as variation and randomness, finding or estimating probabilities, language to communicate about chance, context, and critical questions to reflect upon. Of these five elements, critical questions are linked to context that focusses on the situation and background of the problem. Context here indicates "understanding the role and implications of probabilistic issues and messages in various contexts and in personal and public discourse" (ibid., p.46). Thus, context and critical questions are related in terms of exploration, which is important for solving social problems in the age of VUCA.

As mentioned previously, context plays a significant role in teaching probability and statistics, and its importance has been contended in previous studies (e.g., Cobb & Moore, 1997; Dierdorp et al., 2011; Gil & Ben-Zvi, 2011; Makar et al., 2011; Pfannkuch et al., 2016; Wild & Pfannkuch, 1999). However, as Pfannkuch (2011) pointed out, research on the role of context in teaching probability and statistics is limited. Although additional research has been conducted since Pfannkuch made this observation, a recent survey by Fukuda (2020) suggests that the role of context in teaching probability and statistics at the elementary and secondary education levels has not been sufficiently investigated. In addition, one of the epistemological issues necessary for the future of theory development in statistics and probability education is the following: "there is a need for a deeper theoretical conceptualization of context and contextualizing in statistics education" (Nilsson et al., 2018, p. 374). In summary, Pfannkuch's (2011) point regarding insufficient research on context in statistics education remains relevant. Therefore, this study aimed to determine to what extent pre-service mathematics teachers (hereinafter PSMTs) aspiring to become secondary teachers work on probabilistic and statistical material with an awareness of the role of context. If they are not aware of the role of context, it becomes all the more difficult for students in elementary and secondary school to appreciate context. Thus, the research objective of this study is to clarify the actual situation of the use of contexts in probabilistic problem solving by PSMTs. The related research questions are: To what extent can PSMTs use context in probabilistic problem solving? How do PSMTs use context in this problem solving?

This study builds on Pfannkuch's (2011) argument highlighting the need for more research on the role of context in statistics education. It is important to note that context in this study refers to data-context defined as "the real-world situation from which the problem arose" (Pfannkuch, 2011, p.28), with a particular focus on probability and in alignment with context as defined by Gal (2005).

In S. A. Peters, L. Zapata-Cardona, F. Bonafini, & A. Fan (Eds.), *Bridging the Gap: Empowering & Educating Today's Learners in Statistics. Proceedings of the 11th International Conference on Teaching Statistics (ICOTS11 2022), Rosario, Argentina.* International Association for Statistical Education. iase-web.org

METHOD

We focussed on the Tuesday Birthday Problem, which comprises probabilistic and statistical material that requires awareness of context. The problem was first presented by puzzler Gary Foshee at the ninth *Gathering 4 Gardner* (World Congress of Puzzles) in Atlanta in 2010 (Taylor & Stacey, 2014). We edited the problem to suit our purposes, as shown in Figure 1.

> Mr. A has two children. We know one is a boy born on a Tuesday. What is the probability that the other child is a boy? The probability of being born male or female is 1/2 for each.

Figure 1. Tuesday Birthday Problem

To solve the Tuesday Birthday Problem, it is necessary to reduce the sample space; therefore, this problem requires a conditional probability. As for the condition in the conditional probability sought in this problem, at first glance, the context of one child being born on a Tuesday does not seem to be relevant for solving the problem. However, as demonstrated in the example answer in Figure 2, the context provides key information and a condition for the conditional probability required for solving the problem. Therefore, the Tuesday Birthday Problem is probabilistic and statistical material that requires one to judge whether the context presents a condition or not.

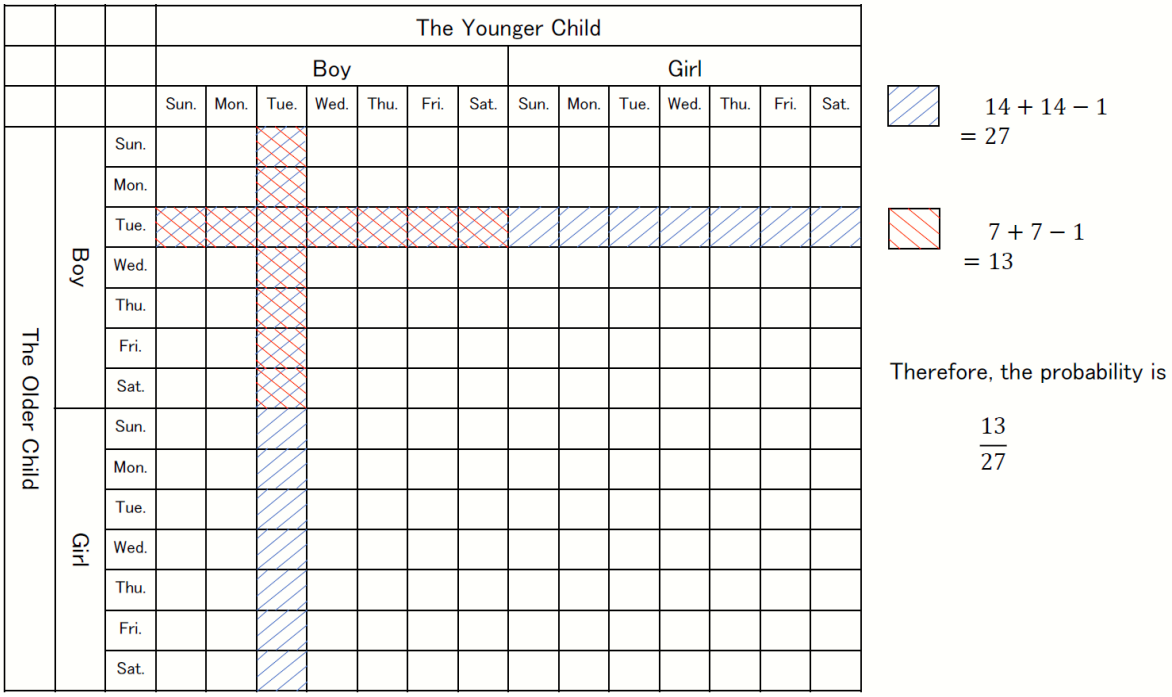


Figure 2. Example answer to the Tuesday Birthday Problem

Using the Tuesday Birthday Problem, we investigated the actual use of context among PSMTs using survey data. Of the surveys administered to 135 PSMTs, 98 responses were included in the analysis. We excluded solution-only responses and answers that we could not decipher. The survey questionnaires were administered during several lectures. They were administered either in person or submitted as assignments due to COVID-19 pandemic restrictions. In particular, in-person surveys were conducted on December 21, 2020, and surveys in the form of an assignment submission were collected on November 9–10, 2020; November 10–11, 2020; and January 21–22, 2021. Data were analysed through the classification of survey responses based on the answer types related to context. Specifically, we analysed responses from three perspectives: (a) whether the idea of conditional probability was generated; (b) whether the necessary and sufficient context for the condition was used; and (c) whether the context was used to obtain the correct answer.

RESULTS

Of the 98 responses, 63 (64.3%) generated the idea of conditional probability, which implies a reduction of the sample space. For example, Figure 3 shows PSMT-A's answer, wherein the sought probability is set at 1/2 on the basis that the sexes of the two children are independent; this indicates that the context of one child being a boy is not taken as a condition.

A さんの 2 人の子供について、それぞれの子供が男か女かは独立である。つまり、一人が男の子であるからと言って、もう一人の性別には影響しない。したがって、男か女かどうかの 2 択であるので、もう一人も男の子である確率は 1/2 である。

With regard to A's two children, the sex of each child is independent of the other. In other words, the fact that one is a boy does not affect the gender of the other. Therefore, the probability that the other one is also a boy is 1/2, because the two options are whether the child is a boy or a girl.

Figure 3. PSMT-A's answer (translated by the authors)

Further, 29 (46.0%) of the 63 participants who generated the idea of conditional probability used the necessary and sufficient context for the condition. PSMT-B's answer (Figure 4) shows that he/she created a tree diagram with the respective sex of the older and younger child, with no possibility of both being girls (Girl－Girl); this was done by capturing the context that one child is a boy as a condition. Of the remaining three patterns (Boy－Boy), (Boy－Girl), and (Girl－Boy), the only one in which the other is also a boy is reflected in the (Boy－Boy) pattern, and participant PSMT-B set the sought probability as 1/3. Therefore, the context of one child being born on a Tuesday was not included in the condition. For this reason, PSMT-B's answer is insufficient in the condition.

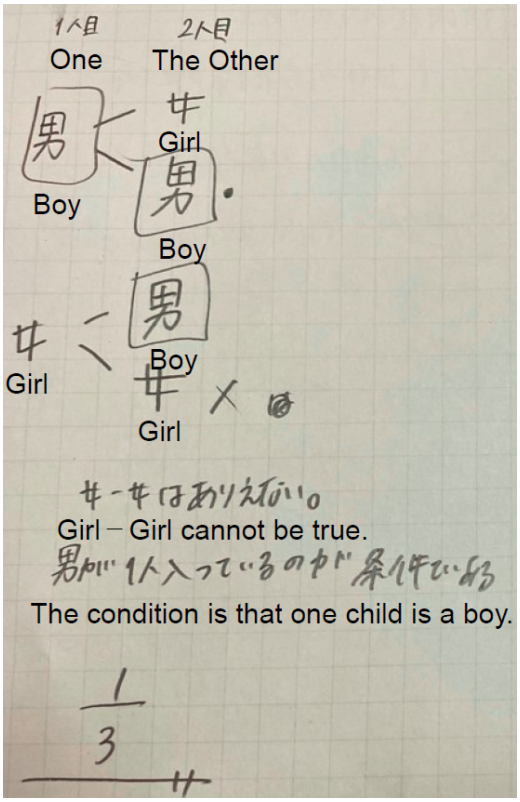
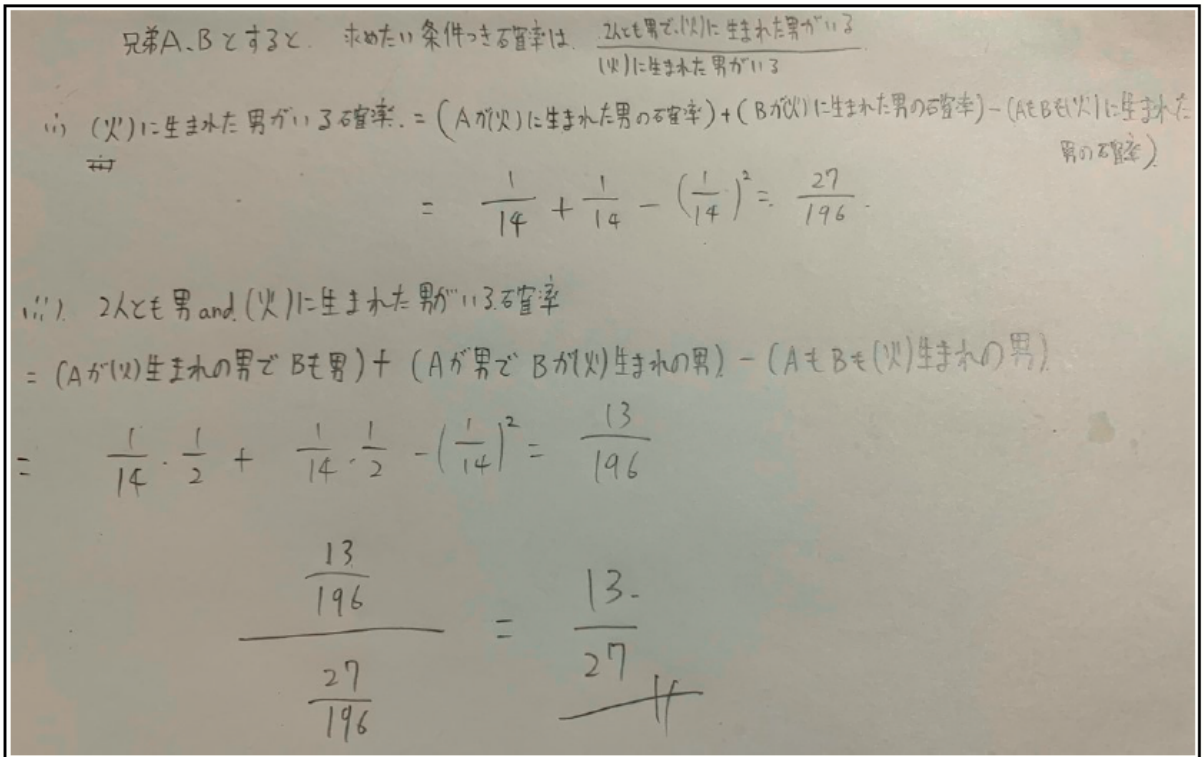


Figure 4. PSMT-B's answer (translated by the authors)

Of the 29 participants who used the context, 19 (65.5%) answered the problem correctly. In these answers, the context that one child is a boy born on a Tuesday was taken as a condition. As for the process of discovering the answer, in some cases, the answer was given by calculations based on the definition of conditional probability, as can be seen in PSMT-C's response (Figure 5). Furthermore, the correct answers were reached through various methods, such as using a table, as described in Figure 2.



I call the siblings A and B.

The sought conditional probability is $\frac{\text{They are both boys and one was born on a Tuesday.}}{\text{One was born on a Tuesday.}}$

(i) The probability that one was born on a Tuesday

= (The probability that A is a boy born on a Tuesday) + (The probability that B is a boy born on a Tuesday) − (The probability that both A and B are boys born on Tuesday)

$= \frac{1}{14} + \frac{1}{14} - \left(\frac{1}{14}\right)^2 = \frac{27}{196}$

(ii) The probability that both are boys and one was born on a Tuesday

= (A is a boy born on a Tuesday and B is a boy) + (A is a boy and B is a boy born on a Tuesday) − (Both A and B are boys born on Tuesday)

$= \frac{1}{14} \cdot \frac{1}{2} + \frac{1}{14} \cdot \frac{1}{2} - \left(\frac{1}{14}\right)^2 = \frac{13}{196}$

$$\frac{\frac{13}{196}}{\frac{27}{196}} = \frac{13}{27}$$

Figure 5. PSMT-C's answer (translated by the authors)

These results suggest that there are three hurdles related to using context when solving the Tuesday Birthday Problem: 'generation of the idea of conditional probability,' 'identification of the necessary and sufficient context for the condition,' and 'correct use of the context.' This theoretical finding can serve as a basis for practicing context-conscious probability and statistics education for students in elementary and secondary school.

FUTURE ISSUES

Two challenges remain for future studies. First, teacher training in probability and statistics on the use of context is necessary because the three hurdles previously discussed in the results apply to PSMTs as well. Second, teaching materials must be developed as part of the probability and statistics curriculum to solve problems such as the Tuesday Birthday Problem correctly for cases in which the use of context is necessary.

ACKNOWLEDGEMENTS

We would like to thank Editage (www.editage.com) for English language editing. This work was supported by the JSPS KAKENHI Grant Numbers JP19K14219 and JP22K13717.

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