DESCRIBING DISTRIBUTIONS WITH AND WITHOUT THE HELP OF TINKERPLOTS—FIRST INSIGHTS INTO A CONTRASTIVE ANALYSIS

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Statistical thinking has become the central goal of statistics education in recent years and has made its way into the primary curriculum. The main educational goal is to guide students from a rather local towards a more global perspective on data and distributions. Because technological tools and software such as TinkerPlots are considered to be valuable support towards developing statistical thinking, this paper will explore differences in primary students' descriptions of distributions as part of their statistical thinking with and without the help of TinkerPlots to illustrate possible potentials and challenges for the use of technological tools within the primary classroom.

DEVELOPING STATISTICAL THINKING—DESCRIBING DISTRIBUTIONS

The development of statistical literacy, reasoning, and thinking is increasingly gaining significance in mathematics education. Because our surroundings ask for a sophisticated understanding of data and their representations as well as the ability to handle big data sets, research in statistics education focuses on establishing theories on how to develop statistical literacy (reasoning and thinking) in view of raising competent data consumers and producers (English, 2014). When aiming to develop statistical literacy, a set of skills is referred to that is needed to "understand … statistical information or research results" (Ben-Zvi & Garfield, 2004, p. 7). Hence, the integration of statistical competences within the curricula have become widespread. There is a consensus within the field of research that the development of statistical literacy is to be set forth from an early age onwards. Therefore, there are more and more implementations of teaching units at primary school level that intend to enhance first statistical concepts, especially through statistical projects.

Central statistical concepts have been identified by Burrill and Biehler (2011) as fundamental ideas of statistics that form the basis for statistics education. Distributions are considered a fundamental idea and can be defined as a statistician's way of handling and analyzing patterns in variations of variables (Wild, 2006). They should be regarded as representing "aggregate[s]" (Konold et al., 2015) with a focus away from individual cases towards a more global, holistic perspective on data. To describe distributions, its components need to be identified and transmitted through language. Arnold and Pfannkuch (2014) have examined said characteristics and formulated a framework that conceptualizes specific features of distributions and their connecting statistical ideas relevant for high school students. Bakker and Gravemeijer (2004) have been able to demonstrate that understanding distributions as aggregates or representations of a whole offers multiple difficulties, especially for young learners. It underlines the importance of further research as well as the need for new teaching materials and practices in order to improve statistical thinking among younger school children.

One above-mentioned development is the integration of technological tools into the statistics classroom. Technology tools have gained broad agreement as their implementation is considered within different frameworks on how to successfully organize the teaching and learning of statistics (e.g., Garfield & Ben-Zvi, 2008). Garfield and Ben-Zvi (2008) have highlighted the potential of technological tools to take over lower-level tasks (i.e., visualizing the data) in order to free mental capacities for higher order tasks such as interpreting data. Further potentials are seen with regards to the possibility of "making statistics visual, interactive and dynamic, focusing on concepts rather than computations, and offering the opportunity to experiment with data to make it engaging for students" (Biehler et al., 2013, p. 648). TinkerPlots is one such technological tool that has already been successfully used within primary contexts (Frischemeier, 2020). Its explicit impact on young learners describing distributions has not been investigated yet.

This paper aims at providing the first findings on differences of primary students' descriptions of distributions with and without the help of TinkerPlots, providing further arguments for the use of technological tools already within primary statistics classrooms. The following research question has therefore arisen: Which differences can be attested between the descriptions of distributions with and without the help of TinkerPlots? The study presented here is framed within a larger project analyzing

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group comparisons among fourth grade students (10–11 years) and will look at learning products from said project.

METHOD

Within the aforementioned project, a teaching unit was designed and implemented to foster reasoning about data and group comparisons in a primary context. In the middle of and at the end of the teaching unit, students were asked to work on individual statistical projects in groups of two to three students and to present their findings in the form of data posters.

THE TEACHING UNIT

The basis for this paper forms a teaching unit that has been implemented as part of a bachelor's thesis (Schäfers, 2017). It was comprised of ten, 45-minute lessons in which the fundamentals of statistics, statistical projects, and the use of TinkerPlots were introduced to a group of twelve, fourth grade students (ages 10–11) from a rural German primary school. As an introduction to statistics, students were familiarized with variables and their values, reading and drawing different types of graphs of distributions, and the components of a statistical project (lessons 1–3). In lessons 4–5, students were asked to manually work on data that they collected from their classmates on quantitative variables of their own choosing and to describe said distributions on a data poster. In the following lessons, TinkerPlots, as a suitable technological tool for a primary classroom, was introduced. These lessons also covered the introduction of hat plots, modal clumps, and group comparisons (lessons 6–8). To end the teaching unit, students were asked to work on their individual group comparisons in small groups with the help of TinkerPlots. Students were asked to investigate differences between boys and girls for quantitative data regarding an individually chosen topic and to document their findings on a data poster for class presentation (Frischemeier, 2020).

The data used for analysis in this study stems from a larger project conducted as the second cycle of design-based research (Cobb et al., 2003). Its original aim was to investigate whether group comparisons could be taught in a fourth-grade class from a German primary school and to develop materials that support primary students' learning processes when comparing groups. Based on the results of the first cycle, adaptations had been made to the teaching materials as well as the use of TinkerPlots.

DATA COLLECTION & ANALYSIS

To answer the research questions, a qualitative approach was chosen to analyze three data posters that displayed the description of a manually created distribution as well as the description of a group comparison realized with the help of TinkerPlots. A structured qualitative content analysis (Mayring, 2014) was applied to the written descriptions of the distributions. For the interest of the paper, three main dimensions were identified. Firstly, the components of the descriptions with regards to their statistical properties are identified. For that, excerpts from the classification by Arnold and Pfannkuch (2014) with regards to concepts of *distributions*, *variability*, and *signal and noise* are used. In a second step, the identified and described features are analyzed as to whether they focus on local, intermediate, or global properties of the distribution (Bakker & Gravemeijer, 2004). For that, the levels of graph comprehension identified by Friel et al. (2001) are also considered as a third dimension. Table 1 portrays the category system developed for the analysis by assigning different categories to the three dimensions and indicating examples from the data for each category.

Because the task posed to the students when working with TinkerPlots required a group comparison, adaptations became inevitable with regards to the data included in the analysis. Only those text passages that describe either the entire distribution or the two sub-distributions (girls and boys) are considered for this paper. Possible impacts of the different types of tasks will be considered in the discussion of the results.

DESCRIBING DISTRIBUTIONS—ROLE OF TECHNOLOGICAL TOOLS

The blue, yellow, and green groups formulated their own statistical questions regarding different contexts and conducted surveys to collect their data. When working on their project manually, they worked with a smaller data set (fewer than 10 survey participants). With the help of TinkerPlots, they collected data from all students in their fourth grade (more than 70 survey participants). Possible effects of different sample sizes will be discussed in the conclusion. To grant transparency without exceeding the space provided, excerpts of only the blue group's data poster as well as a transcript of their descriptions of distributions are presented in Figure 1. An exemplary analysis will be provided for the blue group and brought together with the results of the analyses of the data posters from the other two groups in a next step.

Table 1. Category system for the contrastive analysis of data posters

Dimensions	Categories	Examples from the data
statistical features	individual cases (i.e., minimum, maximum)	"Amongst the girls, the highest value is two hours."
	spread	"Amongst the boys, the values are closer together."
	mode	"Most of the children need two minutes for their way to school."
	majority of the data	"[M]ost boys are between 149cm and 155cm tall."
	modal clumps	"Many values of the boys accumulate at shoe sizes 37, 38 to form a hill."
globality of perspective on data	local	"Some girls don´t spend time on cellphones or tablets at all."
	intermediate	"The hill [modal clump] of the girls is from 30 minutes – 60 minutes"
	global	"You can see that there is a large range between the smallest and the tallest child. The span is 32cm."
levels of graph comprehension	reading the data	"10 minutes are the smallest value for the boys and the highest value is 7 hours."
	reading between the data	"You can see that there is a large range between the smallest and the tallest child. The span is 32cm."
	reading beyond the data	"Some only ride their bikes on school days."

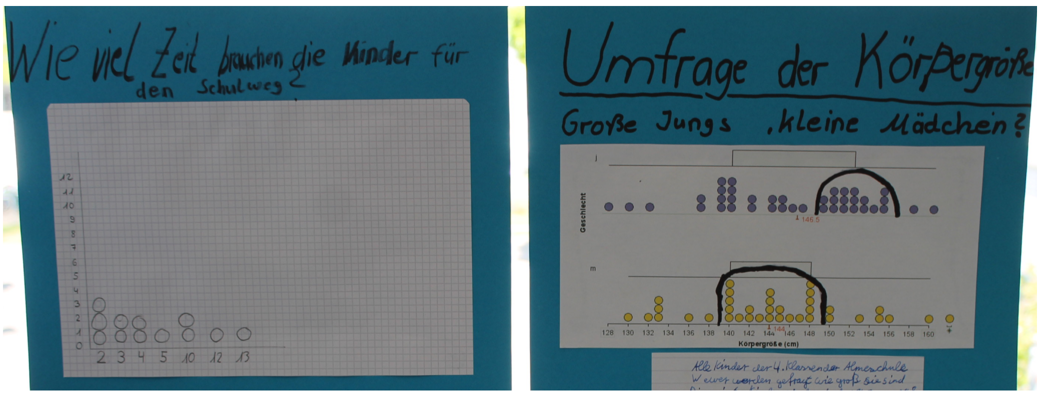

Figure 1. Headlines and statistical displays of the data poster from the blue group

The transcript of the blue group's description of the distribution displayed on the left in Figure 1, which was constructed without the help of TinkerPlots, was as follows.

> How much time do the children need for their way to school? The children of TinkerPlots … have been asked within a survey how much time they need for their way to school. Most of the children need 2 minutes for their way to school. Many have also needed 3, 4 and 10 minutes for their way to school. 3 children have needed 5, 12, and 13 minutes for their way to school.

The transcript of the blue group's description of the distribution displayed on the right in Figure 1, which was constructed with the help of TinkerPlots, was as follows.

> Tall boys, small girls? All children of the fourth grade of the xxxschule … have been asked how tall they are. Most children are between 140 cm and 148 cm tall. … The two tallest children are

160 cm tall. You can see that there is a large range between the smallest and the tallest child. The span is 32 cm. Most girls are 140 cm and 148 cm, most boys are between 149 cm and 155 cm tall. Many values of the boys accumulate at 149 cm–155 cm to form a hill.

RESULTS: DESCRIBING A DISTRIBUTION WITHOUT AND WITH THE USE OF TINKERPLOTS (BLUE GROUP)

When describing the results of the statistical inquiry without TinkerPlots, the students from the blue group referred to all values collected within their project. They identified the mode of the distribution and defined it as the most frequently occurring value. Furthermore, they described further peaks that occurred more than once within the data set. To describe values that appeared more often, students used relative terms ("*[m]ost of the children,*" "*[m]any*") instead of indicating the exact value. A rather local perspective on data appeared for the description of the distribution without the help of TinkerPlots. The students heavily relied on data points that can be read directly from the representation of the data. Regarding the levels of graph comprehension, the descriptions of individual values remained on the level of *reading the data*. The students compared frequencies of different values for their description of the distribution. As they succeeded in comparing the graphical representations of the data set with its different values, they moved towards the stage of *reading between the data* but stayed implicit with regards to precise differences in the variables.

When describing the data set they explored with the help of TinkerPlots, the students from the blue group explored multiple statistical concepts to describe their distributions. They identified a range of values that contained the majority of data points (*range as an interval* as coined by Arnold & Pfannkuch, 2014). They referred to the general range of values within their distributions and numerically defined it ("*You can see that there is a large span between the smallest and the tallest child. The span is 32 cm.*"). Furthermore, they mentioned the highest value while also stating its frequency. This further explored the described range. The students identified the modes of the distributions for the girls as single values ("*Most girls are 140 cm and 148 cm*") while referring to an interval when describing the most frequent heights of the boys ("*most boys are between 149 cm and 155 cm tall*"). Additionally, they described modal clumps as "*hills*" within the data representations in their description of the distribution for boys' heights. Analyzing the considered characteristics of the distributions, the group not only described more characteristics but also more global features of the data set. Although individual values such as the mode and the upper extreme were included within their description (which could be interpreted as *reading the data*), students made use of the portrayed range in data as well as intervals with particular characteristics. When identifying modal clumps within the distributions, they showed an intermediate perspective on data, building up towards the understanding of distributions as an aggregate (Makar, 2014). They showed their ability to *read between the data* as they made use of comparisons and interpretations of relationships between certain values. Numerically defining and interpreting the general range of the values was another indication for a rather global perspective because all data values were taken into consideration. Table 2 portrays the findings from the three different data posters by indicating whether the above-introduced categories are applied in their descriptions.

Table 2. Results of the qualitative content analysis

	Blue Group		Yellow Group		Green Group	
Categories	without TP	with TP	without TP	with TP	without TP	with TP
individual cases	x	x	x	x	x	x
spread		x		x		x
mode	x	x	x		x	
majority of the data		x		x		x
modal clumps		x		x		x
local	x	x	x	x	x	x
intermediate		x		x		x
global		x		x		x
reading the data	x	x	x	x	x	x
reading between the data	(x)	x	(x)	x	(x)	x
reading beyond the data					(x)	

DISCUSSION—DESCRIBING DISTRIBUTIONS WITH AND WITHOUT THE HELP OF TINKERPLOTS

The analysis has shown that there appear to be significant differences between the descriptions with and without the help of TinkerPlots that might offer arguments for the integration of technological tools into the primary classroom. The differences between the descriptions mainly concern the characteristics included as well as their level of globality (see Table 2). When manually describing the distributions, many individual values were referred to within the learning product. In contrast to that, certain values of special meaning were chosen for the description of the data set worked on with the help of TinkerPlots. The identified "hills" (term used for modal clumps within the teaching unit) may be compared to the identified values of higher frequency within the descriptions without the use of technological tools and illustrate an intermediate perspective on data without explicitly stating each value. The students summarize data, which exceeds the level of reading the data. One main difference between the descriptions is consideration of the range, which is not included when students did not use TinkerPlots even though the highest and lowest value of the distributions are equally visible in both data representations. The software did not give the values so that it can be assumed that the students decided to describe the range by themselves. Thus, the students picked up on the notion of variability, which is a crucial statistical idea with high relevance for understanding distributions as aggregates. When comparing the findings from the second dimension of the analysis (globality of the perspective on data), it becomes apparent that the students tended towards a more global perspective on data when working with TinkerPlots. This is supported by a more frequent usage of descriptions that ground on competences that can be interpreted as belonging to the higher graph comprehension level of *reading between the data*. Consideration of single values decreases while intervals are used to describe the aggregate.

CONCLUSION

Statistical literacy has become a topic of focus within the discussions on mathematics education. While claims for its development become more demanding, teaching concepts are being developed within the research community. Even though there are already design elements for the teaching and learning of statistics, more empirical evidence needs to be brought forth in order to support said claims. This paper tried to provide insights into potentials of the use of technological tools in the statistics classroom by analyzing how the use of a technological tool influences descriptions of distributions in a primary context. Within Table 2, it was shown that the students' focus shifts from a rather local description of individual values towards a more global perspective, including the consideration of intervals and spans, with the help of TinkerPlots. This result supports the argument that technological tools such as TinkerPlots may help and center cognitive abilities on higher level tasks as the technology takes over depictions and calculations (Garfield & Ben-Zvi, 2008). It can be stated that even though it supports the findings of a more global perspective with the help of a technological tool, the wrong interpretation of arithmetic means or average also shows that the technological tool does not automatically foster conceptual understanding, which can be seen as a possible challenge. For a next cycle of the design-based research project, a possibility would be to deepen the conceptual knowledge with the help of a technological tool as claimed by Burrill (2019).

Even though the findings of this analysis support theoretical claims in the research field, different possible confounding variables of this sub-project need to be considered and eliminated in further research. Firstly, the tasks posed with and without the help of TinkerPlots were of different forms—the latter included a group comparison. The comparison may influence understanding needed for descriptions and therefore indicate a more expansive approach. Secondly, the sample size of the distributions varied heavily. A bigger sample size might demand a more global perspective on data as the description of every case and value is no longer possible. Furthermore, the work with TinkerPlots took place in the last lessons of the teaching experiment. Students might have profited from already having described distributions in earlier lessons of the teaching unit. Moreover, the different variables chosen for the manual description of a distribution and the group comparison might have had an impact on the comparability. In a next step, these confounding variables should be eliminated by conducting an experiment that includes work with more similar data sets on the same quantitative data and the same task for the descriptions of the distributions at a similar point of time in the learning process.

Regardless of the above-mentioned confounding variables, the qualitative analysis of the three data posters may strengthen the need for a closer analysis of the influence of technological tools on distribution descriptions. It may lead to stronger arguments for their integration in statistics classrooms. This paper wishes to offer first insights into how a contrastive analysis may be designed and encourage research to confirm or contradict the findings presented here.

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