PROJECT-SET MATERIALS FOR THE TEACHING AND LEARNING OF SAMPLING VARIABILITY AND REGRESSION

Anna Bargagliotti[1], Celia Anderson[2], Stephanie Casey[3], Michelle Everson[4], Chris Franklin[5], Rob Gould[6], Randall Groth[7], John Haddock[2], and Ann Watkins[8]
[1]Loyola Marymount University, Los Angeles CA, USA
[2]University of Memphis, Memphis TN, USA
[3]Eastern Michigan University, Ypsilanti MI, USA
[4]University of Minnesota, Minneapolis MN, USA
[5]University of Georgia, Athens GA, USA
[6]University of California, Los Angeles CA, USA
[7]Salisbury University, Salisbury MD, USA
[8]California State Northridge University, Northridge CA, USA
abargag@gmail.com

To address the increased need for statistical literacy across the world, high quality statistical preparation of teachers is needed. Project-SET is an NSF funded project aimed at developing materials to prepare secondary teachers to teach statistics. In this paper, we present results from a professional development (PD) project designed to test whether the materials are effective in preparing teachers to teach sampling variability and regression. Eleven teachers were recruited to take a 45-hour PD that utilized the materials. Participants took the Comprehensive Assessment of Outcomes in Statistics (CAOS) as a pre- and post-test. Results indicate that Project-SET materials were effective in increasing teacher scores. PD participants were also given open-ended assessments. Teacher answers revealed that misunderstandings were present in their thinking.

INTRODUCTION AND BACKGROUND

Due to the increased use of statistics in society today, statistics has become an integral part of mathematics K–12 education. Given the prevalence of statistics in the media and workplace, students require a historically unprecedented level of statistical literacy. Statistics has been included as an important branch of K–12 mathematics education in the United States and abroad, so the need to provide teachers with the understanding needed to teach statistics is an area of great urgency (see Conference Board of the Mathematical Sciences, 2001 Conference Board of the Mathematical Sciences, 2012). There is currently an immediate call to action to provide high quality statistical preparation of teachers.

Project-SET is an NSF funded project (NSF DRK-12 #1119016) aimed at developing materials to prepare teachers to teach statistics. More specifically, Project-SET focuses on the teaching and learning of two fundamental topics— sampling variability and regression. Several studies have shown that teachers have trouble with the concept of variation (Confrey & Makar, 2002; Hammerman & Rubin, 2004; Makar & Confrey, 2004; Watkins, Bargagliotti, & Franklin, 2013) and regression (Casey, 2005; Casey & Wasserman, 2014), and both topics are fundamental in the study of K-12 statistics around the world (Burrill & Biehler, 2011; Franklin et al., 2007; Garfield & Ben-Zvi, 2004).

To enhance teacher learning of these topics, Project-SET developed learning trajectories (LTs), lesson plans/activities, and open-ended assessments. The creation of LTs is a promising research-based way of describing and unpacking how learners understand topics, should be taught topics, and how understanding of a topic should progress (Corcoran, Mosher, & Rogat, 2009; Daro, Mosher & Corcoran, 2011). Project-SET defines a LT as model for successive and gradual thinking about a topic that people typically go through in order to achieve deep understanding of that particular concept. Using the LTs, Project-SET developed a professional development (PD) experience for teachers.

THE LEARNING TRAJECTORIES

Hypothetical LTs were first outlined by Martin Simon (1995) as a means to organize predictions about the anticipated path of student learning. A trajectory provides structure for student learning by outlining a sequence of concepts, sample problems, and teaching tasks (Simon,

In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9, July, 2014), Flagstaff, Arizona, USA.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org

1995; Simon & Tzur, 2004). Once a trajectory is tested with student work (in the case of Project-SET, teacher work), then the trajectory becomes a model for how students (teachers) learn. Example problems can be used at different points along the LT for teaching or assessment purposes, making a LT relevant and ready for implementation (Krainer, 1993; Lappan & Briars, 1995; Simon & Tzur, 2004).

Project-SET constructed and empirically tested two LTs: one for sampling variability and one for regression. The team used an iterative six-step construction process outlined in Table 1 to create the LTs. The process was carried out in two phases: an initial design phase and an implementation and redesign phase (Bargagliotti et al., 2013).

Table 1. Description of LT construction process.

Phase 1: Initial LT Design	
Step 1	The Project-SET team conducted an extensive literature search of 28 articles and books to provide comprehensive background knowledge on the teaching and learning of sampling variability and regression. We formed a guiding literature document.
Step 2	The team completed an internal questionnaire asking how each member defined sampling variability and regression, what the big concepts and ideas were, what range of skills teachers needed, what the prerequisite knowledge should be, and what ending knowledge teachers should have. The answers were carefully studied and synthesized by a subset of 4 members of the team and a first draft of the learning trajectory was crafted. The first draft of the LT was distributed to the team to elicit comments. All comments were incorporated into a second draft.
Step 3	The team created 13 tasks, each aligning with specific portions of the LTs. Eight members of the team (4 expert teachers and 4 researchers) met in person to solve the tasks and discuss the solutions. The problem-solving sessions and team meetings were videotaped. On the basis of the sessions, a third draft of the LT was completed.
Step 4	A final round of comments was elicited to finalize the hypothetical LT. All tasks were edited and developed into full lesson plans (see below for publications).
Phase 2: Implementation and Design	
Step 5	The team designed a 15-week pilot professional development (PD) around the LTs that used the lesson plans to deliver the content. The LTs were validated in the PD (see below for detailed description of the validation procedure).
Step 6	The Project-SET team updated the LTs to reflect the validation results.

Each of the LTs has a loop structure that represents increased levels of sophistication as teachers progress through the loops of the LT to learn the topic. In other words, teachers begin their learning about sampling variability (or regression) at loop 1 and then progress to loop 2 and so on. The sampling variability LT has six loops and the regression LT includes five loops.

The sampling variability LT begins with teachers being introduced to repeated sampling, takes them through the Central Limit Theorem, and ends with formal inference and the construction of confidence intervals.

The regression LT begins by plotting data on scatterplots and informally examining the association between two variables and ends with constructing sampling distributions for the regression parameters.

The LTs can be used sequentially—the sampling variability LT going first followed be the regression LT— or the topics may be separated and the LTs may be used in a stand-alone manner. In general, these LTs provide a map for how teachers can learn about sampling variability and regression.

THE PROFESSIONAL DEVELOPMENT COURSE

The Project-SET PD was designed entirely around the LTs. One loop of a LT was covered per class period (3 hours) in the PD. The PD lasted 15 weeks with 7 weeks specifically dedicated to

sampling variability and 7 weeks to regression. All developed materials used in the PD are free to download on the Project-SET website at www.project-set.com. They include:

- A sample syllabus for the PD for teachers
- The two learning trajectories, one for sampling variability and one for regression
- Sample PowerPoint presentations to introduce each lesson
- Lesson plans/activities aligned with the learning trajectories
- Open-ended assessment tasks to evaluate teacher learning
- Sample homework assignments to reinforce the concepts covered

During each PD meeting, a PowerPoint presentation was used to introduce the concepts outlined in the LT loop being covered that day. Subsequently, the lesson plan/activity aligned with that loop was introduced. The activities typically lasted for about 1-1.5 hours. Technology was used throughout the PD. Of the thirteen Project-SET lesson plans, eight required the teacher participants to use dynamic statistical software. For this implementation of the PD, the teachers utilized Fathom; however, any dynamic software would also work. All other lesson plans used Excel, a graphing calculator, or internet access.

At the end of each meeting of the PD, the participants were assessed using open-ended assessment tasks, designed by the Project-SET team, to gauge whether the teachers had progressed through the loop in the manner outlined in the LT. The assessment tasks were directly adopted and elaborated on from items of the Illustrative MathematicsTM Project (IMP, n.d.). The Project-SET team chose released items that discussed aspects of sampling variability and regression and then developed assessment tasks around them. The team opted to use the IMP items to ensure that the materials were aligned to existing, well-agreed-upon expectations.

Each assessment task was geared toward unpacking the participants' knowledge and understanding of the specific portions of the LT covered during the class period. Each task was graded by two independent graders according to a predesigned rubric. If disagreements between the grades occurred, then the graders discussed the results and settled on an overall grade.

PD Results: Open-Ended Assessments

The designed open-ended assessment tasks uncovered numerous unexpected teacher misunderstandings. Due to the depth with which each topic was covered through the loop structure of the LTs, several teacher misunderstandings surfaced.

For sampling variability, teachers:

- tended to rely solely on the mean and the median to describe a center of a distribution.
- demonstrated a lack of understanding of how to formulate a survey question that would meet the goals of a study.
- showed confusion between how the sample size and number of samples taken affect the sampling distribution. (see Watkins, Bargagliotti, & Franklin, 2014, for details)
- believed that the spread and the shape of a distribution referred to the same thing.
- showed confusion among the real-life situation to be simulated, the model of that situation, and the simulation that implements the model.
- confused the sampling distribution with the population distribution.

For regression, the open-ended tasks uncovered the following unexpected teacher beliefs:

- the residuals summing to zero implies the model is a good fit.
- a "very positive" relationship means there is a very strong correlation.
- misconceiving the connection among the slope, correlation, and residuals.
- falsely believing that the regression coefficient does not have a sampling distribution.

After the PD implementation, all Project-SET materials were updated to include more scaffolding to address the uncovered misunderstandings. The loops of the LTs were adjusted in

order to unpack these particular teacher conceptions. The Project-SET website provides both the hypothesized and the revised LTs.

PD Results: CAOS

A total of eleven high school teachers participated in the pilot PD. To assess the increase in statistical content knowledge for the teacher participants of the PD, the Comprehensive Assessment of Outcomes in Statistics (CAOS) test was used. The CAOS test is a reliable and validated test to evaluate the understanding of statistical concepts (delMas, Garfield, Ooms, & Chance, 2007). Teachers took a subset of items on the pre- and post-test CAOS exam related to sampling variability and regression. The pre-test included 20 items and the post-test included 12 items. The average percent correct for the pre-test was 66.98% while for the post-test it increased to 73.65%. These scores were consistent with prior CAOS outcomes (see delMas et al., 2007). In all cases but one, the teachers' scores increased. Statistically significant differences between pre- and post-scores were observed ($t = -2.51$, $p = .018$) indicating that the teachers' statistical knowledge about sampling variability and regression increased over the course of the pilot PD.

PD Results: Surveys

Teacher participants also took surveys at the beginning, middle, and end of the PD. The surveys asked teachers to comment on their perceptions of the Project-SET materials (e.g., lesson plans and open-ended tasks), on their comfort level with statistics content, and on whether they felt they had adequate activities and resources to teach statistics. All of the teachers' responses improved from the pre- to the post-survey. At the end of the course all respondents replied that they felt very comfortable or somewhat comfortable with statistics content. Several respondents commented on the importance of the use of technology throughout the activities; for example, "I loved the use of technology in this course. Although a bit difficult for each student to use on an individual level, it definitely answered 'what if' questions about large quantities of data or doing something multiple times, repeating samples, etc." Related to the Project-SET–developed activities, the large majority of teachers, eight out of the nine who took the survey (89%) noted that they strongly agreed that the Project-SET PD provided a forum to practice the content and develop more in-depth knowledge about the content. A teacher made the comment: "The activities have really helped me to understand conceptually what I knew procedurally. It has provided several 'aha' moments that I am to take into the classroom."

Taken as a whole, the results from the CAOS, open-ended assessments, and the surveys illustrate the potential effectiveness of the Project-SET materials.

CONCLUSION

Project-SET focused on developing LTs to map out how learners could achieve deep knowledge about sampling variability and regression. To accompany the LTs, lesson plans and open-ended assessments were also designed. Using the loop structure of the LTs, PD was constructed to guide teachers towards conceptual understanding of these topics. The results from the PD were encouraging in that teachers not only increased their content knowledge on the topics but the depth of coverage of the topics allowed for teacher beliefs and misunderstandings to surface. Discussions revolving around the uncovered misunderstandings provided a forum for teachers to discuss their beliefs. It was only through these discussions that teachers were able to reach pivotal moments in their understanding – "a-ha" moments. The Project-SET PD illustrated that LTs provide an effective framework for helping teachers achieve deep understanding.

The developed materials may be directly downloaded from the website and, in turn, implemented in a PD course for teachers. Ideally, the materials would be used as a whole; however, the materials for the two topics could be used separately. For example, a PD that focuses only on regression could merely use the regression materials without the sampling variability. While the PD discussed in this paper lasted 45 hours, because of the misunderstandings uncovered, the materials were all updated to include more scaffolding. Therefore, the materials now encompass an estimated 55-60 hours of PD. These hours could be covered in an intensive summer setting or in weekly meetings or in a course. The materials are flexible in that they can be utilized in different formats.

The goal of the project was to provide standalone materials for those looking for a structure to teach and learn these two fundamental topics in statistics.

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