STUDENTS' CONCEPTIONS OF AVERAGE AND STANDARD DEVIATION

Véronique Dubreil-Frémont, Christelle Chevallier-Gaté and Noëlle Zendrera
L'UNAM Université, Université Catholique de l'Ouest, Angers, France
veronique.dubreil@uco.fr

As statistics teachers, we investigated undergraduate students' conceptual understanding of two major measures of descriptive statistics, average and standard deviation. First, 352 students were asked to define average and standard deviation before their first statistics class. The answers confirmed that most of the students had an algorithmic conception of average and failed to explain what standard deviation is. We then focused on the way students' understanding of average and standard deviation improved, which is still largely unexplored: so we repeated the same experiment after the last statistics lesson of the year. Students' preconceptions seem to have disappeared after statistics teaching; the final results showed a better understanding of average and standard deviation with some lingering difficulties for standard deviation.

INTRODUCTION

As descriptive statistics are largely used in everyday life, it seems important to understand common statistical measures such as the mean or the standard deviation.

In France, statistics are taught before college, by mathematics teachers. The aim of this study is twofold: first, evaluate knowledge of undergraduate students of descriptive statistics before the introductory statistics course; and second, examine their evolution at the end of the course, in order to question our way of teaching statistics. It completes our previous work on the mean (Zendrera, 2010; Dubreil-Frémont et. al., 2012) and on social representations of statistics (Bihan-Poudec, 2012).

METHOD

Undergraduate students in social sciences were asked to fill in a questionnaire before and after a descriptive statistics course.

The questionnaire combined multiple-choice and open-ended questions on their understanding of mean and standard deviation.

In the multiple-choice questions, students had to choose between 4 answers to characterize the way they think they can deal with the mean and the standard deviation (see Table 1).

Table 1. Students' choice

Statistical measures	Abilities			
	"I understand it"	"I think I get it"	"I vaguely see what it is about"	"Never heard of it"
Mean				
Standard deviation				

The aim of the open-ended questions was to describe separately what mean and standard deviation are. The answers were then examined.

Students used up to 3 different answers to define mean or standard deviation; some of them were right but others were wrong.

A total 352 students were tested before the course and 318 after.

RESULTS

What do Students Know About the Mean?

Multiple-choice question about the mean: Students' abilities concerning the mean are illustrated in Figure 1.

Before their first statistics lesson, most of the students answered they thought they were able to comprehend a mean. At the end of the course, the median value is "I understand it". These results seem to show that students improved their understanding of the mean.

In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9, July, 2014), Flagstaff, Arizona, USA.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org

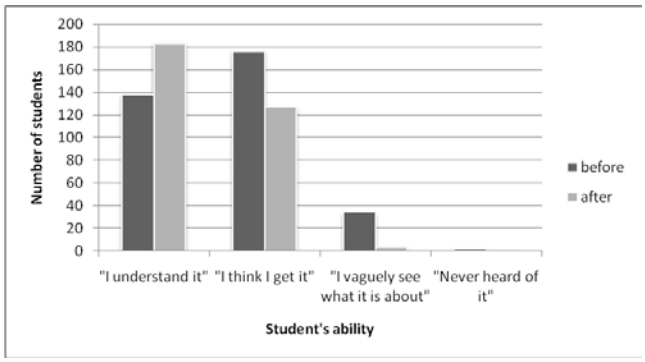

Figure 1. Evolution of students' answers on the way they understand the mean

Open-ended question on the mean: Among the 352 students tested before their first statistics lesson, 19% (67) did not give any definition on the mean, 66.8% (235) were able to define mean and 14.2% (50) gave a wrong explanation for the mean. After the course, among the 318 students tested, 10.1% (32) didn't give any definition and 16.4% (52) gave wrong ones, but most of them (73.6%; 234) succeeded in defining what a mean is, another argument which confirms that students improved their understanding of the mean at the end of a descriptive statistics course.

Table 2. Students' conceptions of the mean and their definitions

Conceptions of the mean	Before the course		After the course	
	Total	Percentage	Total	Percentage
algorithmic definition	211	61.2	175	48.7
summary	40	11.6	68	18.9
midpoint	11	3.2	22	6.1
egalitarian conception	6	1.7	13	3.6
"center of gravity"			8	2.2
point of balance			4	1.1
tautology	26	7.5	22	6.1
median	7	2.0	6	1.7
midrange	1	0.3		
mode			2	0.6
range			1	0.3
other	43	12.5	38	10.6
Total	345	100	359	100

algorithmic definition	sum of data divided by the number of data
summary	summary of a data set
midpoint	middle that represents a set of data
egalitarian conception	value that each one receives if the sum is divided equally among all members of the group
point of balance	the sum of negative deviations from that point is equal to the sum of positive deviations from that point
median	value that separate the higher half of a data set, from the lower half
midrange	mean of the lowest and highest values
mode	value that occurs with a greatest frequency
range	difference between the lowest and highest values

The 285 students' answers about the mean before the course and the 286 ones after the course were then analyzed and classified into several approaches (see Table 2).

These students' conceptions are in agreement with other studies on students' understanding of the mean (Pollatsek et. al., 1981; Mokros & Russel, 1995; Bihan-Poudec, 2010; Watson & Moritz, 2000).

Whenever students are tested, most of the definitions given of the mean were the algorithmic one: 61.2% before the course and 48.7% after. We can also see that the percentage of good definitions (algorithmic definition, summary, midpoint, egalitarian conception, "center of gravity", point of balance) improved from 77.7% before the course to 80.8% after it. So, students before their statistics course in college mostly had an algorithmic conception of the mean, probably because they have been taught by mathematics teachers in middle and high school. Then, after the course, their conceptions became more connected to data sets.

What do Students Know About Standard Deviation?

Multiple-choice question about standard deviation: Students' abilities concerning standard deviation are illustrated in Figure 2.

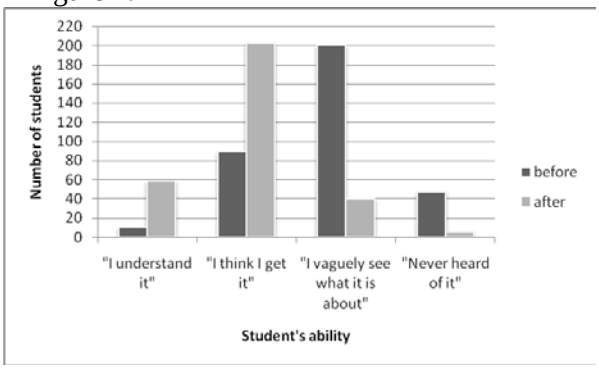

Figure 2. Evolution of students' answers on their understanding of standard deviation

At the beginning of the statistics course, most of students vaguely saw what standard deviation is. Then, after the course, the median answer became "I think I get it". So, first, we can see that the mean seems to be better understood than standard deviation. And second, students seem to improve their understanding of standard deviation.

The answers to the open-ended question confirm that students made progress. The first time, 71.6% (252) were not able to tell anything about standard deviation, only 4% (14) gave a good explanation for it and 24.4% (86) who tried to explain failed. The second time, 38.1% (121) were still unable to tell anything about standard deviation, and 27.7% (88) who tried it failed, but 34.3% (109) were now able to speak about it.

Definitions of standard deviation were also analyzed and classified as shown in Table 3.

Table 3. Students' conceptions of standard deviation

Conceptions of standard deviation	Before the course		After the course	
	Total	Percentage	Total	Percentage
measure of dispersion	9	8.3	34	14.3
square root of variance	11	10.2	109	45.8
tautology	1	0.9		
variance			3	1.3
range	28	25.9	9	3.8
midrange	1	0.9		
interdecile range	2	1.9	1	0.4
interquartile range	2	1.9	3	1.3
other	54	50	79	33.2
Total	108	100	238	100

Before the course, only 18.5% of the answers gave good definitions of standard deviation (measure of dispersion and square root of variance); this rate increased to 60.1% after the course. Compared to the mean answers, these percentages illustrate the difficulties of understanding standard deviation.

CONCLUSION

This study dealt with students' conceptions of the mean and standard deviation before and after the statistics course. The results showed that students have a better understanding of the mean than of standard deviation, whenever they are tested. They confirmed that students improved their conceptions of mean and standard deviation at the end of the course. This is in agreement with previous studies (Garfield & Ahlgren, 1988; Strauss & Bichler, 1988; Gattuso & Mary, 1996; delMas & Liu, 2005; Shaughnessy, 2007).

The next step is the analysis of the answers, student by student: are the answers of the multiple-choice questions correlated to those of open-ended questions? what do the answers become after the course? Then we will compare their declarations with their ability to perform.

REFERENCES

Bihan-Poudec, A. (2010). L'enseignement de la statistique : en premier lieu, l'apprenant. *Statistics Education Research Journal, 9*(2), 88-103. http://iase-web.org/Publications.php?p=SERJ.

Bihan-Poudec, A. (2012). Statistiques ou statistique ? Que peut apporter la théorie des représentations sociales à la compréhension de son enseignement? *Statistique et Enseignement, 3*(1), 37-62. http://publications-sfds.math.cnrs.fr/ojs/index.php/StatEns/article/view/113.

delMas, R., & Liu, Y. (2005). Exploring students' conceptions of the standard deviation *Statistics Education Research Journal*, *4*(1), 55-82. http://iase-web.org/Publications.php?p=SERJ.

Dubreil-Frémont, V., Bihan-Poudec, A., & Chevallier-Gaté, C. (2012). Quelles conceptions de la moyenne ont les étudiants en Sciences Humaines et Sociales ? *Communication au 3e colloque francophone international sur l'enseignement de la statistique*, Angers, 12 septembre.

Garfield, J., & Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. *Journal for Research in Mathematics Education*, *19*(1), 44-63.

Gattuso, L., & Mary, C. (1996). Development of concepts of the arithmetic average from high school to university. In L. Puig and A. Gutiérrez (Eds.), *Proceedings of the 20th International Conference for the Psychology of Mathematics Education* (pp. 401-408). Valenzia, Spain.

Mokros, J., & Russell, S. (1995). Children's concepts of average and representativeness. *Journal for Research in Mathematics Education*, *26*(1), 20-39.

Pollatsek, A., Lima, S., & Well, A.D. (1981). Concept or computation: Students' understanding of the mean. *Educational Studies in Mathematics*, *12*, 191-204.

Shaughnessy, J. M. (2007), Research on statistics learning and reasoning. In F. Lester (Ed.), *Second handbook of research on mathematics learning and teaching* (pp. 957-1010). Greenwich, CT: Information Age Publishing and National Council of Teachers of Mathematics.

Strauss, S., & Bichler, E. (1988), The development of children's concepts of the arithmetic average. *Journal for Research in Mathematics Education*, *19*(1), 64-80.

Watson, J. M., & Moritz, J. B. (2000). The longitudinal development of understanding of average. *Mathematical Thinking and Learning, 2*(1&2), 11-50.

Zendrera, N. (2010). *Enseignement et apprentissage des tests d'hypothèses paramétriques : difficultés rencontrées par des étudiants en sciences humaines. Une contribution à l'éducation statistique*. Thèse doctorale en Education (PhD), U. de Sherbrooke.