

MIDDLE AND HIGH SCHOOL TEACHERS' TRANSFORMATIVE LEARNING OF CENTER

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We describe a project to design and implement professional development and investigate how dilemma, critical reflection, and rational discourse affect middle and high school teachers' reasoning about center. Framed by transformative learning theory, the study highlights how teachers' engagement with activities focused on these elements can enhance understandings of traditionally problematic content. Results reveal that critically reflecting on activities, engaging in rational discourse while working on activities, and examining multiple perspectives through multiple representations and conceptualizations broadened teachers' perspectives on the mean. This study contributes to teacher education by identifying circumstances conducive to deepening statistical understandings and supporting reasoning in increasingly sophisticated ways using a theoretical frame for adult learning to investigate teacher learning.

BACKGROUND

The proliferation of data in our technological world brings about an urgent need for schools to graduate statistically literate students. Considerable research work offers insights into how students learn statistics content in conceptually meaningful ways. Translating these insights into generalized classroom practices, however, necessitates teachers who are knowledgeable about the content and ways to teach the content. Anecdotal evidence suggests that many teachers lack sufficient experiences with statistics content to realize current educational visions in statistics.

Although considerable research focuses on center, evidence suggests that teachers' knowledge related to this foundational topic remains largely procedural (e.g., Leavy & O'Loughlin, 2006) and similar to that of their students (Groth & Bergner, 2006; Jacobbe, 2012). Continued research is needed to design instruction and activities that are effective in facilitating teachers' development of conceptual understandings as one step towards achieving larger educational goals. In this paper, we describe an investigation of professional development (PD) designed to support middle and high school mathematics teachers' reasoning and learning in statistics. We focus on measures of central tendency to address the research question: How do dilemma, critical reflection, and rational discourse affect teachers' reasoning about measures of central tendency?

FRAMEWORK

Our work is framed by transformative learning theory, which is based on constructivist assumptions. Within this framework, powerful learning results from transforming meaning perspectives, which consist of broad predispositions formed from culture and prior experiences (Mezirow, 1991). Perspective transformation begins with one or more events that trigger disorienting dilemmas to prompt examination of broad presuppositions or with series of incremental dilemmas that prompt examination of particular knowledge, beliefs, or attitudes. We use dilemma to refer to both types. Critical reflection begins by critiquing presuppositions upon which knowledge from prior learning are built. Upon reflection, individuals may engage in rational discourse—dialogue with oneself and with others to examine alternative perspectives and to assess expectations, knowledge, beliefs, and attitudes—to develop and act on plans to resolve dilemmas.

The PD program incorporated characteristics of “high quality” PD such as sustained duration and focus on developing content knowledge (e.g., Garet, Porter, Desimone, Birman, & Yoon, 2001). The program included a one-week intensive summer experience with six follow-up sessions during the school year. Teachers actively experienced K–12 statistics content as learners and experienced powerful pedagogical strategies for teaching statistics. Teachers had opportunities to engage in rational discourse with other teachers and opportunities to reflect critically on their activity and practices to enhance their skills and knowledge.

We designed PD tasks with planned triggers for dilemmas in typical areas of struggle within an overarching framework of reasoning about variation (Peters, 2011). The collection of

activities to investigate reasoning about measures of central tendency began with teachers defining mean, median, and mode. Next, using hypothetical student work for data from a random sample of 15 children's ages displayed in case value plots, histograms, and dot plots, teachers considered "typical" ages and correct and incorrect ways in which to represent and interpret "typical." Teachers then used weights and physical number balances centered at 10 and numbered from 0 to 20 to explore mean as a balance point. They first explored data sets with one value and then different configurations of data sets with two values, four values, five values, and so on, all with mean 10. Next, teachers revisited the context of age to compare sample characteristics with population characteristics and to consider "typical" for ages represented quantitatively and categorically. They also considered: connections among histograms, dot plots, box plots, and case value plots; measures of central tendency in terms of larger distributions; and relationships between shape and measures. In one of their last activities, teachers considered multiple blood pressure readings to consider center as a signal within the noise of data. After completing the series of activities, teachers revisited their definitions for mean, median, and mode, and the focus shifted to examining the definitions in terms of properties of mathematical objects. We asked teachers to specify the properties revealed in different definitions, particularly algorithmic definitions, and to make connections among conceptualizations of mean as balance point, fair share, algorithm, and signal within the noise of data.

METHODS AND DATA SOURCES

Eleven middle school (MS) and nine high school (HS) teachers with between one and 30 years of teaching experience participated in the summer PD program; 17 teachers continued during the school year. Multiple data sources included semi-structured interviews, recordings of PD sessions, written work, and classroom observations. Semi-structured interviews were conducted midway through and at the conclusion of the PD program to provide insights into how dilemma, critical reflection, and rational discourse affected reasoning. Teachers also responded to statistics problems with multiple entry points to examine their statistical reasoning. We gained further insight into teachers' reasoning from PD session recordings and teachers' written work. We observed, recorded, and took field notes for two teacher-identified statistics lessons to determine alignment of instruction with PD content. Field notes documented displayed writing and images, referents for comments, and content-related nonverbal movements. We completed post-observation interviews to ascertain teachers' perceptions of the lessons and goals; unanticipated questions, responses, and events; and how PD work informed lessons.

To analyze the data, we first examined PD aspects designed to encourage dilemma, critical reflection, and rational discourse by identifying relevant transcript passages. We searched written work and remaining transcripts for evidence of the constructs, paying attention to indications of insights, questions, or confusion; thoughts and reasoning beyond the immediately observable; content-related interactions with others; and references to the preceding as potential evidence for dilemma, critical reflection, and rational discourse, respectively. We similarly examined transcripts from classroom observations, post-observation interviews, and semi-structured interviews. We used meaning condensation (Kvale, 1996) to compress data events into succinct formulations that maintained pertinent information about the construct(s) and examined events more critically by searching for evidence of triggering events and dilemmas related to central tendency, questioning of presuppositions related to central tendency, and assessment of content-related ideas. We conducted additional passes through the data using the constant comparative method (Glaser & Strauss, 1967) to extract descriptions of teachers' reasoning in relation to the constructs.

RESULTS

Preliminary analyses suggest that dilemma, rational discourse, and critical reflection aided teachers in deepening their understandings of measures of central tendency, particularly for the mean. Different teachers identified different elements as particularly effective for their learning.

Dilemma

PD elements designed to provoke dilemma included presenting teachers with hypothetical student reasoning, confronting incomplete prior knowledge, and representing data in multiple ways.

When teachers encountered hypothetical student work with a case value plot, most teachers resisted the representation. When asked to reconsider how the graph could be used to find the mean, however, some teachers seemed surprised to realize that the representation highlighted mean as fair share. For example, Nathan noted, “Oh, I see what’s going on...these are all plusses [bars extending beyond mean] and these are all minuses [bars not extending to the mean], and they have to balance out.” Although Nathan seemed to have a robust understanding of mean, he was able to use this novel [to him] representation to connect the mean to the idea of fair share or balance.

Dilemmas often are provoked when confronting situations contradictory to prior experience or knowledge. Several teachers, for example, identified dilemmas when they encountered inadequacies in their definitions of mean. Perhaps not surprisingly, initial definitions mostly consisted of algorithmic statements such as Jean’s definition for mean as “a calculated value where the sum of the data is divided by the number of values.” After participating in the PD activities, some teachers’ definitions changed to include mention of deviations, balance, or leveling. When asked to specify the properties revealed in different definitions, particularly algorithmic definitions, teachers distinguished between procedural and conceptual descriptions. The preferred definition became the point at which deviations balance: the sum of distances from the mean for values less than the mean equals the sum of distances from the mean for values greater than the mean. Jean, for example, noted, “[this definition] lends itself to the conceptual understanding of mean.” Ruthie observed, “to derive that one $[\bar{x} = \sum x_i / n \text{ from } \sum (x_i - \bar{x}) = 0]$, I had to start with the sum of the deviations...putting the importance on the deviations.” Discussing the definition of mean was memorable for teachers. Kit later noted, “I got such a better understanding of the mean as a balancing point...the idea of really emphasizing it as the balancing point...taking it beyond the formula just really stuck with me.”

Dilemmas may stimulate thought. Many teachers mentioned encountering the idea of mean as a point of balance prior to the PD. After using physical number balances and seeing the resulting balance, however, teachers professed enlightenment when nonsymmetrical more so than symmetric configurations balanced. For example, Laura stated that three weights at 12 and one weight at 4 would not balance. Her next utterance, “oh,” suggested surprise with the resulting balance. Her partner, Tim, observed, “I wouldn’t have [expected that] either, but it sure does that.” Ruthie, a HS statistics teacher noted, “wow, it really is the balancing point...I understood it was the balancing point, but...to actually visually see it was eye opening.” Exploring mean as a balance point seems to have had a lasting effect on teachers, as many teachers reasoned about mean as a balance point in their interviews and explicitly identified this activity as influencing their reasoning.

Rational Discourse

PD elements to engage teachers in rational discourse included presenting alternative representations and perspectives and presenting teachers with complex tasks to interact and communicate with teachers at similar and different grade levels. Dilemmas seemingly inspired teachers to seek knowledge, often provoking them to seek insights from data displays or from other perspectives. Implicit in the previous section was the importance of multiple representations for reasoning about data and considering measures of central tendency. Tim, for example, noted: “we looked at [center] in different ways...different types of graphs...a variety of ways that we looked at those measures...was really good.” Valenta, who expressed prior statistical consideration mostly in terms of calculation credited “looking at the sets of data...the idea of then seeing...in a set of data and what does that tell you?” for broadening her perspective on measures.

Teachers professed benefit from hearing others’ perspectives. For example, Ruthie noted the importance of interaction: “I think of things algebraically...whenever you can get with someone who doesn’t think in that way, it helps you...because I have a hard time not doing it that way.” Others attributed benefit to “leveling” discussions with alternative perspectives: “important about the groups was being able to discuss different ideas...thinking individually, talking as a group, and then talking as a whole group...refining your thoughts as you go.”

The PD included MS and HS teachers, and teachers at both levels expressed benefit from working with each other. Brooke, a MS teacher, indicated that from working with HS teachers, in addition to “seeing [how] what am I doing with my sixth graders [is] going to affect them in later mathematics,” she could have many of her questions answered. She also believed the HS teachers

learned: “when I was asking him the questions, he said ‘oh, you’re making me think about this.’” Ruthie, a HS teacher, indicated that she gained pedagogical content knowledge: “there were a lot of things this summer that helped me see things maybe through a kid’s eyes...especially having the middle school teachers there. And they’d say, well my kids would think of it this way.”

Critical Reflection

PD activities engaged teachers in reflection by consistently asking higher-level questions, requesting justification, and employing reflective prompts. When asked what aspects of the PD were effective for her learning, Brooke indicated the reflection that activities prompted: “it was the thinking that I did...this is something that I struggle with...I needed it to be where I struggle so that I can come up with the questions to ask to move myself along.” Nathan indicated surprise and joy at “the level of depth where we are talking about okay, why are we thinking this way? Why are we looking at things this way? How else might we look at things?” For him, justification and considering alternative perspectives were important—aspects identified by Laura as well: “show me another way...It was always the technique and the strategy and how you arrived at that answer.” Yvette identified the importance of writing in response to reflective prompts as the most important aspect of the PD for her learning. “I hate to write, but you know, to have to write about it kind of pushed me...it made me sit down and think and make me put down in words...it makes you reflect.” Although not easy for teachers, prompts for deep thought apparently prompted teachers to confront their understandings and to enhance their understandings through reflection.

CONCLUSION

Dilemma seemingly inspired teachers to seek knowledge, often provoking them to seek insights from displays or alternative perspectives through rational discourse. Multiple representations and interactions with others enabled consideration of multiple perspectives, conceptions, and representations of concepts such as mean. Reflection, through justification and consolidation of thoughts, brought further enlightenment and sustainability to thoughts. The PD seemingly affected teachers’ statistical knowledge and how they taught statistics and mathematics more generally. Brooke, for example, indicated that she once dreaded teaching statistics but now is “excited and want[s] to teach it.” Nathan used more graphs in his teaching, “starting as a dot plot and...constantly changing and seeing how viewpoints change.” Jean described general changes: “we talk about big ideas now all the time instead of focusing on what the numbers are...it’s representations.” However, not every teacher seemingly experienced similar enlightenment such as one novice teacher who was looking for ready-made activities and expressed dissatisfaction with the PD. Overall, this study contributes to teacher education by identifying circumstances conducive to deepening statistical understandings and supporting reasoning in increasingly sophisticated ways but raises questions about what support might affect broader teacher populations.

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