TEACHING STATISTICS CONCEPTS THROUGH STOCK MARKET CONTEXTS

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Most students of statistics will learn that variability is the key to separating transient differences from reproducible differences. But many students will not understand the role of variability in the financial markets. Concepts of "risk" and "diversification" are often misused in this context, and this misuse can be blamed on a faulty application of basic statistics. In this talk, I describe how to introduce students to portfolio management though virtual portfolios, and how to assess risk and diversification. The talk will also show how to introduce students to the idea of options without getting into the arcane details. An exposure to these ideas will help students to leverage their understanding of basic statistics in selling their expertise to future employers. Some programs in R can be made available to teachers who wish to use this approach.

INTRODUCTION

I taught statistics for 40 years at various levels, but only in the last few years of my teaching career did I say anything about the stock market. It turns out that statistics ideas can be very useful in understanding the stock market, and many students will benefit from an introduction to the connection between basic statistics ideas and stock market applications. The key to this connection is the understanding of variability in time series; the near-stationarity of stock-market prices make this advanced topic accessible to beginners, since trends can be ignored in the short term. Statistical tests and estimates that incorporate measures of variability are the bread and butter of basic statistics, but sometimes the importance of variability as a descriptive measure in its own right is not fully explored. The evaluation of stock-price time series is a perfect context for filling this gap. The need to deal with risk and diversification in stock market portfolios requires a good understanding of variability, and students in undergraduate programs in statistics should have that understanding before graduation.

RATIONALE FOR THIS APPROACH

The GAISE Report (2007) recommended, amongst other things, the importance of stressing conceptual understanding, and this paper is considering an instance where misunderstanding of a very basic concept has become widespread. The measurement of "risk" by variability is convenient for the financial analyst but is not faithful to the common meaning of "risk". If statistics education is really for everyone, we need to be careful how we use everyday language when we prepare students for the real world.

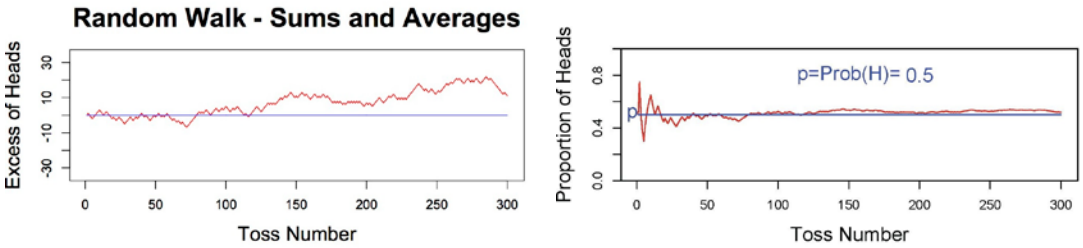
VARIABILITY IN A STOCK MARKET CONTEXT

The variability we are considering is the change in the closing price of a stock from one time period to the next, usually one day to the next day. An investor buys a stock at one price and sells the stock at a later time, and financial advisors pay a lot of attention to the amount of variation in the stock price over time. We examine the role of this variation in advising the investor.

Stock Price Variability Is Not Risk

The first thing to realize about the series of closing prices of a particular stock is that it is very hard to predict whether the next day's price will be higher or lower than today's price. Naïve investors think they see patterns that will provide an edge in this prediction. The symmetric random walk is a model that students can understand, and it is clear from the model that there is no predictability in this series. And yet, even this series seem to have suggestive patterns helpful for prediction, as can be simply demonstrated. The first graph shows a typical result from 300 tosses of a fair coin – the apparent upward trend is not useful for prediction. The second graph confirms that the long run probability of heads is 0.5 – there is nothing wrong with the simulator. These apparent trends up or down are very common even when there is no reproducible underlying trend.

In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9, July, 2014), Flagstaff, Arizona, USA.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org



Thus, a study of the past price pattern in a symmetric random walk is of no help for short-term forecasting of that random walk. Since the symmetric random walk has clearly no useful trend for forecasting, any similar trends in real word stock prices are also not to be trusted for prediction. It is more likely to help with longer-term forecasting, since slight departures from symmetry would be revealed. But the time required for this approach to be useful may require several years of data.

When an investor buys a stock, he or she hopes to make some profit from the investment, either as income (dividends) or as capital gain (higher price on resale). Of course, in order to obtain a capital gain, the stock has to be sold at a higher price. There is a risk that this will not happen in the time horizon the investor allows for this investment. This could be because the investor does not sell the stock when the price has increased (waiting for a even higher price) or because the stock price has declined since the purchase date during the time horizon. Thus there is a risk of a loss from the investment.

How do you measure the risk of loss from an investment in a stock? There are many considerations: general level of market prices at purchase time, susceptibility of the business model to competition, world upheavals, technological changes, ability of company management, fiducial responsibility of management, etc. However, the only way risk is measured by retail investment advisors is by variability of the stock price (daily, or monthly, for example). Even statisticians follow this traditional view of risk: "Therefore, risk is synonymous with variance and if investors are risk averse they want to minimize risk." (Christou 2008). The standard deviation of stock price is what they call "risk" and yet it clearly does not measure the likelihood of loss for the investor. There is no simple way to measure true "risk".

It is true that stock price variability does measure something. For each stock, many investors buy and sell the stock depending on their guess of the future price of the stock during the time interval they wish to invest. When these assessments change with large amplitude, it does say something about the sensitivity of the stock price to "random" events. For a long-term investor, this kind of variability is almost irrelevant. To the short-term investor or "day trader", this variability is a good opportunity. Of course, the chance that the stock will fall catastrophically in a short time, perhaps to $0, is something that every investor needs to consider, but is not a realistic concern for most established companies. It is even less of a concern for an investor that has a portfolio of say thirty stocks, since a stock could go to zero without crippling the portfolio. The bottom line for this discussion is that stock-price variability does not measure "risk" for either the short-term investor or the long-term investor. What it does measure is sensitivity of the stock price to random future events.

Financial advisors like to claim they can measure risk, since it does seem that risk aversion is a characteristic that will vary among investors, and an advisor should be able to take this into consideration in recommending a course of action for an investor. However, we have noted the factors that affect true risk, and have explained that stock price variability does not measure it. One reason that financial advisors find the measure of variability useful, other than conveying a misleading impression to clients who wish to buy stocks, is that stock-price variability is the key measurement in establishing the price of "options". To appreciate this we need some terminology associated with options.

Stock Price Variability Measures: The Value of Options

For an investor that holds an option, that investor has a choice of whether or not to exercise that option, namely to buy or sell a certain stock at a specified price during a specified time period.

The person selling that option therefore has an obligation to satisfy the choice of the option-holder, and accepting this obligation will require a certain payment by the buyer of the option. An option is specified by naming the underlying stock, the exercise price (called the "strike" price), and the time period over which the option may be exercised. Options are bought and sold on the market just like the stocks that they relate to.

What determines the price of an option? For example, what would have to happen to the price of the underlying stock in order for the holder of the option to benefit? It will turn out that the stock price has to have a large swing up or down, depending on the kind of option. To investigate this further, consider the two kinds of options: CALL options and PUT options. CALL options are opportunities to BUY, whereas PUT options are opportunities to SELL. For a CALL option-holder to benefit from his option, the underlying stock price has to go UP. For a PUT option-holder to benefit from his option, the underlying stock price has to go DOWN. In both cases, the option holder has a better chance of benefitting from the option if the stock price changes, and since the strike price is fixed as specified by the option, the larger the price change in the right direction, the greater the benefit. Holders of CALL options want the price to shoot up, at least temporarily. Holders of PUT options want the price to fall way down, at lease temporarily. In other words, the holder of an option has a better chance of benefitting from the option if the underlying stock price is *variable*. If we discount the chance of the stock price going to \$0, then any stock price movement in one direction will eventually be followed by movement in the other direction, and this is why variability of a stock price will determine the price of the option. For a more detailed introduction to options, see Hull (2012).

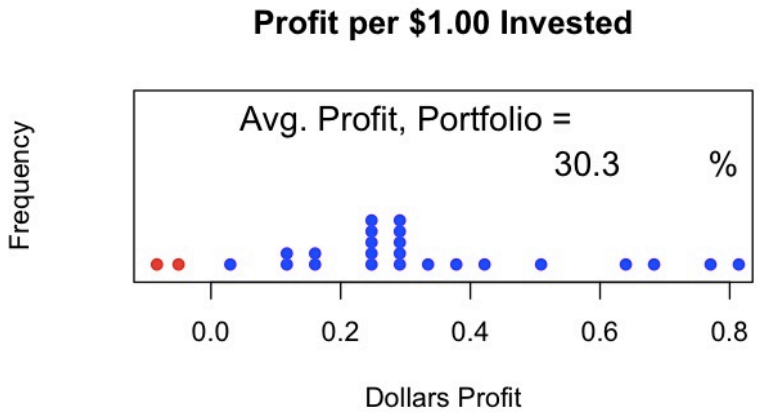
So stock price variability measures the appropriate price of options, but does not measure investment risk for the holder of the stock itself.

Diversification Reduces Variability

That diversification reduces variability is obvious, although the factors that affect the degree of variability reduction are worth considering. Before we get to that, and in view of our emphasis on the idea that "variability is not risk", one could ask why we might want to reduce variability. If diversification does not reduce risk, why bother? The answer is that we are not all long-term investors, and depending on how soon we might need to cash in some investments, the chance that the market value of our portfolio drops to below cost is a concern. In other words, for the medium term investor (say 1-3 years), variability should be reduced as much as possible, while still preserving the best chance of gain. So for the medium term investor, portfolio-market-value variability is a concern, and we will discuss how diversification can help with this.

A truism of the markets is that high reward stocks are often highly variable stocks. Let us consider an artificial situation in which a company RISK1 is a highly variable stock but which can have a big reward. More specifically, suppose the RISK1 share value is \$1.00 now and suppose we know that, in one year, the stock will be worth either \$0, \$0.50, \$1.00 or \$4.00, all with equal probability. Note that there is a 50% chance of losing money with this stock, and only a 25% chance of making a profit. However, when a profit is made, it is a profit of \$3.00 per share, which is a 300% gain.

Now let us indulge our imagination and suppose we have a portfolio of 30 such companies, and let us make the brave assumption that these 30 share values have independent experiences over the next year. This would imply, for example, that if something good or bad happens to one company, there is no link to simultaneous changes in the other 29 companies. In this scenario, even though all the companies have very little chance of making money, the portfolio is almost guaranteed to make money. Here is a typical simulation of the outcome:



In this artificial situation, the portfolio of risky companies produces a fairly secure profit. Thus, in this case, diversification has performed a small miracle: reduced variability and reduced chance of loss.

Now consider a more realistic situation in which the experience of one company is similar to the experience of other companies. Obviously, if all the companies experience the same share effect the portfolio will have the same variability as the individual stock. But in real life there is some degree of independence between stocks in a portfolio, so the tendency is for the portfolio to have a lower variability than an individual stock. Thus diversification tends to reduce the variability of a portfolio. And, if the average stock price of the stocks in the portfolio increase, then of course the portfolio value will increase, but this is not assured by the diversification.

In other words, diversification reduces variability of a portfolio, and this is useful for an investor with a medium-term time horizon. Such an investor can give up a small potential for a large gain in one stock, for reduced variability, and more modest gain, in a portfolio.

IMPORTANCE FOR TEACHING STATISTICS

Statistics courses cannot fail to impress the student with the importance of measuring variability. Formulas for variance and standard deviation dominate discussions of estimation and hypothesis testing. But much of the discussion revolves around the assessment of mean differences, and the variability is only a tool in this assessment. It is not easy to convince students that the measure of variability itself is a useful descriptive tool. Do we talk about the accuracy of clocks, the reduction of variability in quality control, the comparison of measuring instruments, or the predictability of time series? These are all fairly specific applications, and may not impress the student as having general importance. However the applications considered in the context of the stock market are relevant to anyone hoping to accumulate wealth, and young statisticians may well have this in their career goal. The idea of using a context of great practical importance to familiarize students with a technical area of great theoretical importance seems like a good idea.

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