DEVELOPING CONCEPTUAL UNDERSTANDING: THE ROLE OF INTERACTIVE DYNAMIC TECHNOLOGY

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The U. S. Common Core State Standards recommend that all students should analyze univariate and bivariate data, two-way contingency tables with categorical data, margin of error and whether there is a significant difference between two proportions. This paper describes a project to develop materials to support the teaching and learning of these "hard-to-teach/hard to learn" topics in introductory statistics. The materials, intended to complement core course materials, consist of interactive dynamic documents, student worksheets, and teacher notes that build foundations for the concepts and confront typical student misconceptions. They were designed to reflect the content and pedagogy advocated by GAISE where dynamic interactive technology provides the opportunity to rethink how students might come to better understand central statistical concepts.

THE PROBLEM

For years researchers have documented fragile and incomplete understanding of core statistical concepts by students (e.g., Tversky & Kahneman, 1971; Vallecillos, 1995); some concepts in statistics are clearly "tough to learn" and thus, tough to teach. Population distributions and sampling distributions of sample statistics are often confused. Why can't you add variances? Is 30 a "large enough" sample? Each of these topics and many others are based on critical assumptions and involve nuanced interpretations. Statistical reasoning and sense making depend on conceptual understanding of big ideas, such as data, variation, distribution, representations, association and modeling relationships, probability models for data generating processes, sampling and inference (Burrill & Biehler, 2011). But for many students these ideas remain elusive.

INTERACTIVE DYNAMIC TECHNOLOGY

Research suggests learning occurs when students engage in concrete experiences, observe reflectively, develop abstract conceptualizations based upon their reflection, and actively test the abstraction (Zull, 2002). Dynamic interactive technology provides an environment in which these kinds of learning opportunities can take place. This perspective is supported by a number of studies that suggest the strategic use of technological tools can enhance the development of procedural skills and proficiencies such as problem solving and reasoning (Kastberg & Leatham, 2005; Roschelle et al., 2009).

Using dynamic interactive technology students can take a deliberate action, observe the statistical consequence and reflect on the results (for example, change a data value in a spreadsheet and immediately see the consequences in a graph and in the statistical calculations). Existing applets such as those developed by Rossman/Chance provide such platforms but typically without supporting material for students and teachers. Our hypothesis is that a set of coherent, coordinated activities using this "Action/Consequence" principle and targeted at tough to teach, tough to learn statistical concepts can make a difference in student understanding.

BACKGROUND, FOCUS & PHILOSOPHY

A team of statistics teachers and educators designed activities, focused on those concepts identified as tough to teach from an analysis of assessment responses, reviews of the literature and experience of teachers, that would develop and deepen student understanding. An activity consists of an Action/Consequence dynamic interactive document (an "applet"), a student worksheet, and teacher notes. The materials were designed with little or no use of software menus and intended to support lessons from core instructional materials. The student worksheets are available in Word documents enabling teachers to modify them as they think necessary for their students.

The philosophy in the design of the materials is based on the recommendations of Cobb (1992) and the *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) College Report (Aliaga et al., 2010) including an emphasis on statistical thinking, real data, conceptual

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understanding and active learning. The worksheets have carefully crafted questions, motivated by the formative assessment work of Black and his colleagues (2004) who argue there are only two reasons for a question: to probe student thinking to learn what they understand and to push student thinking towards new learning or making connections from prior learning. The interactive documents are designed for either the handheld or computer TI-Nspire™ platform; however, the worksheets and teacher notes could be adapted for use with any interactive dynamic platform with features similar to those required for the activity. The interactive files should:

- Use simple actions that are mathematically/statistically meaningful (enter a value, click, drag);
- Have visible cues for *actions* students can take and for the *consequences* students should be noticing or thinking about; unimportant consequences should be obscured as much as possible;
- Minimize use of text on screen;
- Locate changing quantities as close together as possible.

DEVELOPMENT PROCESS

The writing team identified concepts with which students struggle based on teaching experience, questions peers raised about statistical subtleties, and experiences grading high stakes assessments in statistics. The research, if any, related to the selected topics was surveyed and incorporated into the conceptualization of the activities. The activities were tried in classrooms, reviewed by members from the broader statistics education community, and revised accordingly. Issues that arose in the development included overcoming limitations of the technology functionality, crafting explicit learning objectives, keeping questions focused on the learning objectives, activating pre-requisite knowledge, the role of context.

The activities cover eight major statistics themes: describing and displaying univariate data, normal distributions, describing bivariate data, probability and random variables, sampling and experimentation, sampling distributions, confidence intervals and hypothesis tests. Each unit has four to six "action/consequence" activities that range from an introductory activity on standard deviation to activities investigating the meaning of alpha, *p*-values and power as well as an introduction to blocking. The discussion that follows briefly describes two of the activities.

SAMPLE ACTIVITIES

Case 1: Confidence Intervals

Misconceptions related to confidence intervals detected by Fidler (2006) include thinking the width of a confidence interval is not affected by sample size and believing a 90% confidence interval is wider than a 95% confidence interval (for the same data), and the interval is a statement about the distribution of the data. Students have trouble distinguishing between confidence interval and confidence level. They frequently make statements such as there is a 95% chance the confidence interval includes the sample mean; there is a 95% chance the population mean will be between the limits of the confidence interval; or 95% of the data are included in the interval. To address some of the confusion, the project separated the development of confidence intervals and confidence levels. The confidence interval activities focus on a confidence interval as the set of plausible values of a population parameter based on the observed sample. Students should recognize that a confidence interval for a population parameter from an unknown population is based on knowing how statistics from samples drawn from known populations behave.

Students began with an observed proportion of successes (i.e., 0.61) of a random sample drawn from an unknown population and then inspect simulated sampling distributions of proportions drawn from random samples from a known population (p=0.75 in Figure 1). In each case students respond to questions such as:

1) Based on your experience with 100 samples, suppose a population consisted of 75% men and 25% women. Would you be surprised if the proportion of men in a random sample of 62 people was 0.55? 0.20? Explain your reasoning.
2) Recall your observed proportion of men from Question 2. Would it seem likely that this sample proportion came from a population where the proportion of men was exactly 0.5? Why or why not?

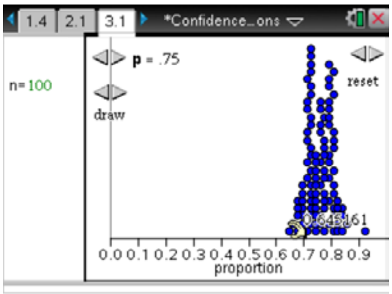


Figure 1: Population proportion = 0.75

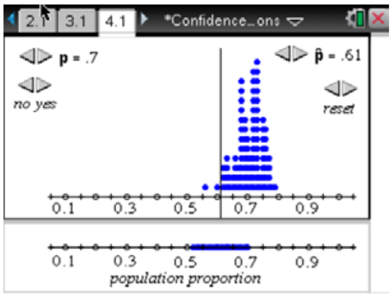


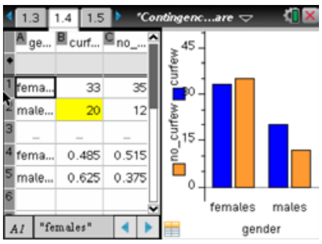
Figure 2: Population proportion = 0.7

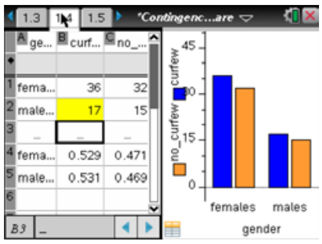
Students generate sampling distributions of known population proportions, *p*, and click "yes" if it seems reasonable that a sample proportion of 0.61 successes could have come from this population. The yes click marks a point on the number line at the bottom of the screen. The set of "yeses" form a "confidence interval"- a set of plausible populations that could have produced a random sample with 0.61 as the proportion of successes (Figures 1, 2). Visualizing the simulated distributions of sample proportions of populations with known proportions of successes establishes the notion of plausible populations as the underlying concept in developing confidence intervals.

Case 2: Two–Way Tables: Significantly different?

Two-way tables are often found, if at all, at the very end of an introductory course. Wild (2006) argues this is unfortunate as too much time is spent on univariate data and not on what is actually important, comparison between groups and relationships between variables. Student difficulties with tables include making decisions based on one cell, ignoring situations where variables are inversely related, using their own beliefs to interpret data (Batanero et al., 1996) and comparing lowest and highest values ignoring the proportionality (Estepa et al., 1999).

Contingency Tables and Chi Square is based on a survey of a randomly selected sample of 100 high school students given the question, "Do you have a curfew?" The activity displays a two-way table and corresponding bar graphs to help students attend to the proportionality (Figure 3). Students test a variety of possible outcomes addressing questions such as, "If gender and curfew were independent, explain what percent of males and of females would have a curfew? (Figure 4)





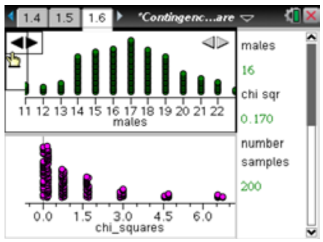


Figure 3: Survey data Figure 4: No difference Figure 5: Simulated distribution

Students generate a simulated chi-square distribution (Figure 5) and respond to questions on how the observed chi-square and number of males in the sample relate to the simulated sampling distribution of the number of males who responded yes, using the chance distributions to make a decision on whether having a curfew and gender are related.

THE ROLE OF THE TEACHER

No matter how powerful the technology or carefully crafted a lesson, the teacher is central in enabling students to learn. Effective use of technology requires teachers to take time to learn the affordances of the particular applet, application, or software; engage students in activities; and provide feedback for students (delMas, Garfield, & Chance, 1999). In addition, teachers should make sure students discuss their observations after an activity so students can focus on observations that were important, become aware of important observations they did not make, and reflect on how important observations are connected (Chance et al., 2007).

CONCLUSION

The materials are intended to provide a solid introduction to statistical concepts and methods and simultaneously increase student interest and success in statistics using dynamic interactive technology to rethink how students might come to better understand central statistical concepts. The questions and tasks posed around the applets can make a difference in whether this approach works for students. The next step is to collect data to investigate the question: what does a set of coherent, coordinated activities using the "Action/Consequence" principle targeted at tough to teach, tough to learn statistical concepts bring to student understanding.

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