TEACHING BOX PLOTS: AN INTERVENTION USING REFUTATIONAL TEXT AND MULTIPLE EXTERNAL REPRESENTATIONS

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Despite the omnipresence of box plots in education and research, many misinterpretations have been reported on this representation for data distributions. Previous studies did not succeed in remediating these misinterpretations, leading to the present study in which two teaching methods are tested: refutational text and multiple external representations (MERs). Refutational text explicitly refutes an incorrect conception and provides an alternative, correct conception. MERs combine various external representations in order to improve the interpretation or understanding. These teaching methods were not only used separately, but also in combination. A posttest showed that students in the control condition scored weakest, and students in the combination condition scored best. The students in the MERs and refutational text conditions scored in between. The implications of these results for theory and educational practice are discussed.

INTRODUCTION

Box plots are often used to graphically depict the distribution of one or more quantitative variables. Recent studies have shown, however, that they are not as easy to interpret as often thought (e.g., Bakker, Biehler, & Konold, 2005; Lem, Onghena, Verschaffel, & Van Dooren, 2012; 2013a; 2013b). Students have more difficulties interpreting box plots than other external representations of data distributions, such as histograms or descriptive statistics (Lem et al., 2013a). Common misinterpretations are, for example, thinking that the median line actually represents the mean, or ignoring the whiskers in the assumption that no data are represented in this part of the box plot (e.g., Lem et al., 2012, 2013a). In this study we focus on one specific misinterpretation: the area misinterpretation (e.g., Lem et al., 2013b). Students who hold this misinterpretation think that a larger area in a box plot represents a larger proportion of data than a smaller area, while a larger area actually represents the same amount of data but spread out over a larger interval, thus representing a lower density.

The goal of the present study is to find a new way to improve students' interpretation of box plots, given that earlier studies were not very successful in improving students' interpretation of the box plot. We test the effect of two instructional techniques: MERs (Multiple External Representations) and refutational texts. The idea to introduce MERs originates from Bakker's (2004) study with the minitool intervention. In the present study we work further on the finding that the use of dot plots to teach box plots helped students to some extent. We will also link the box plot to another data representation, but we now work with histograms as the second representation as this fitted better within the curriculum of the students. With respect to refutational text, we used warnings like Lem et al. (2012b) did, but we extended the warning and made it much more specific to box plots. Moreover, we used refutational text as a format for this warning by explicitly stating the common misinterpretation and refuting it. Refutational text is a frequently used way to replace students' incorrect conceptions by correct conceptions.

METHOD

Participants

Participants in the study were 188 first year students of educational sciences (108 females, 7 males) and speech pathology and audiology (71 females, 2 males) at KU Leuven, Belgium. All students were enrolled in the same introductory statistics course and had been taught about descriptive statistics prior to their participation.

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Materials and Procedure

The intervention was provided in the form of a home study text that was offered to the participants before the teaching of box plots and histograms would take place in class. Four versions of the text were constructed and resulted in four conditions: a control condition, a MERs condition, a refutation condition, and a combination condition. The texts that were used in the three experimental conditions were based on the text of the control condition, except for a few experimental manipulations. In the control condition (n = 45), the text explained histograms and box plots in a standard way based on the textbook used in the introductory statistics course of the participants. The area misinterpretation was not named explicitly and both representations were not linked to each other. In the MERs condition (n = 43), histograms were explained first and were then used to explain box plots by overlaying boxplots onto histograms (see Figure 1 for an example). This way, students were able to actually see the number of data points represented in each of the four parts of the boxplot. In the refutation condition (n = 41), refutational texts were added in two text boxes. In line with the definition of refutational text, these two text boxes first stated the common incorrect interpretation of box plots, followed by an explicit refutation of this interpretation combined with a correct one. In addition, we added a cue to alert the reader of the misinterpretation: In each of the two refutational text boxes we did this by adding a small cue sentence in between the misinterpretation and the correct interpretation: “This is incorrect”. On top of that, we started each text box with “Warning!!” in order to attract students’ attention even more and in the beginning of the instructional text we warned students about the fact that box plots are often misinterpreted by students. In the combination condition (n = 59), finally, box plots were explained using the same overlay on histograms as in the MERs condition. In addition, the text boxes with warnings from the refutation condition were added.

In all four conditions, the same four exercises were included in addition to the instructional text. The exercises were included in order to engage students more in the learning process but also to verify whether students indeed studied the material and whether the possible misinterpretation of histograms would influence the learning about box plots.

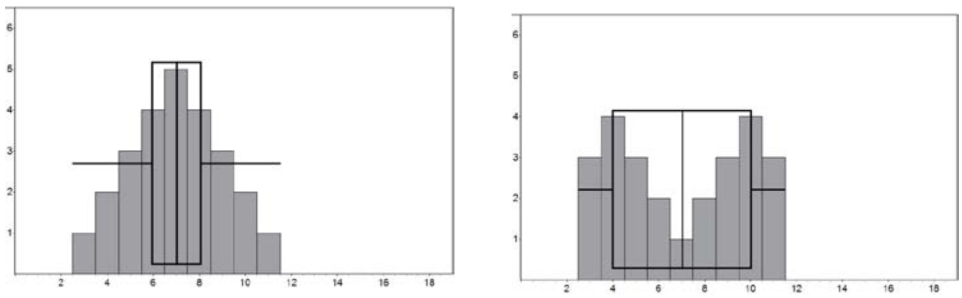

Figure 1. Example of two histograms with a box plot overlay.

RESULTS

A post-test was given to the students consisting of both knowledge items and interpretation items concerning the area misinterpretation. We did not find a statistically significant difference between the conditions concerning the box plot knowledge items.

Three multiple choice-items were used to test whether students interpreted the area of the box plot correctly. We expected that students in the combination condition would score best and that students in the control condition would score worst. We expected the students in the other two conditions to score in between the control and combination condition. A generalized linear mixed model was fitted to all three area items, showing a main effect of condition on accuracy, $\chi 2$ (3,376) = 3.65, p = .013, OR = 0.43 – 0.99. Table 1 shows the percentages of correct responses per item and condition. We see that the control condition in total scored worst (54.8% correct), while the combination condition scored best (73.5%). The MERs condition (67.4%) scored in between, and the refutation condition (73.2%) scored almost as high as the combination condition. Dunnett’s tests showed that the refutation as well as the combination condition differed significantly from the control condition, t(376) = 2.67, p = .022, OR = 0.44, and t(376) = 2.99, p = .009, OR = 0.43, respectively. The difference between the MERs and the control condition was, however, not statistically significant, as were the differences between the three experimental conditions. Accuracy in the control condition was statistically significantly lower than in the three experimental conditions combined, F(1, 376) = 9.60, p = .002, OR = 0.33. This confirms our prediction partly: The combination condition scored better than the control condition, as did the refutation condition, and all experimental conditions combined. The MER condition did not score better than the control condition.

Table 1 *Accuracy rates (in %) for the three area items, per condition.*

	Item 1	Item 2	Item 3	Total
Control	48.9	46.7	68.9	54.8
MERs	55.8	60.5	86.1	67.4
Refutation	73.2	65.9	80.5	73.2
Combination	66.1	67.8	86.4	73.5
Total	61.2	60.6	80.9	67.6

CONCLUSION

In this study we tested two instructional techniques, both separately and in combination, in order to improve undergraduate students' interpretation of box plots. The first technique was the use of multiple external representations, or more specifically, the combined use of histograms and box plots. The second technique we used was refutational text. In a refutational text a common misinterpretation is explicitly invalidated and contrasted with the correct one. We expected that these techniques, and a fortiori their combination, would improve students' interpretation of box plots.

A first important result is that students in all four conditions scored the same on the box plot knowledge items. This suggests that the four conditions were equally successful in teaching students the theory about box plots. However, students did not score the same in all four conditions on the items that could elicit the area misinterpretation. More specifically, we found that students in the control condition scored worse than students in the other three conditions, as predicted. Especially the refutation and the combination conditions scored better than the control condition, with somewhat, but not statistically significant, better results in the combination than in the refutation condition. These results suggest that the use of MERs, refutational text, and their combination worked better to prevent and/or remediate the area misinterpretation than the control condition did. Moreover, it seems that especially refutational text was effective to improve students' interpretation of box plots and that the use of MERs can enlarge this positive effect. However, in one of the area items histograms had to be linked to box plots, which was done best by students in the MER condition. This is not surprising, as the link between the histogram and box plot was very explicitly made in this condition. Which instructional technique works best apparently depends on the task that has to be solved.

We also want to stress that the positive effect of the instructional techniques has to be considered with caution. In neither of the conditions, was the area misinterpretation completely eradicated. The positive effects of refutational text and MERs were at most moderate. Even in the combination condition, on one of the items only two thirds of the students were able to answer correctly.

Various limitations of this study must be taken into account. A first limitation is that the instructional texts were presented as home study texts. We chose to conduct the study this way because of practical reasons and to raise the ecological validity of letting students study a text as if they were studying outside of a research situation. However, the internal validity may have been compromised somewhat due to this choice: We did not have complete control about several factors, such as the time on task or whether students saw or read the instructional texts of other students who were in other conditions. The fact that home study texts rather than direct teaching were used may also explain why relatively moderate effects were found. A follow-up study could be organized in which students have to read the text during one of the practical sessions, or where the different conditions are taught in the actual statistics class. Although this last option would have various practical difficulties, it is possible that the positive trends and effects that we observed in our box plot test for the MERs, refutation, and combination condition would become larger. Second, most students were already taught about box plots in secondary education. It is not clear whether all these students had the same background knowledge, which could have led to a different effect of the intervention texts on different students. Possible solutions for this potential problem could be to replicate the study in secondary education before box plots are taught, or to use a pretest to be able to control for differences in prior knowledge. Third, we did not test the long term effects and transfer ability of the students in the different conditions. This is something that should be done in further research in order to get a better view on the true effects of both MERs and refutational text. A final limitation is that we used questionnaires with multiple-choice questions. Besides the fact that guessing cannot be ruled out this way, this method did not allow us to gather much process data. Future research could use verbal protocol methods to get a better insight into the way students reason when solving the box plot test.

There are various implications of our results for education. First, we have once again shown that the interpretation of box plots is not without difficulties. This means that teachers should pay special attention to them and not treat them like self-evident representations. Second, both MERs and refutational text have proven to be able to yield better interpretation of box plots by students. These teaching techniques can relatively easy be incorporated in statistics classes. Third, the combination of MERs and refutational text seemed to yield even better results than the individual use of both teaching techniques. Teachers can use this to optimize their learning materials.

This study has several theoretical implications. First, with respect to MERs, we can say that this teaching technique seems to have a more positive effect on students' interpretation of box plots than a traditional text about box plots, especially when box plots have to be linked to histograms. Second, we can state the same, and depending on the specific interpretation task, even more strongly so, with respect to refutational text. Third, and most importantly, the combination of these two instructional methods seems to be even more positive for students' interpretation of box plots. This creates opportunities for further research, also in other domains than the interpretation of box plots.

As a general conclusion we can pose that box plots have many interpretation difficulties, but that refutational text seems a viable teaching technique to remediate those difficulties, especially when used in combination with multiple external representations. An important merit of the use of this combination is that it can easily be implemented in the regular curriculum, as it hardly takes any more time and does not require any specific practical procedures than a regular explanation of box plots. More research is however necessary to optimize these teaching techniques, for example by using animation instead of static images, and their learning effect.

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