VERGNAUD'S THEORY APPLIED TO BASIC SCHOOL STUDENTS' STATISTICAL REPRESENTATIONS

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This study addresses the conceptualization of the table and its teaching possibilities at school level. The theoretical framework adopted is the theory of conceptual fields, which allows for a cognitive analysis of conceptualization in learning. Vergnaud (1990, 2013) affirms that mathematics is needed to characterize with minimum ambiguity the knowledge contained in ordinary mathematical competencies and highlights the fact that this knowledge, although intuitive and implicit, should not hide the need for mathematical concepts and theorems for analyzing it. The study explores primary-school children's comprehension and how they make sense of statistical frequency tables by examining their productions when they are faced with a data analysis situation. From a qualitative research standpoint, the mathematical relations underlying the operations students use to resolve a situation were analyzed. The principal result of this study is the identification of different levels of conceptualization of the table for students in the same age group.

INTRODUCTION

The existing literature in mathematics education and statistics education, in general, does not address table learning in children. Some exceptions are the works of Brizuela and Alvarado (2010), Gabucio et al. (2010), Marti (2009), Marti et al. (2010a, 2010b), Ben-Zvi and Sharett-Amir (2005), Brizuela and Lara-Roth (2002). The research of Brizuela and Lara-Roth (op. cit.) showed that 7-year-old students could use tables to resolve problems of an algebraic nature. Although these students had not received direct instruction in the use and configuration of tables, they were able to utilize tables to work on a problem. The tables in Brizuela and Lara-Roth's study were produced without a structure being imposed on the children, which is an element shared by the present study. Marti (2009, p. 12) also explored the use of tables by students and found that, "The process of table construction can change the subjects' a priori knowledge."

In our research, we have considered a list as that which has a one-dimensional character, including enumeration and/or classification -of things, quantities, etc.- and a disposition in columns (vertical reading) or in rows (horizontal reading), does not possess headers, and whose components are separated by spaces and/or punctuation. In contrast, we consider a table to be that which occupies two dimensions -vertical and horizontal-, and is made up of headers and a corpus of data, localized in rows, columns, and cells, which may or may not have marked lines. Additionally, a frequency table is considered in its most basic aspect as a rectangular arrangement with a structure that includes a set of rows and columns (as previously defined) that allows for representing data corresponding to a variable -characteristic of the phenomenon being studied- in an ordered and summarized manner, and facilitates understanding of the behavior of the data and comprehension of the information that can be extracted. In summary, a frequency table is considered to be that which possesses the previous characteristics, explicitly presents the count of each element of the lesson (subcategory), and, eventually, their marginal totals (category count).

Theoretical Perspective for Analyzing Tasks

This study shows the process of table construction in situ and investigates prior conceptions and intuitive strategies prior to teaching. We adopt Vergnaud's theory of conceptual fields (TCF) as a theoretical framework in order to investigate the structure of the concept created and describe its levels of conceptualization (the transformation of a concept in the form it is taught to the form in which it is conceived by the learner).

The analysis of conceptualization begins with *schemes* and continues with an analysis of the activity. The analysis of the conceptualization of tables must be carried out based on the written productions of the students when they resolve the problem. Although a *scheme* is not a behavior, it has the function of generating activity and behavior in a situation. As such, while we cannot

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directly access the mental schema that direct the students' responses in the situation, it is possible to study them through an analysis of the actions and gestures executed, in particular the operational invariants that make the schema work.

An operational invariant is considered to be implicit mathematical knowledge within the *scheme*, which directs the student's recognition of the pertinent elements of the situation and the apprehension of information regarding the situation. In our theoretical framework, the concept of a table, C_{table}(S, OI, SR), includes the situation (S), determined by the task that gives meaning to the concept table, tasks that include ordering, classifying, categorizing, comparing, or registering; the meaning (OI) given by the invariants upon which the division in classes designed to provide order and relations is based; and the signifier (SR), the integration of symbols and language in the visual network, the grid -marked or not- spatially recognizable in the plane, and its properties.

METHODOLOGY

This study uses a qualitative focus to study students' knowledge about the concept table based on experimental data obtained in the resolution of the situation.

Participants

Participants belonged to two third-grade primary school classes in the first semester of 2013 in urban schools chosen based on their accessibility to the professors. One class was made up of 38 students, 16 girls and 22 boys, ages 7 to 8 years. The other class had 42 students, 30 girls and 12 boys, whose ages ranged from 7 to 9 years.

Data Collection

A statistics learning situation was implemented with paper and pencil, looking for the emergence of tables; the data for this study emerges from the productions of the students faced with this situation. The situation is framed in the "Data and Probabilities" theme of the third-grade mathematics course of the Chilean curriculum (MINEDUC, 2012), which calls for students to, "Carry out surveys. Classify and organize the data obtained in tables and visualize it in bar graphs." The goal of the lesson that was implemented was to organize and classify data in order to obtain information. Based on this goal, a context was devised that would be interesting to students, related to the quality of the snacks they eat in school. Based on the snacks brought by the students, a printout was designed with this data displayed iconically. The central question was, "How can we order and organize the data regarding our snacks to find out if we are at risk of contracting a disease?"

Categories of Analysis

Given the importance of systems of representation in the process of conceptualization (Vergnaud, 1990) and the explicitation of concepts, we can expect that in the data analysis situation productions will appear based on systems of representation (Sureda & Otero, 2011). In resolving the snacks situation, we expect three systems of representation: numerical [NSR], referring to counting with numbers; iconic [ISR], referring to constructions with signs similar to the objects represented; and written [WSR], referring to written linguistic forms.

RESULTS

The analysis of student solutions using TCF shows the relation between the students' conceptualizations, systems of representation (SR), and invariant operators (IO). This analysis allows us to recognize the process of conceptualization of the table, which begins with production of lists and becomes progressively more complex in various types of tables, until it reaches frequency tables.

The process detailed in this study shows that student productions in the same situation are guided by different invariant operators, that is, different schemes, sometimes lists, sometimes tables. This shows that, when knowledge of a conceptual field (CF) is incipient or not explicit, contradictory schemes coexist for the same concept. Even when the students only possess list-type schemes, they are used coherently to respond to the initial problem.

The table scheme is not unique, but rather there exists a grand variety of them, differentiated by the representations they use. As such, full command of the CF of tables should incorporate in its teaching the different systems of representation connected to the concept, which until now has been the result of implicit teaching.

The analysis of the results using TCF allows us to distinguish symbolic representations and deliver a partial explanation of their meanings, which give shape to the invariant operators used to face statistical situations that require organizing data. Constant variables were observed for dealing with the proposed situation, which is consistent with a previous epistemological study.

CONCLUSIONS

Considering that the theory of conceptual fields is based on the principle of pragmatic elaboration of knowledge, this study lets us approach the answers to questions such as: How does the notion of tables emerge in students in the first years of school? How do students construct meaning from data? What is the thinking behind the representations that students produce? What representations do students produce when faced with a new data analysis task? What levels of conceptualization are reflected in these representations?

The categorization proposed by Vergnaud takes into account the mathematical structures underlying the situations and the psychological development of the conceptions for dealing with these situations. It is possible to infer that the mathematical concepts used include principally: classification; addition; counting (bijection and cardinality); the commutative and associative properties; existence of the identity element; comparison of numbers (meaning of number); comparison of graphical elements and/or comparison of two cardinals; definition of set by comprehension and by extension; finite set; union of sets; a set as the complement of another set; cardinal and ordinal. Considering the discrete character of frequency tables, the processes activated are those of association, differentiation, and perception of order and quantity. The theorems in use in all the productions were (1) "an equivalence relation determines a partition and a partition defines an equivalence relation," related to the processes of association and differentiation; and (2) “$\text{Card}(A \cup B) = \text{Card}(A) + \text{Card}(B) - \text{Card}(A \cap B)$,” concerning order and quantity.

The conceptualization of tables in these students shows different inherent representations, such as text lists without counting, text lists with counting, tables with icons with and without counting, tables with text with and without counting, and tables with text without individual counts but with marginal totals. This conceptualization of tables in this group of students allows us to shed light on a group of situations: referent, (for example, situations of registering data in a table, of counting in a table, of listing elements belonging to a class); a set of invariant operators, signified, “for example, partition, equivalence relations, and counting that allow for ordering data to obtain information); and a set of representation, signifier, (for example, physical segmentations, rows, columns, cells, headings, written language).

Although in the conceptualization of tables different representations were identified, in observing the productions of the two groups of students from different schools, we observe that the organization of the activity does not vary, as regularities were identified among productions of the same grade level, in the way they address and develop the same situation.

Recognizing the theorems being used that support the process of progressive conceptualization of tables provides us with knowledge to create learning situations that consider continuities and allow for confronting ruptures, to be able to educate students competent in the comprehensive use of the tabular format.

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REFERENCES

Ben-Zvi, D., & Sharett-Amir, Y. (2005). How do Primary School Students Begin to Reason about Distributions? In K. Makar (Ed.), *Reasoning about distribution: A collection of current* *research studies. Proceedings of the Fourth International Research Forum on Statistical Reasoning, Thinking, and Literacy* (SRTL-4), University of Auckland, New Zealand.

Brizuela, B. M., & Alvarado, M. (2010). First graders' work on additive problems with the use of different notational tools. Revista IRICE, 21, 37-43.

Brizuela, B., & Lara-Roth, S. (2002). Additive relations and function tables. *Journal of Mathematical Behavior, 20*(3), 309-319. doi 10.1016/S0732-3123(02)00076-7

Gabucio, F., Martí, E., Enfedaque, J., Gilabert, S., & Konstantinidou, A. (2010). Niveles de comprensión de las tablas en alumnos de primaria y secundaria. *Cultura y Educación*, *22*(2), 183-197.

Martí, E. (2009). Tables as cognitive tools in primary education. En *Representational systems and practices as learning tools* (pp. 133-148). Rotterdam: Sense Publishers.

Martí, E., García-Mila, M., Gabucio, F., & Konstantinidou, K. (2010a). The construction of a double-entry table: a study of primary and secondary school students' difficulties. *European Journal of Psychology of Education*, *26*(2), 215-234.

Martí, E., Pérez, E., & de la Cerda, C. (2010b). Alfabetización gráfica. La apropiación de las tablas como instrumentos cognitivos. *Contextos*, Años IX y X. N° 10.

Ministerio de Educación (2012). *Bases Curriculares 2012*. *Matemática Educación Básica*. Santiago de Chile: Ministerio de Educación.

Sureda, P., & Otero, M. R. (2011). Nociones fundamentales de la Teoría de los Campos Conceptuales. *Revista electrónica de Investigación en Educación en Ciencias*, *6*(1), 124-138.

Vergnaud G. (1990). La théorie des champs conceptuels. *Recherches en didactique des mathématiques*, *10*(23), 133-170. La Pensée Sauvage, Marseille.

Vergnaud, G. (2013). Pourquoi la theorie des champs conceptuels? Por qué la teoría de los campos conceptuales? *Infancia y Aprendizaje*, *36*(2), 131-161. Tandil.